An Emissions Trading Scheme with Auctioning
PRELIMINARY VERSION

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Abstract

This paper models an emissions trading scheme with auctioning in which risk averse polluters and non-polluters respond idiosyncratically to an economy-wide shock. A polluter's willingness to pay for permits increases in her risk aversion and in the shock volatility when aggregate and individual sensitivities to this shock satisfy certain conditions. In addition, there is a region of low sensitivity and high emissions rates where the polluter's willingness to pay for permits decreases in the expected secondary market price. Numerical simulations show that, *ceteris paribus*, dirtier firms overvalue the permits in the auction and become net sellers. When all polluters emit at the same rate, a wealth transfer takes place from the most sensitive to the least sensitive polluters, through the secondary market. The risk aversion exacerbates polluters' sensitivity to the shock when forming their valuations: the more risk averse, the lower the valuations for the same sensitivity. The model predicts that, in a world with polluters only and homogeneous responses to the shock, the secondary market trading volume decreases in the uncertainty faced by the economy due to the tight competition at the initial allocation stage.

Keywords: emissions trading scheme, uniform price auction, secondary market
JEL Clasification: D21, D44, D45, G12, Q55

1 Introduction

Designing schemes to mitigate the effect of climate change has received considerable attention in the past twenty years both from politicians and from economists. Departing from the command and control instruments, which were popular forty years ago, the market-based instruments are becoming more and more used as policy tools for reducing the emissions of greenhouse gases (GHG). One approach to market-based instruments are the cap-and-trade schemes, also called emissions trading schemes. They have been designed throughout the world especially after the ratification of the Kyoto Protocol in 1997. Cap-and-trade programs are favored by policy makers because they give flexibility to firms in complying with the environmental goals by the means of free trade. At the heart of such a trading scheme is the Coase Theorem, which assures that under negligible transaction costs, markets can achieve the

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efficient outcomes, regardless of who holds the rights to emit initially. Hence, the compliance with the environmental regulations will be achieved in a cost effective way.

The major motivation for designing a trading scheme as a policy instrument for mitigating the emissions of GHG is the reduction of the social costs of complying with certain regulatory requirements. Moreover, the regulator needs minimum of information for designing it. In particular, the regulator does not need to know detailed information about the compliance costs or the emissions needs of each plant or installation covered by the scheme. Such considerations have been recognized by the European Union when adopting a cap-and-trade system as its main policy pillar to combat climate change (European Commission (2003)).

Nowadays, the European Union Emission Trading Scheme (EU ETS)[1] for carbon dioxide is the biggest cap-and-trade program in the world, though the beginning of such a scheme was the SO_2 trading scheme in the US under the US Acid Rain Program in 1990. The European Union commenced its ETS in 2005, with the purpose of helping the Member States to comply with the Kyoto Protocol, which became binding in 2008. The scheme has been running in so-called trading phases. The first phase (2005-2007) was the pilot phase of the scheme. The second phase (2008-2012) coincides with the first compliance period of the Kyoto Protocol, during which the Member States are obliged to achieve their emissions targets. One important common element of these two phases is the discretionary nature entailed by the free allocation method[2]. Thus, the national goverments, based on the national allocation plans approved by the Commission, have been distributing the allowances to the individual firms free of charge. This method of initial allocation, called grandfathering, though based on the rule of past emissions, leaves room for firms to exercise their lobbing power over their national governments.

Phase 3 of the scheme will start in 2013 and last until 2020. Apart from a broader scope to include new sectors[3] and a tighter European global cap[4], this phase comes with significant changes regarding its design. The most important design element is, perhaps, the method of initial allocation which will progressively evolve to full auctioning by 2027. One major difference between free allocation and auctioning in the EU ETS is that through free allocation the allowances end-up initially in the hands of the regulated firms only and they would fall in the hands of other individuals, institutions or non-governmental agencies only through the secondary market. In the case of auctioning, however, the non-regulated firms may purchase permits rights at the initial allocation stage, as anyone can bid in the auction conducted by the regulator. Typically, the non-regulated firms participating in the markets for permits are authorized individuals, investment banks or credit institutions who seek to

[1]Established through the Directive 2003/87/EC of the European Parliament and of the Council.

[2]At least 95% and at least 90% of the allowances have been distrubuted for free in the first, respectively second phase of the scheme.

[3]The aviation sector will be included from 2013.

[4]The cap will decrease each year linearly by 1.74% relative to the period 2008-2012.

make profits by engaging in speculating activity on the emissions markets.[5]

This paper is motivated by the change of institutional design of the EU ETS coming with the third Phase. In particular, the design element which is exploited here is the auction as an initial allocation method for the distribution of permits. Although commonly defended as transparent, efficient and void of lobbing power, auctioning, as a method for allocating the permits, still raises several questions, especially if it takes place in an uncertain environment. Although it is reasonable to expect that an efficient auction format will not trigger any wealth redistribution, this only holds true in an environment governed by certainty. In an uncertain environment, wealth redistribution will always arise, if free trade is allowed. Thus, questions such as: who are the predicted winners of the auction; who benefits from wealth transfers in this kind of setting; what is the role of the risk aversion or how do the speculators influence the markets for permits, deserve attention from the policy perspective.

In order to address these questions, I build a model of an emissions trading scheme with heterogeneous firms, where the initial allocation of permits is via auctioning. The agents of the model are risk averse firms and the risk aversion is captured by a constant absolute risk aversion (CARA) utility function of profit. The risk averse behavior of firms, when taking decisions under uncertainty, is advocated by several papers. For example, Sandmo (1971) studies the supply decision of the competitive firm under uncertain demand, arguing that the simple expected profit maximization approach is inadequate. Next, Leland (1972) favors the aversion towards risk approach on the basis that firms are controlled by investors, who are risk averse, or managers who prefer security. Consequently, to the extent that the control over firms is held by risk averse agents, the risk aversion assumption for firms is a fairly plausible one.[6]

The model of this paper is a one-period complete information model and consists of four stages. In the first stage, all firms participate in an auction for the distribution of a fixed supply of permits, in the second stage they trade these permits in a secondary market, in the third stage they take abatement decisions, and lastly the production of the final output takes place. There are two types of firms in this model: polluters, who need to hold a permit for each unit of pollution released, and speculators,[7] who engage in the markets for permits with the purpose of gaining profits from the spread between the auction clearing price and the secondary market price. The presence of the speculators in this model is also in line with the regulations of the third phase of the EU ETS, by which investment firms, credit institutions as well as other authorized persons acting on their own account or on behalf of their clients

[5]It is also common that environmental organizations would purchase emissions permits and retire them, but their power is negligible.

[6]It is also conceivable that a firm may behave in a risk neutral manner in so far as the shareholders, who are risk neutral due to the diversification of their investments, can control the manager and the employees. However, exactly due to the diversification of their portfolios it is implausible to believe that investors can indeed control the firm.

[7]The idea of speculators acting in the permits market is exploited in Colla et al. (2005) in a context of free allocation with two rounds of trading.

are allowed to apply for admission to bid in the auction.[8] In the category of speculators one can also include those firms which, in reality, might be polluters but they are not required to obey the regulations as they are too small polluters or because they belong to a sector which is not under the scheme.

The auction is modeled as a uniform price sealed-bid auction[9] of a perfectly divisible asset, whereby the bidders submit demand schedules and receive permits according to their schedules at the price where the aggregated demand equates the total supply. In this model the supply of permits is fixed and exogenous, as chosen by the regulator based on biological and geological concerns.[10] In the framework of this paper the regulator does not play any strategic role in the sense that she does not make any decision.

The main contribution of this paper is to integrate the auction with the secondary market for permits, explicitly accounting for the production decisions. In addition, I derive and interpret the valuations for permits by firms when forming their bidding strategies.

The main results of the paper can be summarized as follows. First, the presence of the speculators in the permits market does not affect the equilibrium price of the secondary market, assuming that this market is competitive. This price only depends on the number and the fundamentals of the polluters: their costs, their emissions rates and the realized output prices. In addition, it depends on the total number of permits issued by the regulator. Second, with homogeneous sensitivities to the shock, higher uncertainty in the economy induces lower trading volume in the secondary market and lower auction clearing price. Third, in an environment with homogeneous responses to the common shock, the biggest polluters overvalue the licenses in the auction and they become net sellers in the secondary market, regardless of the presence of the speculators. Fourth, with homogeneous emissions rates, an increasing aversion towards risk on the side of the polluters increases the importance of their sensitivity to the economy-wide shock when they form their valuations for the auction: the decreasing valuation as function of firms' sensitivity becomes steeper as the risk aversion increases. Consequently, the more averse the polluters, the larger room for the speculators to win permits in the auction.

[8] For details, see *Article 18* in the COMMISSION REGULATION (EU) No1031/2010 of 12November 2010.

[9] According to *Article 5* of the European Commission's Auctioning Regulations, this is the market institution to be adopted for the initial allocation of permits in the third phase of the EU ETS: "Auctions shall be carried out through an auction format whereby bidders shall submit their bids during one given bidding window without seeing bids submitted by other bidders. Each successful bidder shall pay the same auction clearing price as referred to in Article7 for each allowance regardless of the price bid." (COMMISSION REGULATION (EU) No1031/2010 of 12November 2010).

[10] "The ultimate objective of the United Nations Framework Convention on Climate Change (UNFCCC), which was approved on behalf of the European Community by Council Decision 94/69/EC(5) OJ L 33, 7.2.1994, p. 11., is to stabilize greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. In order to meet that objective, the overall global annual mean surface temperature increase should not exceed 2 degrees Celsius above pre-industrial levels."(DIRECTIVE 2009/29/EC OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL) For example, the cap for 2013 is just slightly bellow 2.04 billion permits and the cap will decrease until 2020 by 37,435,387 permits per year.

From the policy perspective, it appears that allowing for speculators in the auction does not provide incentive for emissions reductions; it only has wealth transfer implications, thus hurting the polluters. The abatement level only depends on the secondary market price and the abatement cost. In addition, in an environment with low uncertainty, the auction leads to high inefficiency in the sense that there is a need for significant transfers in the secondary market in order to achieve the optimal level of final allocations. This situation arises because many firms end up with zero permits in the auction due to the tight competition at this stage.

The paper is organized as follows. Section 2 is devoted to the discussion of the relevant literature. Section 3 describes the model and the agents present in the permits markets. Section 4 solves the model by backward induction. Section 5 discusses the results and analyzes the equilibrium. In Section 6 I analyze the main results numerically and in Section 7 I consider two variants of the model. Finally, Section 8 concludes summarizing the results and outlining the directions for further research.

2 Related Literature

This paper builds on the design features of Phase 3 of the EU ETS and aims at anticipating the implications of the changes in the scheme after 2013. In particular I build a model in which the initial allocation is via full auctioning followed by trade on the secondary market. The model links the output market, governed by uncertainties, with the markets for permits. For compliance with the environmental regulations, polluting firms can engage in investment to reduce the emissions, apart from the trade on the secondary market.

To the best of my knowledge, this is the first paper which explicitly models the auction as initial allocation method together with the secondary market and at the same time integrates the product market. However, there are other papers which model the product market together with the permits market. For instance, Subramanian et al. (2008) construct a model with auctioning, but they omit the secondary market. Their model, under complete information and no uncertainty, has two scenarios: no interaction on the output market and Cournot competition in the output market. In both scenarios they find that the abatement decisions in cleaner industries are more sensitive to the change in the total number of permits than in the dirtier ones. By contrast, in my model I find that the level of abatement is determined by the equality between the permits' price and the marginal cost of abatement. This result is due to the existence of the secondary market in my model. Moreover, in my model, the abatement level by each firm is non-monotonic in the level of dirtiness and decreases with the total number of available permits. By contrast, Subramanian et al. (2008) find that the optimal level of emissions reductions is in fact increasing in the fixed supply of permits.

Another related paper is Colla et al. (2005). They build a model with two types of risk averse traders - firms and speculators - of total measure equal to unity. In their model the

initial allocation is free and thus, only the polluting firms are endowed with permits in the initial stage. The model has two rounds of trade separated by the realization of a common productivity shock, which affects all the polluting firms identically. Agents are homogeneous and therefore, the equilibrium is symmetric. The authors show that the price of the first round of trade increases in the number of speculators if and only if they are less risk averse than the polluting firms. However, the price of the second trading round is indirectly affected by the presence of the speculators through the price of the first trading round. In turn, the first trading round is affected by the speculators through their number and risk tolerance coefficient. In Colla et al. (2005) the spread between the second trading round and the first trading round is positive creating incentive for speculation.

Although my paper has similarities with the the above-mentioned ones, it differs in a few important respects. First, unlike Subramanian et al. (2008), the heterogeneous firms make bidding decision under uncertainty. In addition, in their two asymmetric firms auction game, they impose an ad-hoc linear equilibrium with the intercept equal to the total number of permits, while I derive the unique linear equilibrium for an auction game with N firms. Second, unlike in Colla et al. (2005), in this paper the initial allocation is via auctioning followed by one round of trade. Speculators are present in both markets. In addition, I explicitly model the link between the production and abatement decisions and the markets for permits through the rate of emissions per unit of output. In this paper I explicitly account for firms' idiosyncratic sensitivity to a global shock. Thus, agents with both negative and positive responses to the shock are present in the model. Finally, I allow the speculator to have non-polluting production activity, subject to the same global shock experienced by the polluting firms. This affects their valuation for permits in the auctioning stage.

In terms of methodology, this paper borrows from the finance literature on market microstructure along the lines of Kyle (1989) and Vargas (2003) and it can also be integrated into the auction of divisible goods literature *à la* Wang & Zender (2002). However, it does depart from this literature in that symmetric equilibrium is not feasible here. Therefore, papers on the supply function equilibrium in the electricity markets such as Green (1999), Rudkevich (1999), Rudkevich (2005) and Baldick et al. (2000) provide the basis for the methodological framework in constructing the asymmetric auction equilibrium.

3 The Model

I assume an economy with $N > 2$ polluting firms and $M > 2$ speculators who are engaged in a sequence of decisions, as illustrated in Figure 1.

