Back to the Future: Dynamic Baselines in CGE Modeling

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Abstract

This paper describes the methodology used at the World Bank to develop dynamic scenarios using the Bank's global computable general equilibrium model. The dynamic CGE has been used in a number of contexts over the last 10 years including analysis of international trade policies, international migration, long-term development and structural change, the Bank's poverty forecast and more recently long-term commodity supply and prices and climate change scenarios. Though building dynamic scenarios requires significantly more exogenous inputs than the more standard comparative static analysis, it also leads to a richer set of policy questions that lie at the heart of many development issues.

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1. Introduction

The Development Prospects Group has been in the business of long-term scenario exercises for over a decade. The first application was an analysis of the East Asia crisis that now seems so long ago. Since then, the Group's workhorse computable general equilibrium model, that commonly goes by the name of Linkage, has been used to support studies on a variety of different issues of major concern to the World Bank and the broader community--including long-term prospects for poverty and achieving the Millennium Development Goals, international trade policies and their effects on growth and trade, international migration, long-term structural change with an emphasis on demographics, labor markets and income distribution, agricultural development and more recently with climate change. This latter has been associated with the development of a new CGE model that has a much greater focus on energy markets, greenhouse gas emissions and includes a simple climate module. The new model--though similar to Linkage in many ways--has been named the ENVISAGE model.

This paper presents the elements that lie behind the construction of our dynamic baselines. It mostly describes our methodology and has little in the way of results or findings. The dynamics underlying both models are almost identical and will not be distinguished in this document unless necessary for clarity. Both models are recursive dynamic so in many ways the construction of the baseline scenarios is relatively straightforward--though some of the ideas underlying our baseline may also be of interest to modelers using forward-looking methodology.

2. The core model

Underlying both models is the 2004-based Release 7 of the GTAP[1] database that divides the world economy into 113 countries/regions (of which 95 are countries) and 57 commodities. For modeling

1 More on the GTAP data can be found at www.gtap.org.

purposes the underlying database is typically aggregated to a more manageable set of regions and sectors with a focused selection of both depending on the objectives of the particular study.

Within any given time period the models are virtually comparative static models with all dynamic elements fixed before a model solution (i.e. are determined outside of the model).[2] Production is based on a standard nested CES production structure that largely replicates standard capital-labor substitution but allows for significant flexibility across sectors and inputs--including land, energy and sector-specific inputs such as agricultural chemicals. A vintage production structure is employed that allows for partial capital mobility across sectors in the short-term, or a putty-semi-putty technology. Vintage capital is associated with lower production flexibility, whereas new capital is more flexible thus aggregate flexibility depends on the share of vintage capital in total capital, with greater flexibility associated with those economies with the highest savings rate. Factor payments accrue to a single representative household in each region and the latter allocates income between savings and expenditures on goods and services. The models have a number of optional household demand structures. These include the Constant Difference in Elasticity (CDE), the linear and extended linear expenditure system (LES and ELES) and the AIDADS utility function. Household demand specification also allows for consumed goods to differ from produced goods with the two linked through a consumer transition matrix that allows for product substitution. Trade is specified using the ubiquitous Armington assumption[3]—though the model allows for homogeneous commodities as well. Government plays a relatively passive role—collecting taxes and spending on goods and services. The government's fiscal balance is fixed in any given year and the household direct tax schedule shifts to achieve the fiscal target. The latter implies that changes in indirect

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2 There are some exceptions linked to the periodicity of the model. For example, the capital stock can normally be pre-determined before the beginning of the period since it is based on the sum of the depreciated capital stock and the previous period's investment. However, when running the model where the periodicity is greater than one year, assumptions need to be made on the path of investment in the intervening years for which the model is not solved. These assumptions typically rely on the contemporaneous rate of investment as well as the lagged rate. In this case, the capital stock is not pre-determined but will be a function of contemporaneous investment and hence endogenous. This is further explained in a section below. Unlike many dynamic models, Linkage and Envisage have been coded to be independent of the periodicity of the model, this provides a degree of flexibility that can be useful for specific applications.

