

**Public Policy and Growth in Canada:**  
**An Applied Endogenous Growth Model with Human and**  
**Knowledge Capital Accumulation\***

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## **Abstract**

We analytically investigate and assess the interactions between knowledge driven growth, acquisition of human capital, and the role of strategic public policy for the Canadian economy within the context of a general equilibrium model. For the purpose of characterization of a fully advanced economy, sources of economic growth are endogenized through the simultaneous interaction of two activities: technological innovations and human capital formation. Both of these activities have cross spill-over effects onto each other. With the aid of the modeling framework utilized in this paper, we aim to contribute to the intense discussion on “Canada’s Innovation/R&D puzzle” from a macroeconomic perspective, with a focus on the role of strategic public policies. To this end, we investigate alternative public policies aimed at fostering the development of human capital (investment in education, learning, basic research) and those at enhancing total factor productivity through investments in innovation; and study the impact of these public subsidization policies on patterns of growth, investment and capital accumulation, social welfare, and burden on government budget. Our results suggest that the re-allocation effects triggered by public subsidization policies on higher education and, alternatively on Industry/business R&D might be contributing to the current discussion that Canadian economy is falling short of it’s potential in (business) technological innovation and R&D production.

Keywords: Endogenous growth, education, R&D, general equilibrium, taxation

JEL Classification Codes: O41, O51, O30, O15, H20

## **1 Introduction**

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It is a well-established fact, which was first put through by Solow (1956), that a growth model which solely depends on accumulation of physical capital is bound to be constrained by the diminishing returns. Recent advances in the “new growth theory” identify and emphasize the roles of research and development (R&D) activities and accumulation of human capital as the key determinants in explaining disparity across countries in income per capita, productivity, and the rate of growth. Investment in education (high quality labor) directly elevate the the productivity of the labor force, and provide significant externalities for growth. As well, R&D activities conducted by both private and public sector raise the available knowledge level of the economy and elicit capital accumulation. Thus, economic growth is fed by two sources which nourish each other: education and R&D capital accumulation. For a fully industrialized economy, both practices have cross spill-over effects onto each other.

Such crucial roles attributed to research and development activities and accumulation of human capital in explaining economic growth have led to construction of economic models which allow for limitless growth of per capita income, and in which long run performance depends on structural parameters and domestic and foreign fiscal policies. Some theories consider capital accumulation, which became a broader concept with the inclusion of human capital, as the engine of growth. (Jones and Manuelli, 1990, King and Rebelo, 1993, and Rebelo, 1991). Another approach attributes a leading role to externalities in growth process. Each firm’s physical (Arrow, 1962) and human (Lucas, 1988) capital investment unintentionally contributes to the productivity of other firms’ capitals. Pioneered by Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), a third approach focuses on the effect of human capital on economic growth by triggering technological development and adoption of new technologies. The new growth literature that follows

the paths of the above mentioned literature, developed models in which private industrial development, capital variety production, and technical skill dispersion lead to growth, depending on the importance of representation of knowledge-led economic conditions. In a paper that trace the stages of development for an economy, Funke and Strulik (2000) show that following the chronology of contributions to the theory of economic growth, one can actually come up with the representation of different stages of development: the first stage defining growth based on physical capital accumulation subject to diminishing returns, and the last stage defining an innovative economy with accumulation of human capital. A series of empirical and theoretical models have also been developed that attempt to reconcile Romerian/Krugmanesque R&D-driven growth along with Lucasian human capital formation. It is by the contribution of these studies including Arnold (1998), Riberio-Thompson (2009), Sequeira (2008, 2011), Keller (2005) and Galor (2005) that the cross spill-over effects of R&D production and human capital formation are well-pronounced for an advanced economy.