Initially, the regulator announces the emissions cap, $\overline{E}$, which is exogenous to the model. All the information in the model is common knowledge. In the first stage of the game, i.e. the auction, bidder j submits her demand schedules, $D_j(\nu)$, as the number of permits

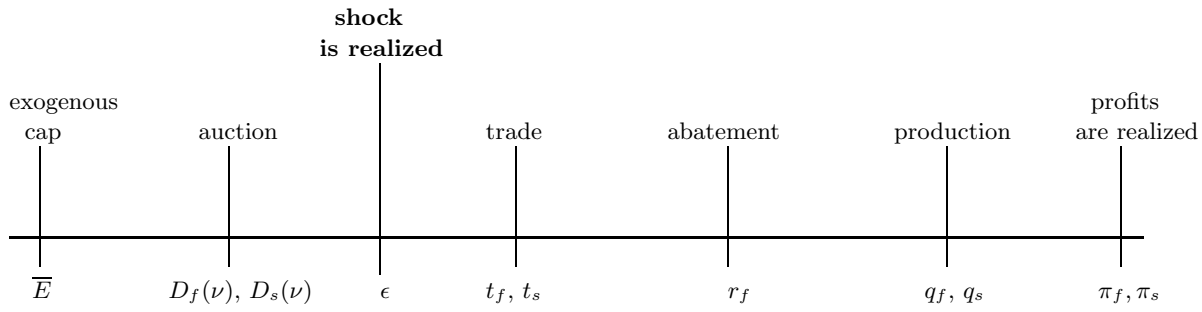

Figure 1: The sequence of decisions

she would like to purchase at any price ν. The regulator collects all the individual demands, aggregates them and computes the clearing price ν^* as the point where the aggregated demand equates the fixed supply $\overline{E}$. At this stage each bidder receives the initial allocation of permits according to her bidding schedule and the auction clearing price. However, when firms bid for their initial endowment of permits, they face uncertainty regarding their output demand. The uncertainty is incorporated into a common shock ϵ, which affects the whole economy. For tractability, I assume that ϵ is normally distributed with mean zero and variance σ^2.[11] Nevertheless, each firm j responds to the aggregate shock in an idiosyncratic manner, as it will be clear shorty. This uncertainty is resolved after the initial distribution of permits is completed and before the secondary market takes place.

Firms can buy and sell permits in the secondary market. Hence, the secondary market has the role of correcting the misallocations from the first stage, when the real needs for permits were unknown. I denote the price of the secondary market by λ and I model this market as a Walrasian one, which clears at the price λ^* such that the excess demand is zero. Firms are closing their positions on permits through the secondary market such that in the end all the polluters comply with the environmental regulations.[12]

In the third stage the abatement investment level is decided by the polluting firms, and lastly the production decisions are made. Note that I model the abatement decision after the final allocation of permits is known and before the production decisions are taken. In this case the abatement investment cycle is relatively fast and firms have the possibility to adapt their investment after the permits markets' outcomes are realized.

In essence, the order of the last three stages of the game is not relevant because of the

[11]The variance will be chosen such that negative prices are ruled out.

[12]Penalties for non-compliance are excluded from the model, as a non-compliant firm had to pay a fine of 40 euro/tonne of CO_2 in 2005-2007 period and 100 euro/tonne of CO_2 in 2007-2012 phase. In addition to the fine, firms have to purchase and surrender the amount of permits they are short of in the following year of the scheme. For the next trading phase the fine will be indexed with the inflation rate. Note that these conditions do not give the option of non-compliance; therefore, this case is excluded from the analyses.

additive structure of the model. However, it is important that they all come after the realization of the shock. One variant of the timing of this game is when abatement decisions take place under uncertainty, before the auction. This would reflect the long term abatement decisions at a lower frequency than the auction and it is, in fact, closer to the reality of the EU ETS because it is expected that several auctions will be conducted during one calendar year. However, this approach complicates the analytical tractability of the model and it is left for further research.

The two types of firms active in the permits markets, together with their actions are described in more detail below.

3.1 Polluting Firms

The polluting firms are the regulated firms which are obliged to hold a number of permits equal to the emissions discharged through their production process. Each polluter $f = 1, \ldots, N$ is characterized by a certain level of emissions per unit of output $k_f > 0$, which denotes the emissions rate. Thus, the polluting firms are heterogeneous with respect to their level of dirtiness. In order to keep the focus on the permits markets, I assume that all firms are price takers on their respective product markets[13] and the output price is given by

$$p_f = \gamma_f + \alpha_f \epsilon,$$

where $\gamma_f > 0$ is a constant and α_f is firm's idiosyncratic sensitivity to the overall shock.[14] Given the assumed zero mean of the shock, γ_f is firm's f expected output price. Heterogeneity in γ_f captures the case of industry-representative firms, i.e. when the polluters belong to different industries, thus facing different demands in expectations. If γ_f's are the same across firms, then we have the case of one competitive industry.

The sensitivity parameter α_f can be positive or negative, determining the direction in which firm's product price moves relative to the common shock. Hence, under the assumed probability distribution of the aggregate shock, firm f believes that her sales price is a normally distributed random variable with mean γ_f and variance $\alpha_f^2\sigma^2$. In order to assure that the probability of p_f being negative is negligible, I will maintain the following assumption:

Assumption 3.1 *For each firm $f = 1, \ldots, N$, the variance $\alpha_f^2\sigma^2$ is low and γ_f is large.*

Note that all output prices are correlated due to the common shock.

In order to keep the analysis simple, I assume that all firms have the same increasing and convex production cost function, $c(q_f) = bq_f^2$, where $b > 0$ and q_f is the output of firm f.

[13] Note that the firms are not competitors on their output markets but they interact through the markets for emissions permits.

[14] This approach to idiosyncratic shocks is also used in Fevrier & Linnemer (2004), relating to marginal costs.

This assumption, although somewhat unrealistic, does not harm the analyses, as the focus of the paper is on the permits markets, rather than the output market.

In addition to the emission permits purchased in the auction or in the secondary market, the polluters can also engage in reducing emissions, thus relaxing their emissions constraint and increasing their production capacity. However, the reduction of emissions, which is generally called abatement, is costly to the firm. I assume that the firms are homogeneous with respect to the abatement cost through the parameter θ and that the cost is quadratic, as given by the function θr_f^2, where r_f denotes the emissions reductions by firm f.

The interpretation of the abatement cost in the context of this model is, for instance, investment in a filter which reduces the emissions at the end of the product line, investment in CO_2 capture and storage facilities, or investment in green projects generating certificates which can be used against the discharged emissions.[15] All these types of investment have the effect of reducing the total business-as-usual emissions, $k_f q_f$, by the amount r_f.

In this model permits are not bankable. This implies that they bear no value at the end of the trading period. Therefore, the net supply (demand) in the secondary market by firm f is the absolute value of $t_f = k_f q_f - r_f - D_f$, i.e. the total amount of emissions discharged less her level of emissions reductions and the amount of permits purchased in the auction. Thus, a positive t_f represents a net need for permits, so f is a net buyer, while a negative t_f is a net surplus, and the firm is a net seller. Obviously, if $t_f = 0$ the firm does not participate in the secondary market. Therefore, any expenditure on permits on the secondary market by one firm represents revenue for another firm. Note that the only tradable instrument in this model is the emissions permit issued by the regulator.

I this point I can formulate the profit function of firm $f = 1, \ldots, N$:

$$\Pi_f = p_f q_f - b q_f^2 - \theta r_f^2 - \lambda(k_f q_f - D_f - r_f) - \nu D_f, \tag{1}$$

Hence, firms derive profit from output sales less the cost of production and the cost of abatement, minus (plus) expenses (revenue) from purchasing (selling) permit in the secondary market and minus expenses for purchasing permits in the auction. As already mentioned, firms maximize a CARA utility function of profits. Let $\rho_F > 0$ be the common constant absolute risk aversion coefficient for all the polluting firms. Finally, each polluting firm $f = 1, \ldots, N$ maximizes the following utility function:

$$U_f(\Pi_f) = -\exp(-\rho_F \Pi_f). \tag{2}$$

[15] Since these types of investments have the same cost regardless of the industry, the assumption of homogeneity about the cost parameter θ is well justified.

3.2 Speculators

Unlike the polluters, the speculators are not required to hold permits for their production. Hence, in this model, technically the speculators are firms with $k_s = 0$, where s is an index for speculators. Consequently, they do not engage in abatement activity, as that is not a source of profit for these firms. Recall that the emissions reductions are not tradable in this model.

In this model the speculator can derive profit from a non-polluting production activity apart from their speculative activity on permits' markets. As in the case of the polluting firms, I assume that they do not compete on their output markets, but they face the same uncertainty with regard to their product demand before the auctioning stage. Let

$$p_s = \gamma_s + \alpha_s \epsilon$$

be their output price, where the parameters γ_s and α_s have the same interpretation as in the case of the polluters. For non-negative p_s I make the same assumption as in the case of the polluters:

Assumption 3.2 *For each firm $s = 1, \ldots, M$, the variance $\alpha_s^2 \sigma^2$ is low and γ_s is large.*

A special case of the speculating firms is the case of pure speculators, who do not engage in any production, i.e. $\gamma_s = \alpha_s = 0$ and $q_s = 0$.

Hence, the profit of any firm $s = 1, \ldots, M$ is given by:

$$\Pi_s = p_s q_s - b q_s^2 + \lambda D_s - \nu D_s. \tag{3}$$

Unlike Colla et al. (2005), I allow for the speculators to engage in some production activity. As their production is not a source of pollution, it is independent of the markets for permits. Thus, one may regard their net revenue from sales, $p_s q_s - b q_s^2$, as an uncertain income realized after the auction. As it shall be shown further, the presence of the production activity has an affect on the valuation for permits, in the auctioning stage, by this type of firms.

Finally, each speculator maximizes a CARA utility function of profits,

$$U_s(\Pi_s) = -\exp(-\rho_S \Pi_s), \forall s = 1, \ldots, M, \tag{4}$$

where $\rho_S > 0$ is the coefficient of risk aversion common to all speculators.

The behavior of the speculators in this model can be related to the auction with re-sale literature, where the re-sale price of the auctioned asset is uncertain. Therefore, they can be regarded as bidders for an asset, which has a random post-auction value λ. In this respect, see, for example, Kyle (1989), Vargas (2003) and Keloharju et al. (2005).

4 Solving the Model

The problem for each firm is to find the optimal production decision and the optimal bidding in the auctioning stage. In addition, the polluting firms have to solve for the optimal abatement decisions. I assume that the permits are perfectly divisible and therefore, the utility functions are differentiable with respect to each decision variable. Hence, in this paper I only consider continuously differentiable strategies. The solution of the game is found by the backward induction method, starting from the production stage. In each stage agents decide optimally, anticipating the outcomes of the subsequent stages.

4.1 Production

Given the anticipated initial allocations of permits and the decisions regarding the abatement volumes, firms solve for the optimal amount of output to be produced. At this stage they do not face any uncertainty. Therefore, maximizing (2) or (4) is equivalent with maximizing Π_j with respect to q_j, where $j \in \{f, s\}$.[16]

The first order condition gives the interior solution

$$q_f = \frac{1}{2b}(p_f - \lambda k_f), \forall f = 1, \ldots, N, \tag{5}$$

for polluting firms and

$$q_s = \frac{1}{2b}p_s, \forall s = 1, \ldots, M, \tag{6}$$

for speculators.

Note that in order for a polluting firm f to produce a non-negative amount of final output, $p_f - \lambda k_f \geq 0$ should hold.[17] If this inequality does not hold, the firm is driven out of the market. This is, obviously, an interesting case, but it is out of the scope of this paper. Therefore, I postpone it for further research.

It appears as no surprise that the supply of the final output does not depend on the initial distribution of permits. Obviously, this is due to the fact that post-auction costless trade is incorporated into the model, hence the efficient outcome is achieved. The optimal production does not depend on the auction clearing price, either, reflecting the fact that this is a sunk cost for the rational firm. Eventually, the output supply of a polluting firm only depends on the output price, the marginal cost of production, the level of dirtiness and the price of the permits in the secondary market. However, as it will become clear in Section 4.3, the output level also depends on the abatement cost, the fixed supply of permits in the market and the number of polluters, through the secondary market price.

[16] Henceforth, let the index $j \in \{f, s\}$ indicate any firm, regardless of her type (polluter or speculator).

[17] Following Vives (1984), for this, it suffices to choose firm's parameters such that $\frac{1}{2b}(\gamma_f - k_f E[\lambda])$ is large enough, where $E[\lambda]$ is the expected price of the secondary market, and the variance of the shock is low enough. Note that this does not contradict Assumptions 3.1 and 3.2 about firms' individual prices.

Thus, the supply of final output is increasing in its price and decreasing in the production cost. Note the difference between equations (5) and (6), reflecting the fact that the polluting firms are constrained by the environmental regulations. For them the amount of final product is decreasing in the price of permits and in the level of dirtiness. However, it must be pointed out that the appearance of the secondary market price, λ, in equation (5) captures the opportunity cost of re-selling the permits on the secondary market. Note that despite the fact that firms have abatement possibility, their production decisions do not coincide with the business-as-usual production.

4.2 Abatement

At this stage the uncertainty is already resolved. Therefore, maximizing the CARA utility function is equivalent to maximizing the profit. Since speculators do not pollute and the amounts of emissions reductions are not tradable, they do not engage in this activity. Hence, at this stage only the polluting firms decide their abatement levels through the variable r_f, given the initial allocations and the outcome of the trade in the secondary market. Anticipating the production level given by (5), firm $f = 1, \ldots, N$ has to solve the following optimization problem:

$$\begin{aligned} \max_{r_f} \Pi_f =& p_f q_f - b q_f^2 - \theta r_f^2 - \lambda(k_f q_f - D_f - r_f) - \nu D_f \\ s.t.\ \ & q_f = \frac{1}{2b}(p_f - \lambda k_f) \geq 0 \\ & 0 \leq r_f \leq k_f q_f \end{aligned} \tag{7}$$

Preserving the assumption about the positiveness of the total production by any firm f, the Kuhn-Tucker conditions provide the following optimal level of abatement for firm f:

$$r_f = \begin{cases} \dfrac{\lambda}{2\theta}, & \text{if} \quad 2\theta(k_f q_f) \geq \lambda \\ k_f q_f, & \text{if} \quad 2\theta(k_f q_f) < \lambda \end{cases} \tag{8}$$

Hence, a firm for which the marginal abatement cost at the optimal production level is above the secondary market price, i.e. $2\theta(k_f q_f) \geq \lambda$, will abate only up to the point where the marginal abatement cost equals the secondary market price, that is $r_f = \dfrac{\lambda}{2\theta}$. This is the supply of emissions reduction, which is increasing in the value of the permits on the secondary market, λ, and decreasing in the cost of abatement, θ. Hence, these are the only parameters the firm takes into account when deciding her abatement level. Note that a positive price in the secondary market assures positive levels of abatement.