3 See Armington 1969.

taxes (for example tariffs or carbon taxes) are recycled in lump-sum fashion to households. Investment is savings driven and savings rates are influenced by the overall growth rate as well as demographic factors such as dependency ratios. The current account balance for each region is fixed in any given. An ex ante shift in either import demand or export supply influences the real exchange rate. Thus, for example, if a country is forced to import more food due to climate damages to its agriculture, this would normally entail a real exchange rate depreciation that increases demand for its exports in order to pay for the additional food imports.

3. Dynamics

Supply side

The supply side of the models has three key drivers at the heart of the dynamic scenarios. The first is demographics, which describes the growth of total population as well as the labor force. Like many others, our baseline uses the medium variant from the UN populations forecast, with the growth of the labor force equated to the growth of the working age population (defined as those between 15 and 65 years of age). The demographic module is completely exogenous--including the international migration module when it is activated. There is much more that could be done linking demographics with the economics of the model. Fertility rates could be linked to economic variables such as per capita income or the level of expenditures on health care and/or population policies. Similarly, mortality rates could be linked to the same variables. The split between skilled and unskilled workers is also based on exogenous assumptions but could similarly be linked to model outcomes.[4]

The second key driver is savings and investment that determines the overall level of the capital stock (along with the rate of depreciation). In our models the savings function is partially determined by demographics with savings rising as dependency ratios (both under 15 and over 65) fall. The parameters

[4] Examples of these linkages can be found in studies based on the World Bank's single country dynamic model known as MAMS--Lofgren xxxx.

of the savings function have initially been taken from estimates produced by Serven et al 20xx. The out of sample properties of these estimates sometimes lead to extreme savings rates in which case the initial estimates are adjusted to lineup with prior assumptions on long-term savings trend. In most scenarios investment is strictly savings driven. The motion equation for the capital stock takes the following standard form:

$$(1) \qquad K_t = (1-\delta)K_{t-1} + I_{t-1}$$

where K is the aggregate capital stock, δ is the rate of capital depreciation and I is the volume of investment. If the periodicity of the model is a single year then the contemporaneous capital stock is pre-determined, i.e. it is exogenous. Annex 1 describes some useful formulas that generalize equation (1) when the periodicity of the model is not one year.

The third driver is productivity. In our scenarios we differentiate productivity across broad sectors: agriculture, energy, manufacturing, and services. The exogenous component of productivity growth in agriculture is currently calibrated to 2.1 percentage points per year, consistent with recent trends (World Bank, 2008). The endogenous component of agricultural productivity growth comes from a linear damage function, which links increases in global temperature to declines in agricultural TFP and is calibrated with an average of Cline's estimates with and without carbon fertilization (Cline, 2007). Productivity in agriculture is assumed to be neutral and affects all inputs (both factor and intermediate goods) uniformly, i.e. the production possibility frontier moves out homothetically relative to all inputs. Productivity in other sectors is unaffected by climate change, and is calibrated through 2015 to match the World Bank's medium- and long-term forecast for per capita growth (at a country level). We impose an exogenous wedge between manufacturing and services productivity growth with the assumption that productivity is higher in manufacturing than in services.[5] Productivity growth in manufacturing and services is strictly

[5] The standard wedge is 2 percentage points. Hence if productivity growth is 1 percent in services it is 3 percent in manufacturing.

labor augmenting (and typically uniform across skilled and unskilled workers). The dynamic calibration between the base year and 2015 can then be described by the following equation:

(2) $$Y_t = F_t(v_t, \pi^s; \pi^a, \varphi^m)$$

where Y is aggregate GDP (for a region/country), and F is the production function that depends on the vector of factors (v), services productivity (π^s), agricultural productivity (π^a) and the wedge between manufacturing and services productivity (φ^m). In the baseline calibration Y is exogenous and the model is used to calibrate π^s that is consistent with the assumed growth rate of (per capita) GDP. In subsequent alternative scenarios (policy shocks or differences in other exogenous variables) productivity in all sectors is fixed and it is GDP that is an endogenous outcome of the simulation. Another way of looking at this is that the model is used as a dialogue (or diagnosis) tool to evaluate the plausibility of the exogenous assumptions. If the calibrated productivity parameter takes on perverse or implausible values than the exogenous assumptions on the GDP growth rate will be adjusted in order to find a plausible level of productivity at both the sectoral and aggregate level.