Such contributions of course, bring the issues of innovation, R&D production, human capital formation and optimal design of public policies that takes into account the interaction among the mechanisms that contribute to the generation of economic growth. Barro (1991), Tanzi and Chu (1998) and Jung and Thorbecke are among the studies that emphasize the importance of both the size and efficiency of public education expenditures in improving economic growth. Following a similar path, Kim (1998), in an endogenous growth model with financial, physical and human capital that is calibrated both to the US and the East Asian NIC economies, evaluates the contribution of different taxation schemes on growth rate differences. Sener (2008), in an R&D driven endogenous growth model investigates the

effectiveness of R&D subsidy vs. taxation policies to promote growth. Such contributions emphasize the role of public policies in terms of investments in R&D and in education, fiscal debt management, and the inter-household and inter-generational burden of taxation to the fore.

The main purpose of this study is to analytically investigate and assess the interactions between knowledge driven growth, acquisition of human capital, and the role of strategic public policy for the Canadian economy within the context of an endogenous growth, general equilibrium model. We investigate alternative public policies aimed at fostering the development of human capital (such as investments in education and learning) and those at enhancing total factor productivity through investments in innovation (such as subsidies to R&D); and study the impact of various public policies on patterns of growth, along with their likely consequences from the points of view of per capita income growth, social welfare, burden to government budget and economic efficiency.<sup>1</sup>

The analytical model simulates the “production - creation of income- and demand generation” components of the national economy under market constraints in applied general equilibrium context. In the model, four production industries, labor markets that consist of

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<sup>1</sup>Clearly, the potential determinants of long run growth are numerous and a single model, based on the experience of a selected number of countries cannot capture all of the long run dynamics of the history of real world economies. For example, in his review of the growth experience of the East Asian countries, Stiglitz (1996) suggests that the determinants of growth are generally caused by a host of market failures that vary by country and by the level of development. This view implies that models focusing on a single or narrowly based determinant of growth are unlikely to explain the experience of a large number of countries. Keeping in mind the gulf that still appears to exist between the various theories of growth and the lack of empirical evidence to support one category of theory over another, it is nevertheless possible to empirically explore the effects of human capital formation, technological spillovers and the production of capital varieties on growth. In this context, attention can also be focused on the extent to which a decentralized market economy provides adequate incentives for the accumulation of production technology, and how variations in economic structures, institutions and public policies might translate into different rates of productivity gains.

formal (human capital) and informal labor force, and public sector balances are decomposed by means of algebraic equations. Production process is portrayed as an augmented Cobb-Douglas type of production function that utilizes both skilled (human capital) and unskilled labor and physical capital varieties. Industrial production increases with accumulation of physical capital. Physical capital becomes available through knowledge capital (R&D). Knowledge capital investments are performed by oligopolistic entities and oligopolistic profits are used to finance R&D investments. In the meantime, fixed costs enable increasing returns to scale in physical capital accumulation and allow growth process to be sustained endogenously.

As a result of the research activities at available knowledge level, stock of differentiated capital expands; in other words, with technological improvement, varieties of differentiated capital goods raise. Each “*intermediate capital input*” is obtained as a result of R&D activity or associated with a patent or blueprint. Technological spillover effects can be generated by human capital acquisition and R&D activities through “*learning via varieties*”, rather than physical capital investment.

Furthermore, accumulation of knowledge capital depends on the production of human capital. Accumulation of human capital is solved endogenously by inter-household dynamic inter-temporal consumption optimization behavior; and nourished by externality effects of both R&D production and public expenditures on education.<sup>2</sup> Thus, three main forces that

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<sup>2</sup>Externality effect from R&D to human capital has been touched upon in literature by Eicher (1995) and Keller (1995). Galor (2005) also emphasizes the idea of technology complementing with skills in the production of human capital or contact with technology through accumulated human capital. As a theoretical contribution, unlike the standard endogenous growth models where the steady state growth rate is not affected by the level of innovative activities but solely on human capital variables, the model of learning with existing knowledge generates a steady state growth rate affected by the level of R&D (relative to the level of human capital stock) in the economy. See Sequeira(2008, 2011) and Appendix for further derivations.

affect the path of economic growth emerge: Knowledge capital accumulation, human capital accumulation, and public expenditures on the accumulation of these factors. While first two depend on rational optimization behavior of private investors under market constraints, the last one is determined by the medium/long run expenditures of a rational government to provide stimulus to R&D and education (human capital) investments. Thus, the macroeconomic general equilibrium model used in this study has a unique approach that combines the optimization elements of the private sector and strategic growth objectives of the state. Determination of the optimal public policy tools that enable internalization of these externalities and their relative efficiency lie at the main focus of this study.