Conversely, a firm for which the marginal abatement cost is below the secondary market price will choose to abate the whole amount of emissions necessary for supplying the optimal

level of production, that is $r_f = k_f q_f$.

For convenience, in what follows I assume that all firms operate in the region where the marginal abatement cost is above the secondary market price. If there are firms with the marginal abatement cost below the secondary market price, they should not be motivated to participate in the auction. However, if they do participate, then they do it for pure speculative reasons. Thus, I consider this group of firms as being assimilated into the group of speculators.

4.3 Secondary Market

Given the anticipated production and abatement levels, let us now solve for the equilibrium price of the secondary market. The secondary market takes place after the initial endowments of permits are distributed and after the shock to the economy is realized. Thus, the trading volume for each firm $f = 1, \ldots, N$ is given by the absolute value of

$$t_f(\lambda) = \frac{1}{2b}\left(k_f p_f - \frac{b + \theta k_f^2}{\theta}\lambda\right) - D_f, \ \forall\lambda. \tag{9}$$

Thus, positive amount in (9) represents demand of permits and negative amount represents supply of permits. The speculators, on the other hand, have to close their positions. Since at the end of the game any permit held has zero value, they will necessarily sell any amount they had earned in the auctioning stage. Consequently, for each $s = 1, \ldots, M$,

$$t_s(\lambda) = -D_s, \ \forall\lambda \tag{10}$$

represents supply of permits.

Hence, the net position of a polluting firm f on the secondary market depends on the sign of the function $t_f(\lambda)$ in (9), evaluated at the secondary market equilibrium price. Note that for a net seller, the more she earns in the auction, the more she supplies in the secondary market and the relationship is one-to-one. Conversely, for a net buyer, the more she earns in the auction, the less she demands in the secondary market. Again, the relationship is one-to-one.

The market clearing condition imposes the excess demand of permits to be zero. Thus, the equilibrium price on the secondary market is given by

$$\lambda^* = \left\{\lambda \mid \sum_{f=1}^{N}\left(\frac{1}{2b}\left(k_f p_f - \frac{b + \theta k_f^2}{\theta}\lambda\right) - D_f\right) - \sum_{s=1}^{M} D_s = 0, \ \lambda \in R_+\right\}.$$

I assume that the entire supply of permits is distributed in the auction. In addition, players anticipate that the whole supply of permits, $\overline{E}$ will be distributed in the auction at the auction

clearing price such that $\sum_{i=1}^{N+M} D_i = \overline{E}$. Hence, the secondary market equilibrium price is:

$$\lambda^* = \frac{\sum_{f=1}^{N} k_f p_f - 2b\overline{E}}{\frac{Nb}{\theta} + \sum_{f=1}^{N} k_f^2}. \tag{11}$$

It is worth noting that the secondary market is essentially modeled as a pure exchange economy and thus its equilibrium is the Walrasian equilibrium. The participants to this type of market institution do not make strategic decisions.

The assumed support of the global shock ϵ ensures that this price is positive. It is obvious from equation (11) that λ^* is increasing in the abatement cost parameter, θ. Hence, the more expensive it is to reduce emissions, the more expensive the permits become. Also, as the number of permits issued by the regulator increases, the permits' price in the secondary market drops. Consequently, as equation (8) shows, the amount of abatement is decreasing in the number of available permits. This result is in contrast with both Subramanian et al. (2008) and Colla et al. (2005), who find that the abatement investment is increasing in the number of permits issued by the regulator. It is also interesting to observe that only the output prices of the polluting firms affect the price of permits. As the final product of a speculator does not require usage of permits, its price will not influence the secondary market price. Hence, the presence of the speculators in the market has no effect on the secondary market price. However, as it will be showed later, their participation in the auction does affect the clearing price.

Given all the information available, before the shock is realized, all market participants expect that the secondary market equilibrium price is:

$$E[\lambda^*] = \frac{\sum_{f=1}^{N} k_f \gamma_f - 2b\overline{E}}{\frac{Nb}{\theta} + \sum_{f=1}^{N} k_f^2}, \tag{12}$$

where the expectation is taken over the random variable ϵ. Substituting (12) in (11) the secondary market price can be written as the sum of two components:

$$\lambda^* = E[\lambda^*] + \Omega\epsilon, \tag{13}$$

where $\Omega = \frac{\sum_{f=1}^{N} \alpha_f k_f}{\frac{Nb}{\theta} + \sum_{f=1}^{N} k_f^2}$ is the sensitivity of the secondary market price to the economy-wide shock, ϵ. Hence, like the final output prices, the secondary market price has a deterministic component, given by its expected value $E[\lambda^*]$, and a random component given by $\Omega\epsilon$.

Note that Ω can be both positive and negative, as α_f's may be both positive and negative numbers. In essence, Ω is a weighted average of firms' responsiveness to the common shock experienced by the economy and its sign depends on the sign and the magnitude of the sensitivity to the shock of the most polluting firms. Thus, I interpret a positive Ω as the case

when the biggest polluters have a positive sensitivity to the common shock. Hence, based on equation (13) we have the following proposition, which does not require a formal proof:

Proposition 4.1 *The secondary market equilibrium price increases in the economy-wide shock if and only if the product demand of the biggest polluters follows the direction of the shock.*

Thus, for a positive shock to the economy, most of the firms would like to produce more as their demands are boosted. Therefore, the demand for permits also increases, resulting in a higher secondary market price. If however, the positive shock is actually bad news for the firms (in the case of a negative Ω), firms are not willing to buy any more permits, but rather they are willing to sell the available permits, thus increasing the supply in the secondary market. Consequently, the price is depressed.

As the secondary market price has been established, one assumption is due here. In order for the expected price to be positive the following assumption is needed:

Assumption 4.2 $\sum_{f=1}^{N} k_f \gamma_f > 2b\overline{E}$.

One way to interpret the above inequality is by dividing both sides by the positive amount $\sum_{f=1}^{N} k_f$, which is the sum of all emissions rates in the economy. Further, let us assume that there is no abatement activity in this economy and no speculators, but there is possibility for trade. Therefore, the total emissions in the economy must equal the total available permits, i.e. $\sum_{f=1}^{N} k_f q_f = \overline{E}$. Thus, the above inequality becomes

$$\frac{\sum_{f=1}^{N} k_f \gamma_f}{\sum_{f=1}^{N} k_f} > 2b \frac{\sum_{f=1}^{N} k_f q_f}{\sum_{f=1}^{N} k_f}, \tag{14}$$

which is an economy-wide condition. The left hand side of (14) represents the expected weighted mean of the output price, while the right hand side is the marginal cost of the weighted average output where the weights are the emissions rates. Therefore, condition (14) says that the expected output price should exceed its marginal cost. The reason why we have inequality is that the right hand side of this condition does not incorporate the cost of buying the emissions permits from the regulator.

4.4 Auction

At this stage the regulator conducts the uniform price sealed-bid auction for the initial distribution of permits. In a uniform price auction, each bidder submits an order to purchase a certain quantity for a given price. A bidder may submit several such price-quantity orders, forming her demand (bidding) schedule. The auctioneer then aggregates these schedules to create an aggregated demand curve, determining for each price the total quantity demanded

at that price. The auctioneer sets the price so that the quantity demanded equals the available supply.

In this model the firms have to form and submit their bidding schedules to the regulator, who is the auctioneer of permits. When doing this, their output price, as well as the valuation for emissions permits in the secondary market, are uncertain. Because the secondary market price is uncertain before firms decide on their bids, we have here an auction with uncertain valuations. Consequently, in the case of the speculators, the equilibrium price of the secondary market, λ^*, can be interpreted as the post-auction value of a permit. As ϵ is normally distributed, from (13) it follows that λ^* is also normally distributed with the expectation given by (12) and the variance equal to $\Omega^2\sigma^2$.

After substituting for q_j, $j \in \{f, s\}$, r_f and λ^* from (5), (6), (8) and (13), respectively, into the profit functions given by (1) and (3), the latter can be written as

$$\Pi_j(D_j(\nu), \epsilon) = \hat{A}_j\epsilon^2 + \left(\Omega D_j(\nu) + \hat{B}_j\right)\epsilon + \left(E[\lambda^*] - \nu\right) D_j(\nu) + \hat{C}_j, \tag{15}$$

where for convenience I define:

(i) $\hat{A}_f = \frac{1}{4b}\left((\alpha_f - \Omega k_f)^2 + \frac{\Omega^2 b}{\theta}\right)$ and $\hat{A}_s = \frac{1}{4b}\alpha_s^2$;
(ii) $\hat{B}_f = \frac{1}{2b}\left(\gamma_f - k_f E[\lambda^*]\right)\left(\alpha_f - \Omega k_f\right) + \frac{1}{2\theta}\Omega E[\lambda^*]$ and $\hat{B}_s = \frac{1}{2b}\gamma_s\alpha_s$;
(iii) $\hat{C}_f = \frac{1}{4b}\left(\gamma_f - k_f E[\lambda^*]\right)^2 + \frac{1}{4\theta}\left(E[\lambda^*]\right)^2$ and $\hat{C}_s = \frac{1}{4b}\gamma_s^2$.

Note that, regardless of the choice of the underlying parameters, $\hat{A}_f$, $\hat{A}_s$, $\hat{C}_f$ and $\hat{C}_s$ are non-negative quantities.

As equation (15) shows, the profit function of a polluting firm is convex in the global shock ϵ. Likewise, the profit function of the speculators is convex in ϵ, unless they have no production activity in which case $\hat{A}_s = 0$ and the profit is increasing in the shock, provided that $\Omega > 0$. It appears that for a polluting firm a strong positive shock to the whole economy is needed for her profit to increase, regardless of the sign and the magnitude of her sensitivity to this shock.

Recall that both the polluting firms and the speculators are risk averse. Because of the uncertainty they face at this stage, all firms form expectations regarding their utilities. The derivation of $E(U(\Pi_j))$ can be found in Appendix A. As it turns out, maximizing the expectation of $U(\Pi_j(D_j(\nu), \epsilon))$ is equivalent to maximizing the following objective function:

$$\widehat{U}_j(D_j(\nu)) = \left(E[\lambda^*] - 2\kappa_j\Omega\hat{B}_j - \nu\right) D_j(\nu) - \kappa_j\Omega^2 D_j^2(\nu) - \kappa_j\hat{B}_j^2 + \hat{C}_j, \tag{16}$$

where, for the sake of conciseness, I denote $\kappa_j = \dfrac{\rho\sigma^2}{2 + 4\rho\sigma^2\hat{A}_j} > 0$. This is a mean-variance derived utility function. Parameter κ_j captures the disutility of the firm from bearing the uncertainty at this stage of the game. The higher the uncertainty or the higher the risk aversion, the larger the disutility: $\partial\kappa_j/\partial\sigma^2 > 0$ and $\partial\kappa_j/\partial\rho > 0$, where ρ refers to ρ_F and ρ_S

for a polluter and a speculator, respectively.

The objective function in (16) reveals two interesting facts. First, the quadratic term in D_j is the result of the risk aversion of bidder j relative to the uncertainty governing her decision process. Second, we are facing the problem of a uniform price auction with heterogeneous bidders. Thus, bidders have individual valuations and each winning bidder pays the same unit price, ν, for the permits, in the auction. Following Kyle (1989), I can interpret

$$v_j(D_j) = E[\lambda^*] - 2\kappa_j \Omega \hat{B}_j - 2\kappa_j \Omega^2 D_j,\ j \in \{f, s\} \tag{17}$$

as the marginal valuation for the permits of bidder j.[18] This is both the result of the risk aversion of the bidders and of the two-fold usage of permits: as an input in the production process and as an asset with a re-sale market.

Accounting for the definition of $\hat{B}_f$ and grouping around $E[\lambda^*]$, the marginal valuation of a polluter, defined in (17), can further be decomposed as,

$$v_f(D_f) = \left(1 - \frac{\Omega^2 \kappa_f}{\theta} + \frac{\kappa_f \Omega}{b} k_f (\alpha_f - \Omega k_f)\right) E[\lambda^*] - \frac{\kappa_f \Omega}{b} (\alpha_f - \Omega k_f) \gamma_f - 2\kappa_f \Omega^2 D_f, \tag{18}$$

Similarly, $\hat{B}_s$ gives the following marginal valuation of a speculator:

$$v_s(D_s) = E[\lambda^*] - \frac{\kappa_s \Omega \alpha_s}{b} \gamma_s - 2\kappa_s \Omega^2 D_s. \tag{19}$$

Because κ_j's are positive, the marginal valuations are decreasing in the permits holding. This is consistent with the auction and market microstructure literature.[19] The marginal valuations are endogenous since there is a re-sale opportunity, which is reflected in the first terms on the right-hand side of equations (18) and (19).

The constant terms in (18) and (19) represent the willingness to pay (WTP) for permits in the auction. In essence, if no production activity took place and if permits were assets with no use value, the WTP would simply be the expected re-sale price $E[\lambda^*]$. However, this is altered due to the dual nature of a permit: an asset for speculation and an input in the production process.

Along the lines of Biais (2005), the constant terms of (18) and (19) define the expected fundamental value of a permit. Obviously, the double purpose of a permit is reflected differently in its fundamental value, for a polluter and a speculator, respectively. The weight a polluter assigns to the expected secondary market price can be larger, smaller or equal to unity (the coefficient which multiplies $E[\lambda^*]$). A speculator puts full weight on the expected re-sale price. Moreover, if the speculators did not engage in any production activity, i.e. $\alpha_s = \gamma_s = 0$, their valuations would be given just by the expected secondary market price,

[18] This is derived from the first order conditions of (16).