The assumptions on per capita growth had been largely judgmental until recently though obviously based on recent performance. To take some of the ad hoc aspect of these judgments out, a change was undertaken several years ago that relied more on letting data talk. Specifically, very long times series on per worker output (at an individual country level) are passed through a Hodrick-Presecott filter to determine potential output. The potential output series is merged with actual history to provide a smooth transition between observed output, the World Bank's medium term forecast (2-3 years ahead) and the longer-term forecast through 2015. The country based forecast is then aggregated to the GTAP level of aggregation to provide our exogenous assumptions on per capita growth through 2015.

In the climate change scenarios, after 2015, productivity growth in the U.S. is calibrated to achieve a long-term average (2004-2100) growth in real GDP per capita of 1.2 percent per year—with faster growth in the first half of the century—while productivity in other countries/regions is calibrated based on simple convergence assumptions. In our more standard scenarios say through 2025 or 2030, we fix all productivity factors (save perhaps for climate damage related factors) to that estimated in 2015 and allow GDP to be determined by the model.

Finally, energy efficiency (sometimes referred to as the autonomous energy efficiency improvement or AEEI factor) grows exogenously at a rate of 1 percent per annum in intermediate and household demand.

Demand side

Investment demand--discussed above--is largely savings driven and the expenditure share parameters are fixed. Government demand is linked to GDP growth in the baseline and is fixed to baseline levels in alternative scenarios. Similar to investment the expenditure share parameters are exogenous.

Household demand is somewhat trickier and depends to some extent on which demand system is being used.[6] The standard utility function has been the ELES. Though it has income elasticities that can differ from 1, in the long-run, it converges relatively rapidly to a Cobb-Douglas specification even when the subsistence parameters are defined on a per capita basis.[7] The convergence of all income elasticities to unit value is particular unfortunate. To overcome this, the parameters of the demand system are re-calibrated between time periods so as to line up with exogenous assumptions regarding income elasticities. For the high-income countries this has typically meant keeping them more or less constant at their base year level. For developing countries, the income elasticities are assumed to converge at some rate to the high-income country level and the parameters of the consumer demand system are calibrated between periods to line up with this convergence property. At the moment, nothing similar is done for the CDE demand system. Due to its specification, this essentially implies that the income elasticities remain around their base year level.

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[6] There is also an inconsistency in the dynamic treatment of the household demand parameters that should be addressed in the future.

[7] Income growth is typically greater than population growth.

The AIDADS demand system--initially proposed by Rimmer and Powell 1992 and implemented in the GTAP system by Hertel and co-authors (20xx,...) has attractive properties for dynamic scenarios as attractive Engel properties naturally emerge from its specification and generally do not need dynamic adjustment. At the same time it is a system that has proven to be difficult to calibrate/estimate--particularly in a manner that maintains aggregation flexibility across commodities. Work is ongoing at the World Bank to implement AIDADS that preserves the flexibility of the other demand systems. This should be eased somewhat from our consumer demand specification that relies on the transition matrix approach.

Miscellaneous

The production parameters--beyond the changes engendered by productivity--are not fixed to base year levels due to the use of the vintage specification. Before each time period the production share parameters for installed, or *old*, capital, are re-calibrated to reflect the malleable nature of capital. The details on this are explained in van der Mensbrugghe 2006. Despite the malleable nature of capital, the re-calibration of the production parameters leads to a small and slow evolution of these parameters.

In most baseline scenarios the trade parameters are held fixed at their initially calibrated level. We have tested an alternative based on work undertaken at the Dutch-based CPB (CPB 19xx) that allows for the trade parameters to adjust over time. The basic idea is that the trade parameters in each time period are re-calibrated to the value shares of the previous time period. One way to interpret this is that changes in market share have permanency, i.e. once an exporter has gained market share in a country, it is somehow easier to keep or grow this market share in the future. In the long-run, this implies that traded goods converge, albeit slowly, to a Hecksher-Ohlin world of homogeneous goods, or that the long-run trade elasticity is higher than the short-run trade elasticity. This adjustment procedure has rarely been used.