We calibrate the model to the “Canadian economy” and is solved both for the transition and the steady-state path of the economic variables under the general equilibrium setting. Hence, to the best of our knowledge, our model is the first model to be calibrated to a real economy, that is capable of internalizing both the effect of human capital on the production of R&D as well as the feedback effect of R&D, or the stock of knowledge on human capital production. We also explicitly model the government accounts to be able to have a well-defined platform to compare the effects of alternative scenarios on the key variables of the economy. Calibrated to the Canadian economy, the model follows Russo (2004), Ghosh (2007) and Annabi, Harvey and Lan (2008) in analyzing alternative policies to promote growth under an endogenous growth setting within the Canadian context.

Remaining pages of this paper are designed in five sections. In the second section, we present R&D and human capital data, and provide a synopsis of the discussion on the characteristics of the “Canadian innovation puzzle”. Analytical and algebraic set up of the

model is presented in the third section, while policy analyses are conducted in the fourth section. In the fifth section, we summarize the main findings of the study and conclude. Data set and calibration strategy of the algebraic model are introduced in a separate Appendix section in deeper detail.

## **2 Growth with respect to R&D and Human Capital Accumulation in the Canadian Economy: Facts and Figures**

It has been well documented by now that increasing the rate of productivity growth is the key to achieving sustainable increase in living standards, and innovation and human capital accumulation are keys to achieve productivity improvements. Yet, there still lies significant diversity over the best policies to encourage the keys to achieve productivity improvements. It is the concern of this paper to investigate the accumulation and the interaction between the accumulation of human capital and R&D, their interaction and cross-spillover effects and the role of strategic public (subsidization) policies within the Canadian context.

Canada's "Innovation/R&D Puzzle" is now a phenomenon that has raised concerns and has been intensively discussed for decades in both *policy* and academic circles. Based on the comparative OECD data, reports by Science, Technology and Innovation Council<sup>3</sup> emphasize that Canadian economy (with respect to the OECD, First World or G-7 countries, based on the selected indicator) has been in a "low ranking" position in terms of performance in R&D in general; but especially show "low ranking" in Industry/business expenditure on R&D, percentage of total R&D performed by Industry/business and Industry/business

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<sup>3</sup>*State of the Nation* (2008, 2010).

investment in machinery and equipment (Table 1). On the other hand, Canada has been within the “high ranking” group in basic research performance, R&D conducted by universities, and education level of its workforce (but with “low ranking” in the number of advanced degree graduates in the Industry/business).

Table 1 reveals that, according to the data compiled from OECD, Canada’s Gross Domestic Expenditures on R&D (GERD) to GDP ratio lies on average at 1.99% in 2003-2009 period.<sup>4</sup> This ratio is lower than OECD average and higher than the EU-15 average. But, it should also be noted that the GERD/GDP ratio for Canada was 2.04% in 2003 and with a steady decrease, it dropped down to 1.96% in 2007 and to 1.87% in 2008. Concomitantly, the GERD/GDP ratio for the EU-15 has been increasing steadily, reaching to 2.05% in 2009.

What turns out to be noteworthy in terms of the discussion on the Canadian growth path is that, compared to the OECD and the EU-15 averages, the share of Industry/business sector in both total R&D expenditures (BERD/Total R&D Exp.), and in total R&D production ( R&D Performed by Industry-business/ Total R&D Performed) is relatively lower in Canada. With reference to studies that emphasize the role of the increased BERD intensity on growth<sup>5</sup>, most commentators interpret that such lower numbers contribute to lower innovation activity, lower R&D levels, lower investment in machinery and equipment, and “lower than potential” performance of the Canadian economy (Munroe-Blum and Mackinnon, 2009; Stewart, 2011).