[19] See, for example, Kyle (1989) and Biais (2005).

which only depends on the fundamentals of the polluting firms, as it was shown above. This reflects the pure speculation behavior, which brings us back to the situation analyzed in Kyle (1989), where there is no use valued for the traded asset. It is also important to stress that if firms are risk neutral, that is $\kappa_j = 0$, then a permit is valued at its after-auction price, regardless of the firm being a speculator or a polluter.

4.4.1 Equilibrium Bids

In the model of this paper firms are heterogeneous in several dimensions, therefore the auction equilibria will be asymmetric. The bidding strategies are based on the endogenously determined individual marginal valuations, which are decreasing functions of the permits holdings. In this paper the individual valuations are common knowledge among the auction participants, as is the perfectly inelastic supply, $\overline{E}$, auctioned by the regulator. Thus, each bidder j chooses her optimal bidding strategy maximizing the utility in (16) acting as a monopsonist on the residual supply. The equilibrium concept is Nash equilibrium. A strategy for bidder j is a non-increasing schedule $D_j(\nu)$ which specifies the quantity demanded for every price ν. Thus, each bidder solves the following problem:

$$\max_{\nu} \left(E[\lambda^*] - 2\kappa_j \Omega \hat{B}_j - \nu \right) \left(\overline{E} - \sum_{i \neq j} D_i(\nu) \right) - \kappa_j \Omega^2 \left(\overline{E} - \sum_{i \neq j} D_i(\nu) \right)^2, \tag{20}$$

where $\overline{E} - \sum_{i \neq j} D_i(\nu)$ is the residual supply faced by bidder j and the constant terms in (16) have been ignored. After re-arranging, the first order condition for this problem reads:

$$\left(1 - 2\kappa_j \Omega^2 \hat{B}_j \sum_{i \neq j} D_i'(\nu) \right) D_j(\nu) + \left(E[\lambda^*] - 2\kappa_j \Omega \hat{B}_j - \nu \right) \sum_{i \neq j} D_i'(\nu) = 0 \tag{21}$$

Following Green (1999), I focus on linear equilibrium bidding strategies. Assuming that all firms have positive valuations for permits,[20] I let the equilibrium bidding strategy take the form

$$D_j(\nu) = x_j - y_j \nu, \quad x_j,\, y_j \geq 0, \tag{22}$$

Substituting (22) in (21) and grouping around ν yields the following:

$$\left(-\sum_{i \neq j} y_i + y_j + 2\Omega^2 \kappa_j y_j \sum_{i \neq j} y_i \right) \nu + \left(E[\lambda^*] - 2\Omega \kappa_j \hat{B}_j \right) \sum_{i \neq j} y_i - x_j - 2\Omega^2 \kappa_j x_j \sum_{i \neq j} y_i = 0 \tag{23}$$

[20] This assumption assures that all firms submit bids in the auction. If some firms had negative valuations, they would not participate in the auction, thus they would be disregarded by all the other participants at this stage. However, they are present on the secondary market at most as buyers.

Equation (23) holds along any point on the demand schedule if and only if the following two equations are satisfied simultaneously:

$$y_j = (1 - 2\Omega^2 \kappa_j y_j) \sum_{i \neq j} y_i, \ \ \forall j = 1, \ldots, N + M \tag{24}$$

and

$$x_j = \frac{\left(E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j\right) \sum_{i \neq j} y_i}{1 + 2\Omega^2 \kappa_j \sum_{i \neq j} y_i}, \ \ \forall j = 1, \ldots, N + M. \tag{25}$$

Substituting (24) in (25) yields $x_j = \left(E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j\right) y_j$. Therefore, the demand schedule for any bidder j is an affine function. Because the initial endowment cannot be negative, for the price interval where the demand is negative, I restrict it to be zero. Specifically, the demand schedules are kinked function with the kink at the point where the price equals firm's valuation:

$$D_j(\nu) = \begin{cases} y_j \left(E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j - \nu\right), & \text{if } E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j > \nu \\ 0, & \text{if } E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j \leq \nu \end{cases} \tag{26}$$

The slopes $y_j \geq 0$ are solutions of the system of equations given by (24), which cannot be solved analytically. Fortunately, Rudkevich (1999) shows that this system of equations has exactly one non-negative solution.[21] This information is enough to conclude that the auction has a unique linear equilibrium in the form of the piecewise affine functions given by equation (26).[22]

4.4.2 Clearing Price

After the individual demands are submitted, the auctioneer aggregates them and calculates the auction clearing price. The aggregate demand is a horizontal summation of piecewise functions, so it is itself a piecewise function. Figure 2 illustrates an example of this aggregation for the case of 5 bidders. Note how the kinks of the aggregate demand appear at the points where the individual demands become positive, i.e. in the points where the price equals bidders valuations. Also note that it is possible for some bidders to receive zero permits in the auction if their valuations are too low. For instance, this is the case with bidder 1 in the example illustrated in Figure 2.

Definition 4.3 establishes the auction clearing price as the highest price for which the aggregate excess demand is non-negative.

[21] For this type of problem, Subramanian et al. (2008) give equilibria in the form $D_j(\nu) = \overline{E} - y_j \nu$, but this is analytically tractable for two players only.

[22] As the number of bidders increases, i.e. $N + M$ is large, the slopes y_j's converge to the perfect competition slopes given by $\frac{1}{2\kappa_j \Omega^2}$. This case will be analyzed in Section 7.1.

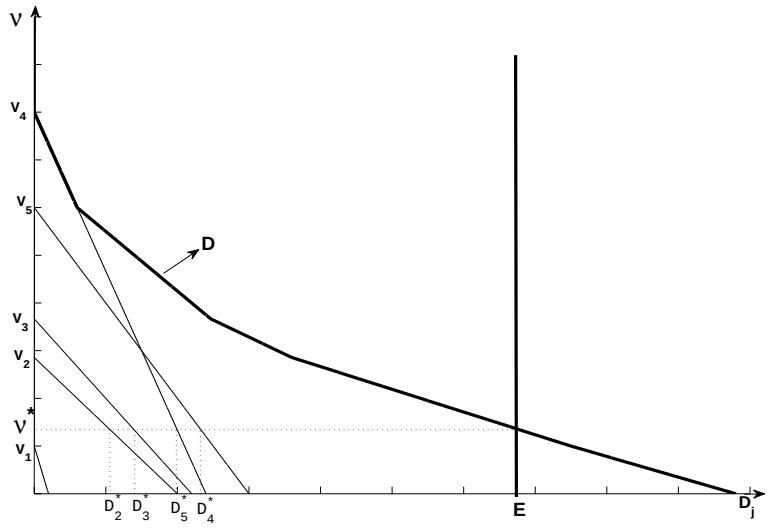

Figure 2: Demand schedules in the auction - example

Definition 4.3 *Let ν^* be the price at which the auction clears. Then, ν^* is defined as:*

$$\nu^* = \begin{cases} \max\{\nu \geq 0 | \sum_{j=1}^{N+M} D_j(\nu) \geq \overline{E}\} & \text{if} \quad \{\nu \geq 0 | \sum_{j=1}^{N+M} D_j(\nu) \geq \overline{E}\} \neq \emptyset \\ 0 & \text{if} \quad \{\nu \geq 0 | \sum_{j=1}^{N+M} D_j(\nu) \geq \overline{E}\} = \emptyset \end{cases} \tag{27}$$

Thus, without a price floor condition, the auction clearing price reads:

$$\nu^* = E[\lambda^*] - \frac{\overline{E}}{\sum_{j=1}^{n+m} y_j} - \frac{2\Omega}{\sum_{j=1}^{n+m} y_j} \left(\sum_{f=1}^{n} y_f \kappa_f \hat{B}_f + \sum_{s=1}^{m} y_s \kappa_s \hat{B}_s \right) \tag{28}$$

where $n \leq N$ and $m \leq M$ is the number of polluting firms and speculators, respectively for which the non-zero branch of the demand function is crossed by the horizontal line that goes through the clearing price. For instance, in the example from Figure 2 this line crosses the demand functions D_2, D_3, D_4 and D_5. Hence, in this simple example they are the firms whose demands make up the aggregated demand at the point where the total supply intersects the aggregated demand.

The n and m firms can be qualified as winning firms, as they win positive amounts of permits in the auction. Substituting the clearing price in the demand schedule, the equilibrium quantity is given by:

$$D_j^*(\nu^*) = \max \left\{ 0, 2\Omega y_j \left(\frac{\sum_{i=1}^{n+m} y_i \kappa_i \hat{B}_i}{\sum_{i=1}^{n+m} y_i} - \kappa_j \hat{B}_j \right) + \frac{\overline{E}}{\sum_{i=1}^{n+m} y_i} y_j \right\}$$

This is the initial allocation of permits at the end of the auctioning stage. It is straightforward, and at the same time not surprising, to see that the higher the total supply of permits, the higher the initial allocations for those who earn permits in the auction.

5 Analytical Results

In this section I derive the main analytical results, which, in essence, are comparative statics of the main parameters of the model. Because of the complexity of the model with multiple levels of heterogeneity and non-monotonicity, for further conclusions I employ a series of numerical examples by reasonably choosing the parameters of the model. The numerical results are discussed in the next section.

First, the endogenously derived valuation for permits deserves further investigation since it characterizes firm's bidding power: the higher the valuation, the more aggressive the bidder. The following propositions describe the properties of the constant marginal valuation for permits.

Proposition 5.1 *The willingness to pay for permits, $E[\lambda^*]-2\Omega\kappa_f\hat{B}_f$, of a polluter increases in the coefficient of risk aversion, ρ_F, and in the variation of the global shock, σ^2 if and only if $\Omega > (<)0$ and $\alpha_f < (>)\Omega\left(k_f - \frac{b}{\theta}\frac{E[\lambda^*]}{\gamma_f - k_f E[\lambda^*]}\right)$*

Essentially, Proposition 5.1 says that the monotonicity of the valuation for permits of a polluting firm depends on the firm's sensitivity to the global shock and on the overall sensitivity of the polluters. If her response to the shock has the same direction with that of the biggest polluters in the economy, her valuation for permits is decreasing both in her risk aversion and in the volatility of the global shock to the economy. Hence, firm f uses the permits markets to hedge the risk she faces on the product market. More precisely, if good news for the rest of the polluters represent bad news for polluter f, then this polluter will bid more aggressively in the auction so that she can sell some of her permits holdings in the secondary market in order to compensate on the losses she suffers on the product market due to an unfavorable shock. She can do this because there will be demand for permits from the firms who experience the opposite effect on their respective product markets.

Proposition 5.2 *The WTP for permits, $E[\lambda^*] - 2\Omega\kappa_f\hat{B}_f$, of a polluter increases in $E[\lambda^*]$ or, equivalently, decreases in $\overline{E}$ if and only if one of the following conditions holds:*

i) $\alpha_f < \alpha_f^{\lambda-}$ *and* $k_f > \bar{k}$ *or,*
ii) $\alpha_f > \alpha_f^{\lambda+}$ *and* $k_f > \bar{k}$ *or,*
iii) $k_f < \bar{k}, \forall\alpha_f$

where $\alpha_f^{\lambda-} = \frac{\rho_F\Omega\sigma k_f - \sqrt{\rho_F^2\Omega^2k_f^2\sigma^2 - 8\rho_F b}}{2\rho_F\sigma}$, $\alpha_f^{\lambda+} = \frac{\rho_F\Omega\sigma k_f + \sqrt{\rho_F^2\Omega^2k_f^2\sigma^2 - 8\rho_F b}}{2\rho_F\sigma}$ *and* $\bar{k} = \sqrt{\frac{8b}{\rho_F\Omega^2\sigma^2}}$

Proof: Let us denote $WTP(E[\lambda^*]) = E[\lambda^*] - 2\Omega\kappa_f\hat{B}_f$ the WTP function of the expected secondary market price of permits. Then, the first derivative is given by

$$\frac{\partial WTP(E[\lambda^*])}{\partial E[\lambda^*]} = 1 - 2\Omega\kappa_f\frac{\partial\hat{B}_f}{\partial E[\lambda^*]}$$

Substituting for κ_f, $\hat{A}_f$ and $\hat{B}_f$, after some algebraic manipulations it arrives at:

$$\frac{\partial WTP(E[\lambda^*])}{\partial E[\lambda^*]} = \rho_F \sigma^2 \alpha_f^2 - \rho_F \sigma^2 \Omega k_f \alpha_f + 2b,$$

which is a second degree polynomial in α_f. Since the coefficient of the quadratic term of this polynomial is positive, it takes positive values outside the roots $\alpha^{\lambda-}$ and $\alpha^{\lambda+}$, provided that the discriminant is positive. This is the situation described in *i)* and *ii)*. If the discriminant is negative, then the polynomial is positive for any value of α_f. This is the situation described in *iii)*. The proof is complete.

The result summarized in Proposition 5.2 is intuitive: a polluter values the permits more if she expects a high price in the secondary market. This is true both for a predicted net seller and a predicted net buyer. For instance, if a polluter expects a buying net position on the secondary market, she would like to secure more permits in the auctioning stage. Thus, the higher the expected secondary market price, the more aggressively she bids in the initial allocation stage. Conversely, if she expects to be a net seller, she may want to increase her permits inventories in order to re-sale them in the secondary market for a high expected price.

However, according to Proposition 5.2, there exists a convex region of relatively high emissions rate ($k_f > \bar{k}$) and low sensitivity to the shock ($\alpha_f \in (\alpha^{\lambda-}, \alpha^{\lambda+})$), where the WTP decreases in the expected secondary market price. The intuition is that, due to the low sensitivity level, such a polluter faces little uncertainty at the auctioning stage, such that her hedging needs are low. Consequently, her speculative reasons are low. At the same time, because the high sensitivity polluters bid aggressively when the expected secondary market price is high, the auction will clear at a high price. Hence, a polluter in this region bids less aggressively exactly for the purpose of lowering the auction clearing price, since she knows her permits needs with a higher precision.