5. Conclusion

Comparative static scenarios are much easier to design and require many less assumptions, but don't necessarily allow for a richer set of scenario designs that can be derived from dynamic scenario building. The World Bank has been using dynamic scenarios for over a decade to quantify issues of particular importance to developing countries--such as the impacts from trade reform or prospects for achieving the MDGs. Other issues have emerged more recently that necessarily require a long-term perspective--how will we feed 9-10 billion people in 30 or 40 years, or what are the development impacts from global warming and policies implemented to limit greenhouse gases. These issues naturally lend themselves to long-term dynamic analysis and it is relatively easy to generate alternative scenarios that provide insights into the robustness of specific policy choices. As both hardware and software improve, our dynamic scenarios shall also improve, as have the comparative static exercises over the last decades. The ability to implement forward looking models with many regions and sectors will be a significant step forward. The climate change literature has also signaled the importance of uncertainty and systematic sensitivity analysis using either partial or full Monte Carlo techniques will eventually become an indispensable complement to long-term analysis.

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Annex: Closed form cumulative expressions

Most dynamic models are easy to describe and implement with fixed periodicity--for example annual. To allow for flexibility in defining our periodicity--within and across studies--we have implemented and coded our models to allow for variable periodicity. Most of the dynamic formulas can be solved in closed form solution--for example the capital accumulation equation--using some assumption about the inter-period growth path. The formulas below derive formulas for three different assumptions and some of their offshoots.

A generic form of an accumulation expression is:

(A-1) $$S_{t+1} = \alpha S_t + \beta D_t$$

where S is some stock with a retention rate of α and D is an impulse to the stock with potential adjustment factor β. Below, we will see how the expression changes if the impulse is contemporaneous rather than lagged.

Solving the equation over multiple periods leads to:

(A-2) $$S_{t+n} = \alpha^n S_t + \beta \sum_{j=0}^{n-1} \alpha^j D_{t+n-j-1}$$

Assuming the model is solved every n periods, the impulse in intermediate years is undetermined. There are several possible solutions that nonetheless lead to closed form solutions for (A-2) that only require knowledge of the end-point values for D.

Case a—Assume D is the average of the end-points

In this case we have:

(A-3) $$D_{t+n-j-1} = 0.5\,(D_t + D_{t+n})$$

The resulting accumulation expression is then:

(A-4) $$\begin{cases} S_{t+n} = \alpha^n S_t + \beta\left[0.5(D_t + D_{t+n})\right]\left[\dfrac{1-\alpha^n}{1-\alpha}\right] & \text{if} \quad \alpha \neq 1 \\ S_{t+n} = \alpha^n S_t + \beta\left[0.5(D_t + D_{t+n})\right] n & \text{if} \quad \alpha = 1 \end{cases}$$

Case b—Assume D grows at a constant annual growth rate between the two periods

(A-5) $D_{t+n-j-1} = (1+g)^{n-j-1} D_t$

where g is calculated from expression (A-5) with j=1. The resulting accumulation expression is then:

(A-6) $S_{t+n} = \alpha^n S_t + \beta\, D_t \frac{(1+g)^n - \alpha^n}{1+g-\alpha} = \alpha^n S_t + \beta\, D_{t+n} \frac{1-\alpha^n (1+g)^{-n}}{1+g-\alpha}$

As can be seen from expression (A-6), the accumulation expression can be calculated from either the lagged or contemporaneous impulse. Expression (A-6) is commonly used for the capital stock equation and can be expressed as:

$$K_{r+n} = (1-\delta)^n K_t + I_t \frac{(1+g)^n - (1-\delta)^n}{g+\delta}$$

When n is equal to 1, it is easy to see that this expression reverts to the standard capital accumulation equation. There could be a perverse case in which g = 1+α,[8] in such a case, expression (A-6) becomes:

$$S_{t+n} = \alpha^n S_t + n\beta\, D_t (1+g)^{n-1} = \alpha^n S_t + n\beta\, D_{t+n} (1+g)^{-1}$$