Proposition 5.3 *The WTP for permits,* $E[\lambda^*] - \dfrac{\kappa_s \Omega \alpha_s}{b}\gamma_s$, *of a speculator*

i) increases in the coefficient of risk aversion, ρ_S *and in the variation of the global shock,* σ^2 *if and only if* $\Omega > (<)0$ *and* $\alpha_s < (>)0$*;*

ii) increases in $E[\lambda^*]$*, or equivalently, decreases in* $\overline{E}$

It is obvious to see that, given $\Omega > 0$, a firm with a positive sensitivity to the shock will undervalue the permits relative to the secondary market price, while a firm with a negative sensitivity will overvalue them relative to the secondary market price. However, in the case of the polluting firms this condition is less clear-cut because the sign of $\hat{B}_f$ depends on the firm's dirtiness and on the output market parameters, which complicates the discussion.

Corollary 5.4 *A speculator values a permit more than its expected price in the secondary market if the sign of her response to the shock,* α_s*, is opposite to that of the aggregate sensitivity of the polluting firms (the sign of* Ω*).*

Hence, good news for the polluters is bad news for the speculators and vice-versa. In a common knowledge environment this makes the speculators overoptimistic about the post-auction value of the permits, and they use the permits markets as a hedging devise against the risk they face in their product markets. If the economy is hit by a positive shock and Ω is positive, this means a higher than expected secondary market price. In addition, the polluters' needs for permits increase, as they experience a boost in their output markets. At the same time, with negative α_s's, the speculators experience a lower than expected outcome from their production activity. In this situation they compensate the loss from the production activity with a gain from re-selling the permits. This kind of scenario determines the speculators to have an aggressive bidding in the auction stage. However, the higher the number of the speculators who have this type of sensitivity, the more likely is that the auction clearing price is higher than the secondary market price inducing losses for the net sellers.

Second, an element of interest both for firms and policy makers is the difference between the prices of the two markets for permits: the auction, as the primary market, and the secondary market. The conventional wisdom advocated in several policy papers is that these two prices should not be significantly different from each other. Moreover, some argue that the secondary market price must serve as a signal for the auction price. However, as I argue below, there is no reason to believe a-priori that this must necessarily be the case.

Let S define the spread between the secondary market price and the auction clearing price, i.e. the difference between the two prices. Then, from (28) it follows that the expected spread is:

$$E[S] = \frac{\overline{E}}{\sum_{j=1}^{n+m} y_j} + \frac{2\Omega}{\sum_{j=1}^{n+m} y_j} \left(\sum_{f=1}^{n} y_f \kappa_f \hat{B}_f + \sum_{s=1}^{m} y_s \kappa_s \hat{B}_s \right) \tag{29}$$

The following proposition discusses the properties of the expected spread between the auction and the secondary market price.

Proposition 5.5 *The expected spread between the secondary market price and the auction equilibrium price, $E[\lambda^*] - \nu^*$*

i) increases in $\overline{E}$, the total number of permits issued, if and only if a certain condition regarding only the parameters of the n winning polluting firms holds;

ii) can be both positive and negative and this depends on the characteristics of all bidding firms.

The following numerical examples show a condition under which this spread is increasing. However, it is obvious that, if the polluters have on average a positive sensitivity, a negative shock will depress the spread between the realized secondary market price and the auction clearing price because the secondary market price decreases after a negative shock, while the auction clearing price is unaffected by the realization of the shock.

6 Numerical Examples

As it has already been seen, it is difficult to obtain unambiguous predictions of the model via analytical manipulations. Moreover, the system of equations in (24) can only be solved numerically. Thus, in order to obtain the outcome of the auction and the trade position of the firms in the secondary market, numerical simulations are necessary. Therefore, in this section I use a few numerical examples in order to asses the predictions of the model and the effect of the different parameters on the outcome variables. The two main levels of heterogeneity in the model are the emissions rates and the idiosyncratic sensitivities to the aggregate shock. Thus, in order to keep the analysis simple, I will firstly fix the sensitivity parameter α_j to be the same for all firms and I allow for heterogeneity in the emissions rates in order to isolate the effect of this parameter on firms' valuations, allocations and market outcomes. Further, keeping all firms at the same level of dirtiness, I assess the impact of firms' idiosyncratic sensitivity to shock on allocations and secondary market trade. Finally, I assume heterogeneity across firms both with respect to their sensitivity levels and the emission rates. In each simulation polluters react positively to good news ($\alpha_f > 0$, $\forall f$), as this is the most realistic assumption. The results are explained below and the graphs can be found in Appendix B.

6.1 Emissions rates

First, I analyze the role of the speculators in a set-up where all agents, polluters and speculators, have the same expected output price and sensitivity to the shock. I consider an universe composed of 70 agents evenly split between polluters and speculators. In order to keep the focus on the emissions rates, I set the expected output price, $\gamma_i = 1000$ and the response to the overall shock, $\alpha_i = 2, \forall i = 1, \ldots, 70$. This makes the speculators identical, but the polluters are heterogeneous with respect to their rates of emissions per unit of output. I choose the emissions rate of each polluter to be increasing in her index, that is $k_f = f/N$ for each f.

Figure 3 shows the outcome of the emissions trading scheme when the speculators are allowed to participate in the market. Since the speculators are identical in the current situation, their valuations are the same and, unsurprisingly, they end up with the same initial endowments of permits. This can be seen on the right side of the lower-left panel in Figure 3. The upper-left panel of the same figure depicts the valuations for permits in the auctioning stage of the polluting firms as a function of their dirtiness level. This graph suggests the clear pattern that, all else equal, dirtier firms value permits more and, hence, they are stronger bidders in the auction. The next graph, in the upper-right panel shows the trading behavior of the polluters in the secondary market together with their endowments from the auction, as a function of the emissions rates. The bars in the negative side of the vertical axis represent sales of permits and the bars on the positive side represent purchases of permits. Hence, an

interesting conclusion is that only the very high polluters become net sellers in the secondary market, while all the rest are net buyers of permits even with the positive shock to the demand of the final output. This denotes the fact that, *ceteris paribus*, the high polluters overvalue the permits in the auctioning stage. By design, all the speculators are net sellers. Finally, the lowest panels depict the valuations and the initial allocations for all firms. On the horizontal axis is indicated the index of the bidder, keeping in mind that polluters are increasingly ordered according to the emissions rates. In the valuations' graph, the thin horizontal line shows the auction clearing price. As it can be seen, in the given situation, re-selling in the secondary market is profitable, i.e. the secondary market price is higher than the auction clearing price. It is worth noting that the auction clearing price is only slightly below the valuations for the speculators, suggesting that they have a high weight in determining this price.

Figure 4 shows the same results in the situation where the speculators would be prohibited from participating in the auction for the distribution of emissions permits. I keep the same parameter values as in Figure 3, except for setting the number of speculators to zero. A simple comparison between the lower-left panels of these two graphs shows that the presence of the speculators does, indeed, drive some of the polluters out of the auction: four more polluters earn zero permits in the auction. Needless to say, in addition to some polluters being driven out, the winning polluters earn fewer permits in the auction when speculators are present. It is also worth noting that the presence of the speculators raises the auction clearing price (the thin horizontal line in the lower-left panels of these figures).

In the example in Figure 3, I assumed that the polluters are more risk averse than the speculators. Therefore, the set of graphs in Figure 5 examines the case where the speculators are as risk averse as the polluting firms and I set the risk aversion coefficients $\rho_F = \rho_S = 0.02$. In this case polluters and speculators are identical with respect to the response and the aversion towards the risk, but the polluters have an additional dimension: the rate of emissions. This determines them to form valuations above those of the speculators and drive them out of the auction. Although, the speculators do not win any permits in the auction, their simple presence drives out one polluter from the set of winners in the auction. This is due to the fact that the bids submitted by the speculators increase the auction clearing price, even if they are not winning bids (compare this with the case of Figure 4 where speculators are absent).

6.2 Sensitivity to the global shock

In order to assess the impact of polluters' sensitivity to the global shock, I fix all the parameters except for their idiosyncratic sensitivity, α_f. Again, the speculators are identical. I set $k_f = 0.9, \forall f = 1, \ldots, N$ such that the level of dirtiness of the polluting firms is relatively high, and $\alpha_s = 2, \forall s = 1, \ldots, M$, which is lower than any of the idiosyncratic sensitivities of

the polluters. Again I consider an economy with $N = 35$ polluters and $M = 35$ speculators.

Recall that firms indexed from 1 to N are the polluters and firms indexed from $N + 1$ to $N + M$ are the speculators participating in the market. Moreover, polluting firms are indexed from the lowest to the highest sensitivity to the aggregate shock. A clear conclusion from Figures 6 and 7 is that polluters' valuations are decreasing in the sensitivity to shock: all else equal, firms with a higher sensitivity to shock value the permits less. Consequently, the bidding aggressiveness of these firms is low, and they earn fewer permits in the auction than the low sensitivity firms. It also becomes apparent from Figures 6 and 7 that the low sensitivity firms are net sellers and the high sensitivity firms are net buyers in the secondary market. As the secondary market price is higher than the auction clearing price (at least in the case of high product prices and a relatively low shock volatility), it seems that without speculators (Figure 7) a wealth transfer is taking place from the high sensitivity firms to the low sensitivity firms through the secondary market.

Figures 8 to 13 depict the situation where the polluters become more and more averse to the aggregate shock, keeping the risk aversion of the speculators at a constant level. In these figures I also allow for the speculators to be heterogeneous with respect to the sensitivity to shock - a full idiosyncratic sensitivity case. The results show that the valuation curve for the polluting firms, as a function of their sensitivity to shock, is becoming steeper and lower the more risk averse they become. The intuition is that the more risk averse the firms are, the higher the importance of the sensitivity, and they are more reluctant to invest in permits in the first place. However, once the shock is realized, firms will need to buy the deficit of permits at a higher price in the secondary market. At the same time, speculators' valuations are virtually unaffected.

Another interesting result of this exercise is that the spread between the secondary market price and the auction clearing price increases as the polluters become more risk averse. This is due to the fact that the auction clearing price is decreasing in polluters' risk aversion, while the secondary market price is independent of this parameter (it only varies with the realization of the shock).

In Figure 8 the polluters and the speculators are identical with respect to all the parameters involved. In particular, for each polluter with a certain sensitivity level there is a speculator with the same sensitivity level. In addition, polluters and speculators have the same risk aversion. However, the fact that the emissions permits act as an input in their production process, induces the polluters to value the permits considerably more than the speculators. Under such conditions, no speculator earns permits in the auction. Thus, allowing the speculators to be idiosyncratically affected by the economy-wide shock may drive them out from the speculation activity if the polluters are not sufficiently risk averse. In order for the speculators to make positive profits, the polluters would need to be sufficiently more risk averse than the speculators.

In the upper-right graphs of the above-mentioned sequence of figures, one can see how the initial valuations for the polluters is dropping, while that of the speculators is lifting as the polluters risk aversion coefficient increases. If in Figures 8 and 9 the speculators do not earn any permits (polluters have a low risk aversion), by Figure 12 the speculators are already winning more of the fixed supply of permits than the polluters do. Finally, in Figure 13 the speculators' initial endowment of permits is well above that of the polluters. Hence, for this particular example, the polluters have to be at least three times more risk averse than the speculators in order for the latter to win a positive number of permits in the auctioning stage.

The main policy implication of this result is that allowing for speculators to enter the primary market for permits is a decision that has to take into account the risk aversion of the speculating firms relative to that of the polluters. In particular, if the polluters are substantially more risk averse than the speculators, the presence of the latter type of firms can harshly hurt the regulated firms. If, on the other hand, this difference is not significant, the presence of the speculators in the auction has the effect of boosting the clearing price, compared to the situation when they are absent, without hurting too much the polluting firms.[23] This trade-off may be worthwhile if the regulator is interested in the auction revenue.

6.3 Trading volume and risk volatility

In this section I discuss the effect of the risk volatility on the trading volume of the secondary market. The simulation results are shown in Figure 14 for a certain parameters choice, where firms' heterogeneity resides only in the emissions rate variable. Note that firms have the same response to the shock. Hence, this particular choice of the parameters set reflects the case where all firms belong to the same industry, i.e. have the same output price, but they have different production technologies as represented by the different emissions rates. The interesting result is depicted in the first quadrant of this figure, where the horizontal axis has the standard deviation of the shock and the vertical axis has the trade as a percentage of the total number of permits auctioned: An increase in the risk volatility faced by the economy results in less trade in the secondary market. At first glance, this result is counterintuitive. However, the explanation lies in the following two graphs of Figure 14. As the uncertainty increases, firms' WTP for permits in the auction are closer to each other (less variation), resulting in more competition in the auction (lower-left panel of Figure 14). Consequently, the auction clears at a very high price (upper-right panel of Figure 14), leaving some of the firms with zero permits after the auction, despite the fact that their WTP is closer to that of the winning firms. Thus, only the highest WTP firms earn a positive number of permits in the auction. Nevertheless, as they face the same output price, the non-winning firms will need to purchase the necessary permits, thus giving rise to an intense trading activity in the secondary market. Conversely, with high uncertainty, the standard deviation of the valuations

[23] To see this, compare Figures 7 and 9.

is decreasing, resulting in a looser competition in the auction and a lower auction clearing price. Hence, more firms have the chance to earn some permits in the auctioning stage and there is less need for redistribution in the secondary market.

In conclusion, for the case of a single industry regulated by a cap to emissions, the lower the uncertainty in the economy, the more inefficient the auction outcome. However, this result is less clear-cut if the firms belong to different industries, i.e. respond differently to the shock. Figure 15 illustrates this case. Although the results concerning the WTP and the auction price carry on from the previous scenario with different emissions rates, the trade result is less obvious, as the after-auction needs for permits are more heterogeneous. However, a somewhat ascending trend in the trading ratio can be observed.

7 Variants of the Model

7.1 Competitive primary market

Until now I have assumed that firms recognize their ability to influence the price in the auction for the distribution of the permits to pollute. Therefore, the auction equilibrium was solved assuming strategic behavior on the side of the firms. However, with a large number of firms, as it is the case in the EU ETS, the assumption that the firms have power to influence the auction clearing price may not hold. Hence, in this section I examine the case of competitive bidding.

7.1.1 Competitive bids

Competitive bidding amounts to each firm submitting a bid equal to her marginal valuation for permits, as it is given by the first order condition of (16). Note that the individual willingness to pay is unchanged as it is intrinsically determined by firm's characteristics and it does not depend on the strategic behavior. Thus, the competitive bid of any firm $j = 1, \ldots, N + M$ is given by

$$D_j(\nu) = \begin{cases} \frac{1}{2\kappa_j \Omega^2} \left(E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j - \nu \right), & \text{if } E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j > \nu \\ 0, & \text{if } E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j \leq \nu \end{cases} \tag{30}$$

The interpretation is that each bidder j is willing to purchase $\frac{1}{2\kappa_j \Omega^2} \left(E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j - \nu \right)$ permits for any price ν up to $E[\lambda^*] - 2\Omega\kappa_j \hat{B}_j$. Note that as κ_j is decreasing in $\hat{A}_j$, the higher the emissions rate or the sensitivity to the shock of firm j, the flatter the slope of her bidding function.

It follows that the clearing price is:

$$\nu^* = E[\lambda^*] - \frac{2\Omega}{\sum_{j=1}^{n+m} \frac{1}{\kappa_j}} \left(\Omega \overline{E} + \sum_{f=1}^{n} \hat{B}_f + \sum_{s=1}^{m} \hat{B}_s \right), \tag{31}$$

where, as previously, n is the number of winning polluters and m is the number of winning speculators.[24] Finally,

$$D_j^*(\nu^*) = \max \left\{ 0, \frac{\overline{E}}{\kappa_j \sum_{i=1}^{n+m} \frac{1}{\kappa_i}} + \frac{1}{\Omega \kappa_j} \left(\frac{\sum_{i=1}^{n+m} \hat{B}_i}{\sum_{i=1}^{n+m} \frac{1}{\kappa_i}} - \kappa_j \hat{B}_j \right) \right\}$$

7.1.2 Discussion

Comparative statics of these results are informative for policy recommendations. Fortunately, the assumption about bidders' competitive behavior in the auction allows more clear-cut conclusions about the auction outcome than those which could be achieved in Section 5. For example, looking at (31) the following proposition can be stated:

Proposition 7.1 *Let us suppose that on average the polluters' output demands follow the direction of the shock, i.e.* $\Omega > 0$. *The auction clearing price is decreasing in the risk aversion of the polluters,* ρ_F, *and in the risk aversion of the speculators,* ρ_S, *if the following two sufficient conditions hold simultanously:*

(i) the polluters' sensitivities to the shock are positive and relatively high: $\alpha_f \geq \Omega k_f$, $\forall f$;
(ii) speculators' product demand follow the direction of the shock: $\alpha_s \geq 0$, $\forall s$.

Proof: First it should be noted that this result depends exclusively on the sign of the quantity in the brackets of equation (31). Second, that the condition $\gamma_f \geq k_f E[\lambda^*]$ assures that in expectation the polluter has non-negative production. This together with *(i)* assures that $\hat{B}_f \geq 0$. In addition, *(ii)* assures that $\hat{B}_s \geq 0$. We know from before that $\frac{\partial \kappa_f}{\partial \rho} > 0$. This completes the proof.

Recall that this result was identified in Section 6.2 for the numerical case considered there, where all firms had positive sensitivities to the shock: as the polluters became more risk averse, the auction clearing price became smaller.

The conclusion of Proposition 7.1 holds true also for the opposite case when all firms have negative responses to the shock. It is obvious that in this case $\Omega < 0$. Hence, if all firms respond in the same direction to the overall shock, then they would be more reluctant to purchase permits in the primary market as the uncertainty about the economy is perceived stronger (higher risk aversion).

The auction clearing price *may*, however, increase in the risk aversion, both that of the polluters and that of the speculators, if some firms respond to the shock in the opposite

[24] These numbers will, normally, be different than those from the strategic bidding case.

direction than the biggest polluters do, i.e. the sign of α_j is opposite to the sign of Ω. For example, if $\Omega \geq 0$ and $\alpha_j \geq \Omega k_f$, $\forall f$, but the speculators have a strongly negative response to the shock, such that $\Omega\overline{E} + \sum_{f=1}^{n} \hat{B}_f + \sum_{s=1}^{m} \hat{B}_s < 0$, then the auction clearing price increases in the risk aversion. This is in opposition with the result found by Colla et al. (2005) who prove that the price of the first round of trading in their model decreases with the risk aversion for any positive supply of permits. However, their first round of trading is a bilateral market where polluters can sell permits to other polluters and to the speculators. In addition, their model assumes identical responses to the shock.

Proposition 7.2 *The auction clearing price is decreasing in the risk volatility, σ^2, if and only if the parameters of the winning firms satisfy the following condition:*

$$\Omega(\Omega\overline{E} + \sum_{f=1}^{n} \hat{B}_f + \sum_{s=1}^{m} \hat{B}_s) > 0.$$

7.2 No abatement

In this section I outline the changes suffered by the model in the case when there is no abatement possibility. First, without the abatement possibility the secondary market price is given by

$$\lambda^*_{\text{no abat}} = \frac{\sum_{f=1}^{N} k_f p_f - 2b\overline{E}}{\sum_{f=1}^{N} k_f^2} = E[\lambda^*_{\text{no abat}}] + \Omega_{\text{no abat}}\epsilon, \tag{32}$$

where $\Omega_{\text{no abat}} = \frac{\sum_{f=1}^{N} k_f \alpha_f}{\sum_{f=1}^{N} k_f^2} > \Omega$.

Compared with equation (11), the price given by equation (32) does not contain the term in θ at the denominator, which, for the same realizations of the output prices, makes this price higher. A by-product of this result is that, without abatement, the amount of final output given by (5) is lower. At the same time, the aggregate sensitivity parameter, $\Omega_{\text{no abat}}$ is higher than Ω, which means that the secondary market price is more volatile if there is no abatement possibility. These results are intuitive since the absence of the abatement possibility has the effect of tightening the environmental constraint, making compliance more expensive.

Second, while the coefficients of the random shock in the profit function do not change for the non-polluters, they do change for the polluters is the following way:

$$\begin{aligned}
\hat{A}_f &= \frac{1}{4b}\left(\alpha_f - k_f\Omega_{\text{no abat}}\right)^2 \\
\hat{B}_f &= \frac{1}{2b}\left(\gamma_f - k_f E[\lambda^*_{\text{no abat}}]\right)\left(\alpha_f - k_f\Omega_{\text{no abat}}\right) \\
\hat{C}_f &= \frac{1}{4b}\left(\gamma_f - k_f E[\lambda^*_{\text{no abat}}]\right)^2
\end{aligned} \tag{33}$$

With the profit function parameters given by (33), the rest of the analyses carries through without any changes.

8 Conclusions

In this paper I built a model that mimics the mechanism of an emissions trading scheme with auctioning, such as the EU ETS as it will be applied from 2013. The model aimed at anticipating the outcome of the main changes in the institutional design of this scheme, i.e the incorporation of the auction as the initial allocation method and the permission of the non-polluting entities to participate in the auction. The latter feature of the scheme is captured by the speculating agents.

In an uncertain environment, in which both the polluters and the speculating firms are present, I find that the secondary market price is unaffected by the presence of the speculators. In fact, the secondary market price is only determined by the characteristics and the number of the polluting firms present in the economy. In other words, the secondary market price reflects only the fundamentals of the polluting firms, which have to comply with the environmental regulations. Moreover, contrary to the conventional wisdom, which often ignores the existence of the secondary market when auction is the distribution method, I find that, with risk averse firms, the secondary market price can be both greater and lower than the auction clearing price. The spread between the two depends on the mix of firms' characteristics as a result of the differences in firms' responses to the shock.

However, given the set-up of this model, in which firms take decisions under uncertainty, it is unclear whether the presence of the speculators in the permits market has a positive or a negative effect over the auction clearing price. This will depend both on the sign of the sensitivity to the shock of the polluting firms and on that of the speculators.

Under the assumptions of this paper, where the production and abatement decisions come after the aggregate shock is realized, speculators have no influence on the secondary market price. Consequently, firms' production and abatement decisions are unaffected by the presence of the speculators in the auction and in the secondary market. The presence of the speculators does, however, increase the trading volume on the secondary market. As a corollary, the introduction of the speculators into the scheme has the effect of converting some potential sellers, from among the polluting firms, into buyers. This has obvious adverse welfare implications for the regulated firms. Regardless of the presence of the speculators, the risk aversion prevents the high sensitivity polluters from securing the necessary permits in the auctions and they become net buyers after the shock is realized. Moreover, the more risk averse they are, the fewer permits they earn in the auction and the auction clearing price drops, allowing the speculators to earn more profits. Therefore, in an economy where the polluters are much more risk averse than the speculators, conducting an auction for the

distribution of the emission rights where the speculators are allowed to participate may not be the best method of allocation, as it may significantly hurt the regulated firms. Hence, from the policy perspective this exercise shows that designing a "fair" benchmark for the free allocation of the fixed supply of the polluting rights may, in fact, be a more suitable method of allocation than the auction. This finding is against the conventional wisdom which regards auctioning as a fair and efficient method for the distribution of these rights. Therefore, when designing an emissions trading scheme with auctioning, the regulator should take into account the regulated firms' attitude towards risk, as well as the characteristics of non-regulated firms allowed to participate in the auction.

It is obvious that the current set-up requires more refinement and a few extensions of the current model are worth attention for further research. First, a multi-period model in which firms are allowed to bank permits from one period to another is a more realistic set-up in line with the actual regulations of the European scheme. Obviously, this would involve dynamic optimization and discounting. Second, cash constraints on permits expenses may replace the assumption about firms' risk aversion, which is a less accountable measure. Moreover, it is common and more plausible that some firms impose such constraints in their annual budgets and this can be empirically observable. Third, relaxing the assumption about positive production constitutes another path of research, which accounts for the case of non-compliance behavior of the polluting firms. Finally, the translation of this model into a laboratory experiment may contribute to detecting possible behavioral implications of the model. It is not excluded that firms, as represented by individuals, may exhibit some form of sunk cost fallacy once they are allocated the permits following the auction. This is a well documented behavioral anomaly in other contexts. However, to the best of my knowledge, there has been no such investigation in the context of emissions permits markets, either from the field, or from the laboratory. The direct implication of the existence of this fallacy for permits markets is an illiquid secondary market, which may severely affect the final production activity of those firms which are predicted net buyers in the market for permits. If this is the case, then efficiency through trade cannot be achieved.

References

Baldick, R., Grant, R. & Kahn, E. (2000), 'Linear supply function equilibrium: Generalizations, application, and limitations', *POWER Working Papers Series PWP-078* .

Biais, Bruno; Glosten, L. S. C. (2005), 'Market microstructure: A survey of microfoundations, empirical results, and policy implications', *Journal of Financial Markets* **8**, 217–264.

Colla, P., Germain, M. & van Steenberghe, V. (2005), 'Environmental policy and speculation on markets for emission permits', *CORE Discussion Paper* .

European Commission, E. (2003), 'Directive 2003/87/EC of the European Parliament and of the Council of 13 October 2003 establishing a scheme for greenhouse gas emission allowance trading within the Community and amending Council Directive 96/61/EC (Text with EEA relevance)'.

Fevrier, P. & Linnemer, L. (2004), 'Idiosyncratic shock in an asymmetric Cournot oligopoly', *International Journal of Industrial Organization* **22**(6), 835–848.

Green, R. (1999), 'The electricity contract market in England and Wales', *The Journal of Industrial Economics* **47**(1), 107–124.

Keloharju, M., Nyborg, K. G. & Rydqvist, K. (2005), 'Strategic behavior and underpricing in uniform price auctions: Evidence from Finnish treasury auctions', *The Journal of Finance* **60**(4), 18651902.

Kyle, A. (1989), 'Informed speculation with imperfect competition', *The Review of Economic Studies* **56**(3), 317–355.

Leland, E. H. (1972), 'Theory of the firm facing uncertain demand', *The American Economic Review* **62**(3), 278–291.

Rudkevich, A. (1999), 'Supply function equilibrium in Poolco type power markets: Learning all the way', *Technical Report Number 0699-1701, Tabors Caramanis and Associates* .

Rudkevich, A. (2005), 'On the supply function equilibrium and its applications in the electricity markets', *Decision Support Systems* **40**, 409–425.

Sandmo, A. (1971), 'On the theory of competitive firm under price uncertainty', *The American Econimic Review* **61**(1), 65–73.

Subramanian, R., Gupta, S. & Talbot, B. (2008), 'Compliance strategies under permits for emissions', *Production and Operations Management* .

Vargas, J. S. (2003), 'Bidder behavior in uniform price auction: Evidence from Argentina'.

Vives, X. (1984), 'Duopoly information equilibrium: Cournot and Bertrand', *Journal of Economic Theory* **34**, 71–94.

Wang, J. J. D. & Zender, J. F. (2002), 'Auctioning divisible goods', *Economic Theory* **19**, 673705.