Case c—Assume D grows linearly between the two periods

(A-7) $D_{t+n-j-1} = D_t + (j-1)\Delta = D_t + (j-1)\frac{D_{t+n} - D_t}{n}$

where Δ is the annual increment to the impulse D calculated from the end-point values for D. The accumulation expression becomes more complicated:

(A-8) $\begin{cases} S_{t+n} = \alpha^n S_t + \beta\,[D_{t+n} - \Delta]\left[\frac{1-\alpha^n}{1-\alpha}\right] - \beta\,\Delta \frac{\alpha\left[1 - n\alpha^{n-1} + (n-1)\alpha^n\right]}{(1-\alpha)^2} & \text{if} \quad \alpha \neq 1 \\ S_{t+n} = \alpha^n S_t + n\beta\,[D_{t+n} - 0.5\Delta(n+1)] & \text{if} \quad \alpha = 1 \end{cases}$

If the impulse is contemporaneous with the stock equation, the generic accumulation expression becomes:

(A-9) $S_{t+n} = \alpha^n S_t + \beta \sum_{j=0}^{n-1} \alpha^j D_{t+n-j}$

8 In the case of the capital stock equation, this equates to an investment growth rate that matches exactly the depreciation rate.

For **Case a**, i.e. using the average of the end-points for the annual impulse, the accumulation expression is identical to (A-4).

For **Case b**, expressions (A-5) and (A-6) are replaced by the following:

(A-5') $$D_{t+n-j} = (1+g)^{n-j} D_t$$

(A-6') $$S_{t+n} = \alpha^n S_t + \beta\, D_t \frac{(1+g)^{n+1} - \alpha^n (1+g)}{1+g-\alpha} = \alpha^n S_t + \beta\, D_{t+n} \frac{(1+g) - \alpha^n (1+g)^{1-n}}{1+g-\alpha}$$

The second term in expression (A-6') is the same as the second term in (A-6) multiplied by the factor (1+g).

For **Case c**, expressions (A-7) and (A-8) are replaced by the following:

(A-7') $$D_{t+n-j} = D_t + j\,\Delta = D_t + j\,\frac{D_{t+n} - D_t}{n}$$

(A-8') $$\begin{cases} S_{t+n} = \alpha^n S_t + \beta\, D_{t+n}\left[\dfrac{1-\alpha^n}{1-\alpha}\right] - \beta\,\Delta\dfrac{\alpha\left[1 - n\alpha^{n-1} + (n-1)\alpha^n\right]}{(1-\alpha)^2} & \text{if} \quad \alpha \neq 1 \\ S_{t+n} = \alpha^n S_t + n\beta\left[D_{t+n} - 0.5\Delta(n-1)\right] & \text{if} \quad \alpha = 1 \end{cases}$$

In the first expression of (A-8') the increment is no longer subtracted from the stock term, D, in the second term of the expression. In the second expression, the factor (n+1) is replaced by the factor (n-1).

The cumulative expressions rely on two well known expression from finite series.[9] Equation (A-6) with the assumption of constant annual compound growth relies on the following:

(A-10) $$\sum_{i=0}^{n-1} b a^i = b\frac{1-a^n}{1-a}$$

The formula simplifies to ($n.b$) if a equals 1. Note that if the absolute value of a is less than 1, at the limit, as n approaches infinity, the sum converges to b/(1-a).

The other expression, with linear growth, is somewhat more complex and relies on the following finite series:

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9 See *Handbook of Mathematical, Scientific, and Engineering Formulas, Tables, Functions, Graphs, Transforms* (1980), p. 304.

(A-2) $$\sum_{i=0}^{n-1}(b+i.d)\,a^i = \frac{b\,(1-a^n)}{1-a} + \frac{ad\left[1-na^{n-1}+(n-1)\,a^n\right]}{(1-a)^2}$$

If *a* equals 1, the formula simplifies to:

(A-2') $$\sum_{i=0}^{n-1}(b+i.d) = n.b + d\,\frac{n(n+1)}{2}$$

If the absolute value of *a* is less than 1, the expression converges towards the following as *n* approaches infinity:

$$\lim_{n\to\infty}\sum_{i=0}^{n-1}(b+i.d)\,a^i = \frac{b}{1-a} + \frac{ad}{(1-a)^2}$$