Appendix A: Derivation of the expected profit

I calculate the expected value of $U(\Pi_j) = -\exp(-\rho(A_j\epsilon^2 + B_j\epsilon + C_j))$ accounting for $\epsilon \sim N(0, \sigma^2)$. The probability density function of ϵ is:

$$f(\epsilon) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) d\epsilon.$$

Thus,

$$\begin{aligned} E(U(\Pi_j)) =& E(-\exp(-\rho(A_j\epsilon^2 + B_j\epsilon + C_j))) \\ &= -\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp(-\rho(A_j\epsilon^2 + B_j\epsilon + C_j) \exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) d\epsilon \end{aligned} \tag{34}$$

Letting $A'_j = \left(\rho A_j + \frac{1}{2\sigma^2}\right) > 0$, we have further:

$$\begin{aligned} E(U(\Pi_j)) = &= -\exp(-\rho C_j)\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-\left(A'_j\epsilon^2 + \rho B_j\epsilon\right)\right] d\epsilon \\ &= -\exp(-\rho C_j)\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-A'_j\left(\epsilon^2 + \frac{\rho B_j}{A'_j}\epsilon\right)\right] d\epsilon \\ &= -\exp(-\rho C_j)\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-A'_j\left(\epsilon + \frac{\rho B_j}{2A'_j}\right)^2 + A'_j\left(\frac{\rho B_j}{2A'_j}\right)^2\right] d\epsilon \\ &= -\exp\left(-\left(\rho C_j - \frac{\rho^2 B_j^2}{4A'_j}\right)\right)\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-A'_j\left(\epsilon + \frac{\rho B_j}{2A'_j}\right)^2\right] d\epsilon \\ &= -\exp\left(-\left(\rho C_j - \frac{\rho^2 B_j^2}{4A'_j}\right)\right)\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-\frac{\left(\epsilon + \frac{\rho B_j}{2A'_j}\right)^2}{\frac{1}{A'_j}}\right] d\epsilon \\ &= -\frac{\exp\left(-\rho\left(C_j - \frac{\rho B_j^2}{4A'_j}\right)\right)}{\sigma\sqrt{2A'_j}} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\frac{1}{2A'_j}}\sqrt{2\pi}} \exp\left[-\frac{\left(\epsilon + \frac{\rho B_j}{2A'_j}\right)^2}{\frac{1}{A'_j}}\right] d\epsilon \end{aligned} \tag{35}$$

Under the last integral we have the probability density function of a normal random variable with mean $-\frac{\rho B_j}{2A'_j}$ and variance $\frac{1}{2A'_j}$. Therefore, its value is 1. Hence, we have

$$E(U(\Pi_j)) = -\frac{1}{\sigma\sqrt{2A'_j}} \exp\left(-\rho\left(C_j - \frac{\rho B_j^2}{4A'_j}\right)\right). \tag{36}$$

Consequently, maximizing $E(U(\Pi_j))$ is equivalent to maximizing

$$C_j - \frac{\rho}{4A'_j}B_j^2 = C_j - \frac{\rho\sigma^2}{2 + 4\rho\sigma^2 A_j}B_j^2. \tag{37}$$

Appendix B: Figures

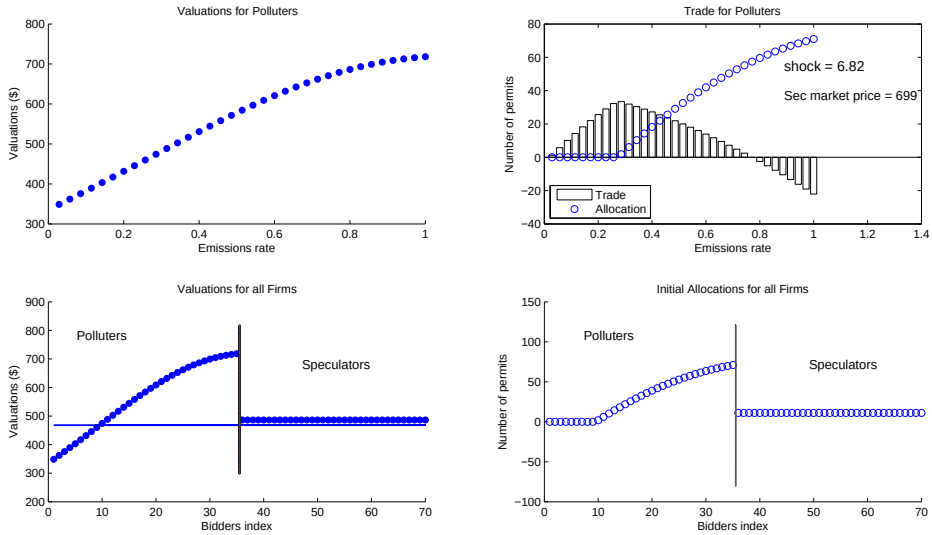

Figure 3: Heterogeneity in emissions rates, scenario with speculators: $\overline{E} = 1500$, $N = 35$, $M = 35$, $\sigma = 5$, $\rho_F = 0.02$, $\rho_S = 0.01$, $b = 3$, $\theta = 100$, $\alpha_f = \alpha_s = 2$, $\gamma_f = \gamma_s = 1000$, $k_f = f/N$

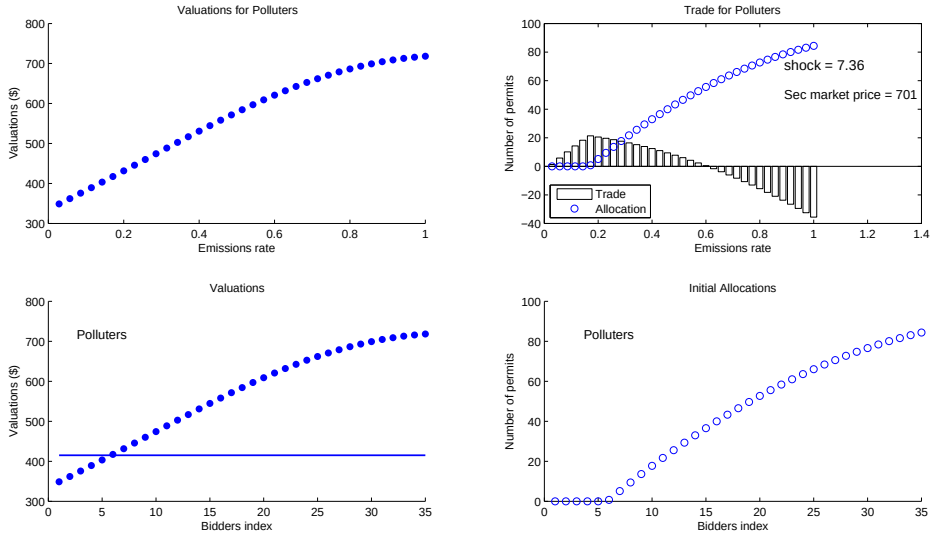

Figure 4: Heterogeneity in emissions rates, scenario without speculators: $\overline{E} = 1500$, $N = 35$, $M = 0$, $\sigma = 5$, $\rho_F = 0.02$, $b = 3$, $\theta = 100$, $\alpha_f = 2$, $\gamma_f = \gamma_s = 1000$, $k_f = f/N$

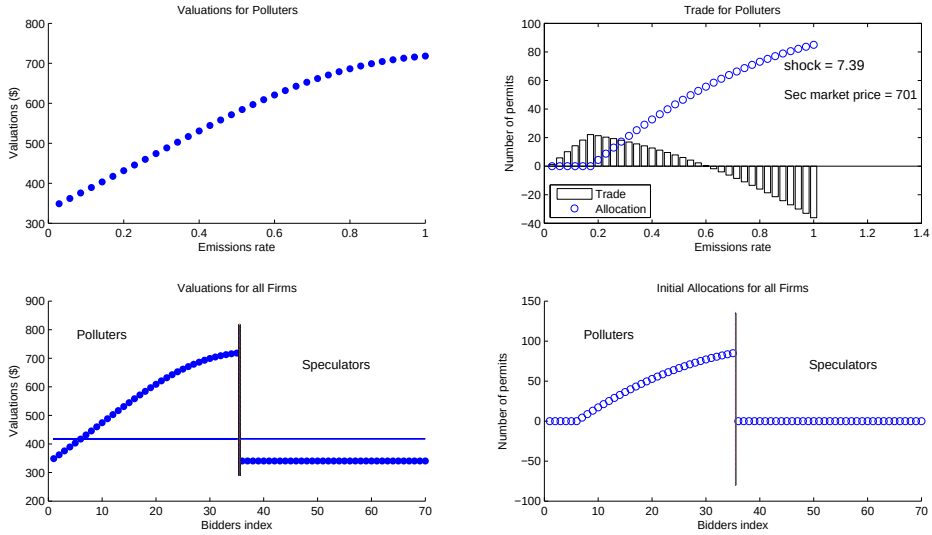

Figure 5: Heterogeneity in emissions rates, equal risk aversion: $\overline{E} = 1500$, $N = 35$, $M = 35$, $\sigma = 5$, $\rho_F = \rho_S = 0.02$, $b = 3$, $\theta = 100$, $\alpha_f = \alpha_s = 2$, $\gamma_f = \gamma_s = 1000$, $k_f = f/N$

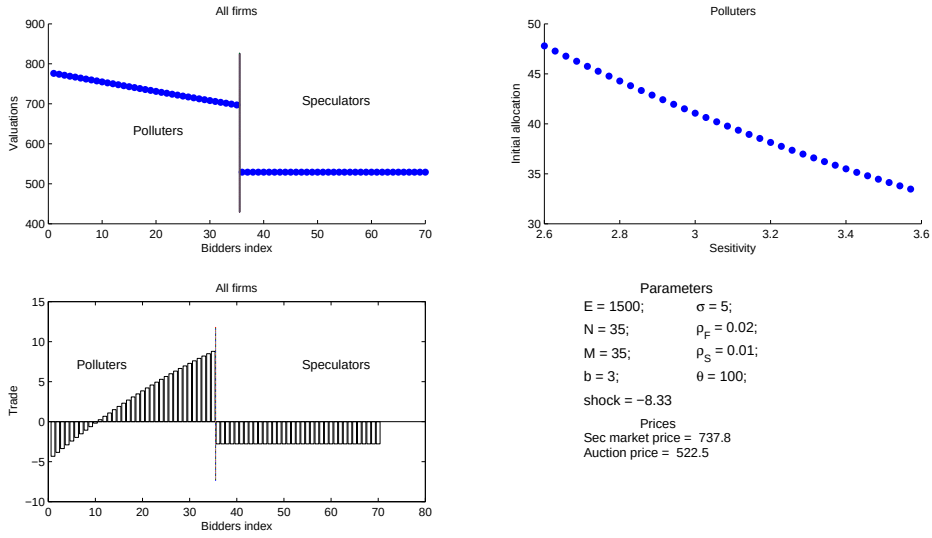

Figure 6: Idiosyncratic shocks for polluters - scenario with speculators: $\alpha_f = (f + 90)/N$, $\alpha_s = 2$, $k_f = 0.9$, $\gamma_f = \gamma_s = 1000$

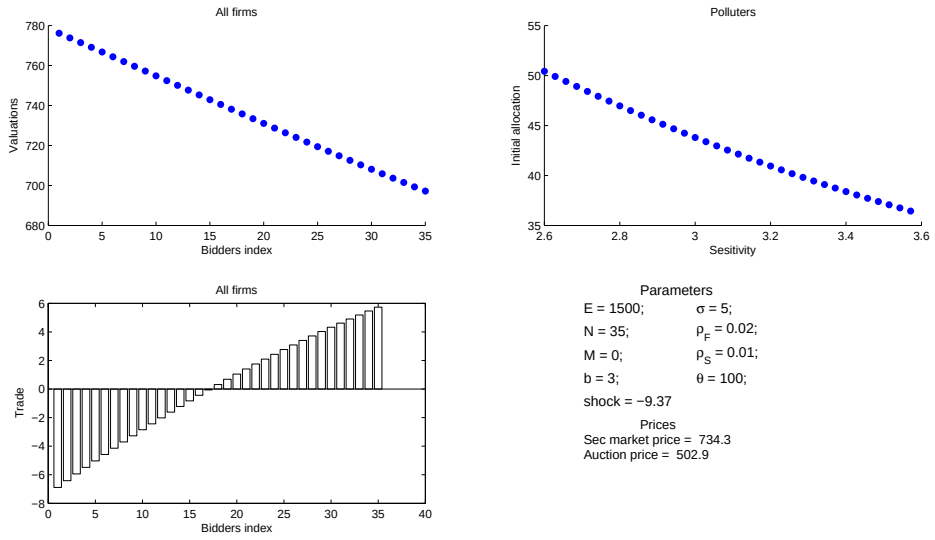

Figure 7: Idiosyncratic shocks for polluters - scenario without speculators: $\alpha_f = (f + 90)/N$, $k_f = 0.9$, $\gamma_f = 1000$

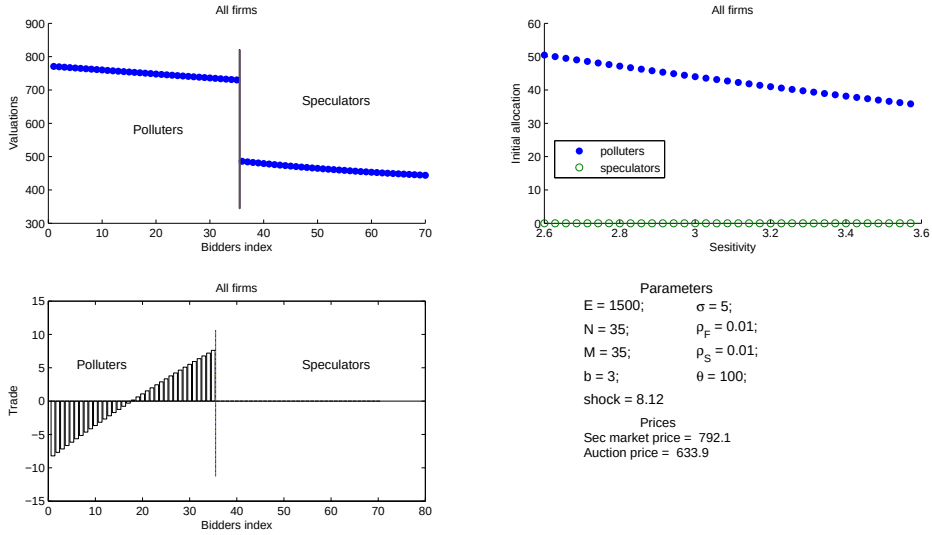

Figure 8: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.01$, $\rho_S = 0.01$, $k_f = 0.9$

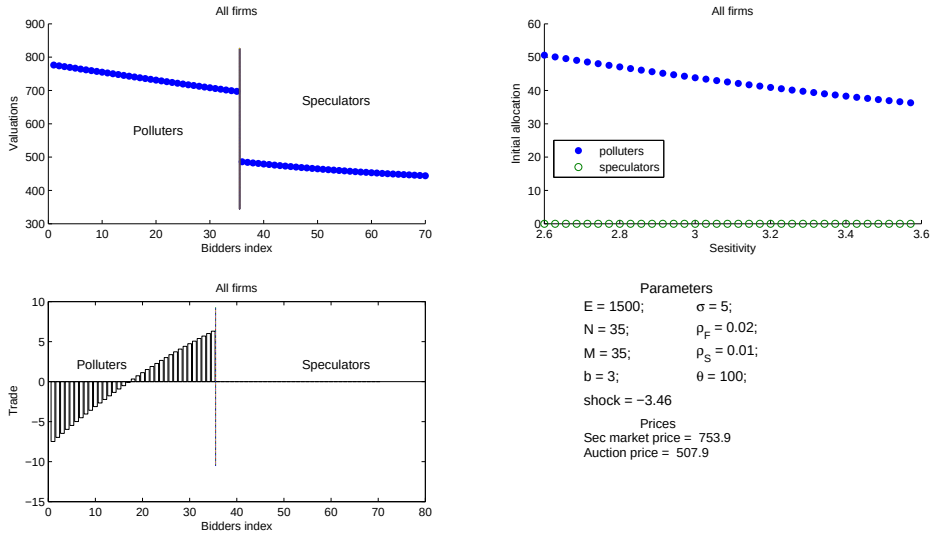

Figure 9: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.02$, $\rho_S = 0.01$, $k_f = 0.9$

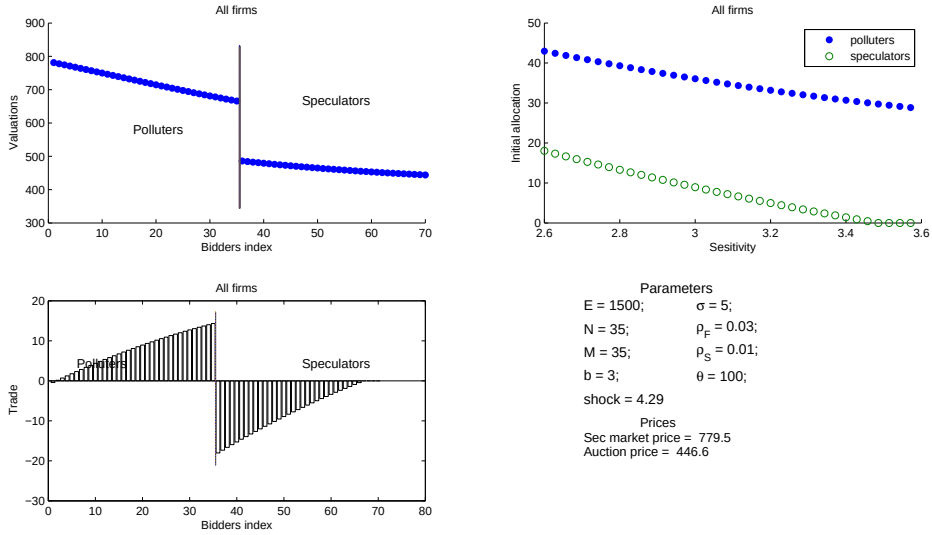

Figure 10: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.03$, $\rho_S = 0.01$, $k_f = 0.9$

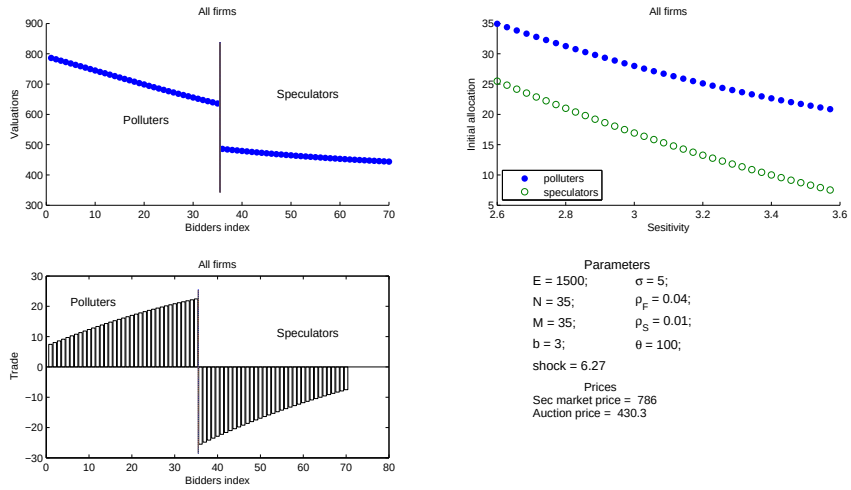

Figure 11: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.04$, $\rho_S = 0.01$, $k_f = 0.9$

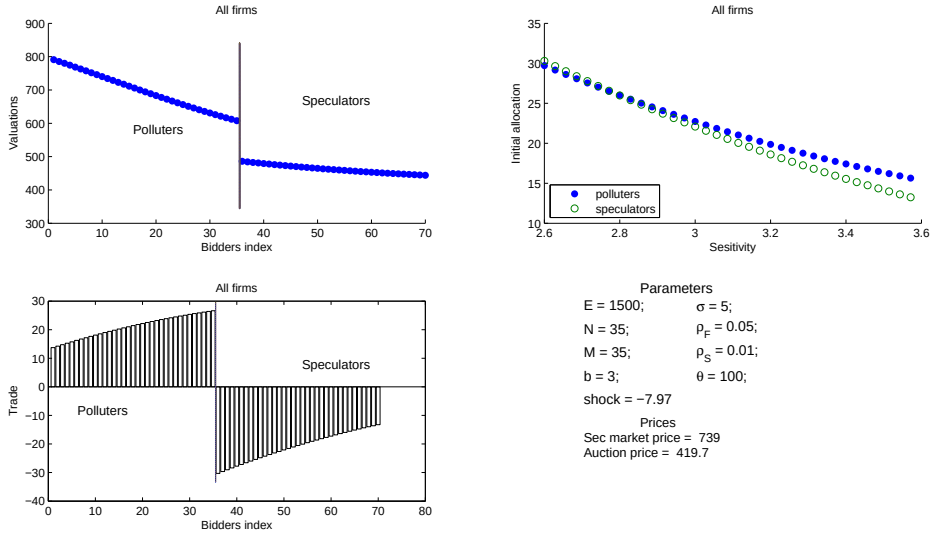

Figure 12: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.05$, $\rho_S = 0.01$, $k_f = 0.9$

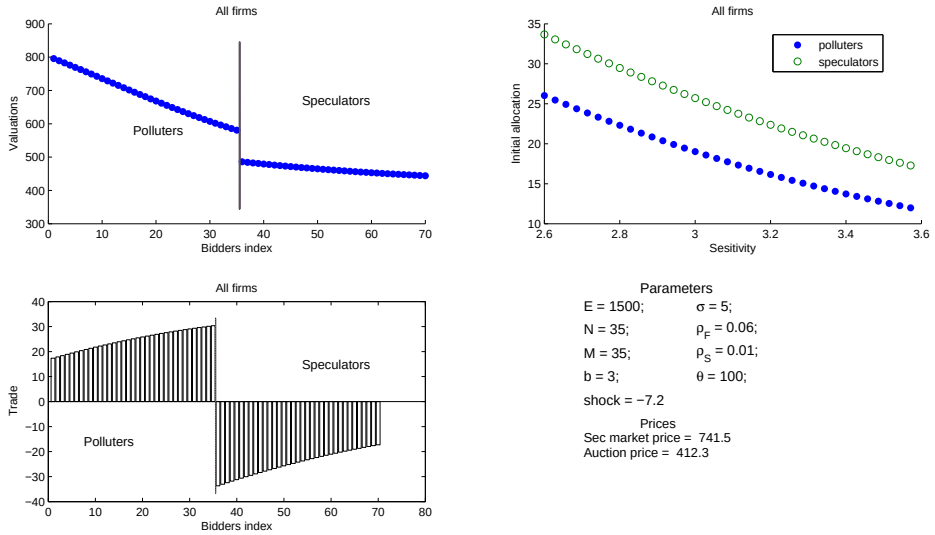

Figure 13: Idiosyncratic shocks for all firms: $\alpha_f = (f + 90)/N$, $\alpha_s = (s + 90)/M$, $\rho_F = 0.06$, $\rho_S = 0.01$, $k_f = 0.9$

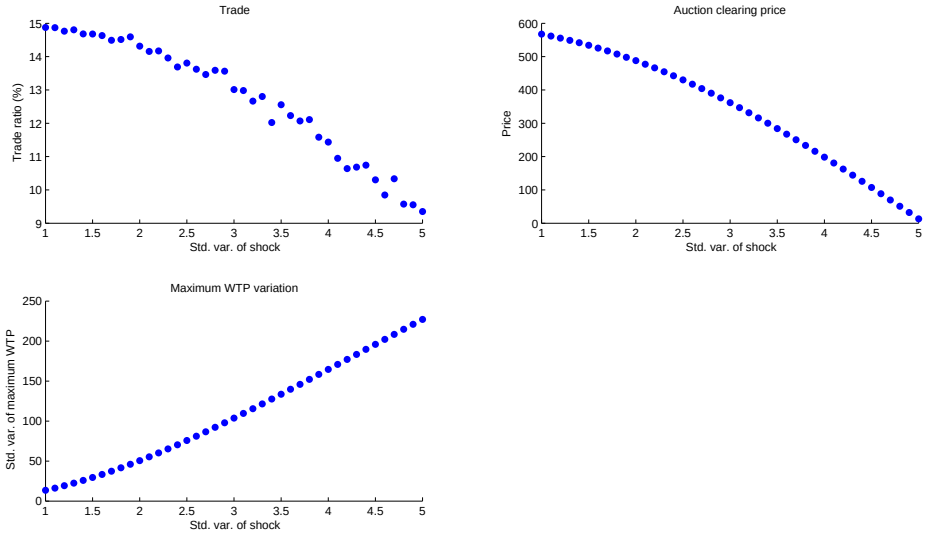

Figure 14: The effect of shock volatility on trade and auction with homogeneous responses to the shock: $N = 50$, $\rho_F = 0.02$, $k_f = f/N$, $\alpha_f = 3$, $\gamma_f = 1000$, $\overline{E} = 2400$, $\theta = 100$, $b = 3$

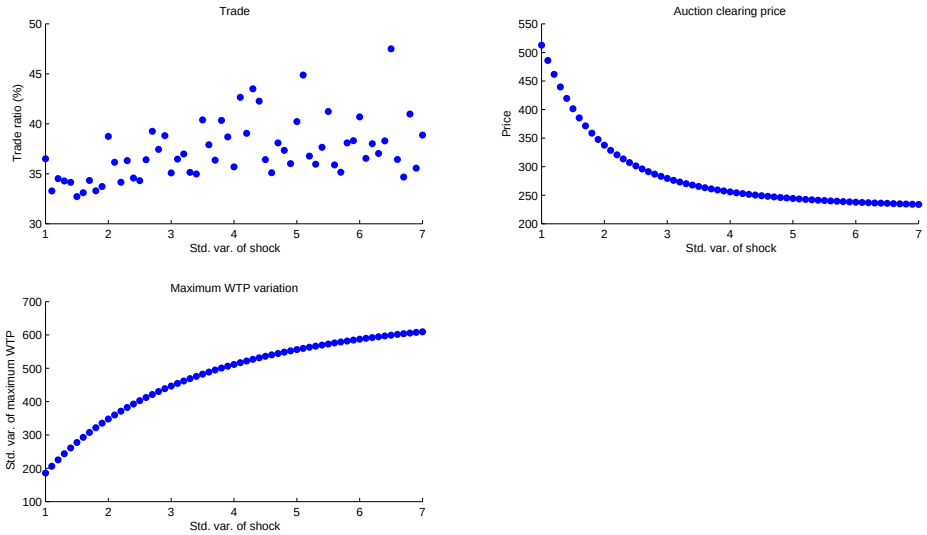

Figure 15: The effect of shock volatility on trade and auction with heterogeneous responses to the shock: $N = 50$, $\rho_F = 0.02$, $k_f = 0.8$, $\alpha_f = f^2/N$, $\gamma_f = 1000$, $\overline{E} = 2400$, $\theta = 100$, $b = 3$

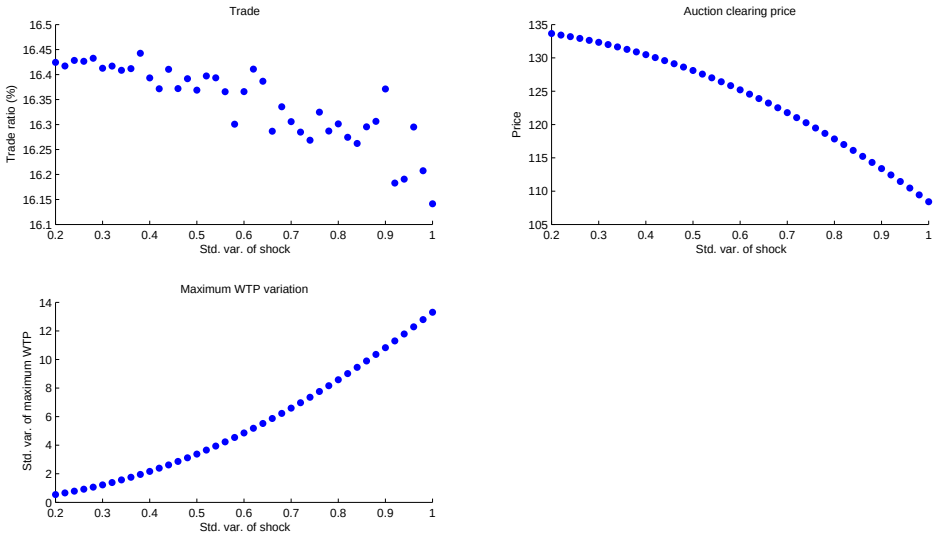

Figure 16: The effect of shock volatility on trade and auction with heterogeneous responses to the shock: $N = 50$, $\rho_F = 0.02$, $k_f = f/N$, $\alpha_f = 3$, $\gamma_f = 1000 - 10f$, $\overline{E} = 2400$, $\theta = 100$, $b = 3$