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<sup>4</sup>OECD Main Science and Technology Indicators Database.

<sup>5</sup>OECD (2003) estimates that an increase in the BERD intensity of 0.1% raises the real output per capital by 1.2%.



One other major point to note within the Canadian context is that not only government support to Industry/business R&D is high, but government itself is a major producer of R&D, especially if one takes into account the R&D produced by Higher Education sector (Table 3).<sup>6</sup> The average percentage of R&D undertaken by the Higher Education sector in Canada in 2003-2009 is 34.6%. This level is almost 17 percentage points higher than the OECD average and 12 percentage points higher than the EU-15 average for the same period. Similarly, in 2005-2008 period, Canada's government funding to Industry/business R&D was higher than that of US and in 2008, it was the second highest among the OECD countries (Table 1). Yet, it has also been much emphasized that almost 90% of this funding is indirect, composing of rather compound tax incentives (such as Scientific Research and Experimental Development Tax Incentive Program), whereas in the US, 80% of government support is direct government funds of Industry/business R&D activities.<sup>7</sup> Such observation is considered as another contributor to the "lower than potential" performance of the Canadian economy in terms of technological innovation and R&D production.

Finally, for the purpose of focusing on the comprehensive set of mechanisms that contribute to the generation of and setting in the dynamics of economic growth for a fully industrialized economy, as represented in this paper by the accumulation of human capital, accumulation of R&D and their cross spill-over effects, data on human capital for the Canadian economy should also be mentioned. Here, it should be highlighted that in terms of human capital as a factor of production, and as a critical resource for the

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<sup>6</sup>One should note that most universities engaged in R&D are publicly funded and overseen by federal, provincial or local government in Canada.

<sup>7</sup>Science, Technology and Innovation Council, *State of the Nation* (2008).

interaction with and the absorption of R&D for a fully industrialized economy, Canada has been highly prosperous. Human capital, of course, is a resource that requires increased and sustained investment for achieving increased and sustained living standards. Canada has been identified with having one of the best educated workforces in the world and has been leading the OECD economies in those aged 25-64 who have completed some form of education (Table 2). However, it has also been noted that despite this first ranking in educated labor force, the employment of high-quality labor in Industry/business sector is relatively low (Munroe-Blum and MacKinnon, 2009).

In a nutshell, Canada’s relative position in terms of the modes and mechanisms of economic growth for a fully advanced country is identified by low ranking in Industry/business expenditure and performance of R&D, middle ranking in Industry/business investment in machinery and equipment, high ranking in human capital (with lower number of advanced degree graduates in Industry/business) and high ranking in government support to R&D (with high reliance on indirect mechanisms in its support to Industry/business R&D). In this paper, we aim to contribute to the debate around such figures, by analyzing alternative government (subsidization) policies aimed at fostering development of human capital (investment in education and learning) and enhancing investment in R&D. To this end, and briefly within the specifics of our model, we organize this study around the most conducive questions concerning public subsidization policy for enhancing growth and social welfare: promotion of human capital formation through subsidies to education expenditures or promotion of Industry/business R&D through (direct) subsidies to R&D investment) and the role of re-allocation effects on human capital triggered by such policies, within the context of “Canada’s Innovation/R&D puzzle”.

### 3 Model Structure

The model is a direct application of the recent advances in the new growth theory literature and is built on the complementarities between R&D-driven technological change and human capital acquisition. The model is presented in five sub-sections, starting with the final output production, concluding with the conditions for equilibrium and discussion of the macroeconomic identities.

#### 3.1 Production Activities

The economy is presumed to be open, and is *small* in the world markets. It accommodates four activities in the aggregate, three of which are *production* activities: (i) production of a final good,  $Y$ ; (ii) production of capital input varieties,  $k(i)$  to be used as inputs in the production of  $Y$ ; and (iii) production of R&D (blueprints, ideas, etc..). A final activity further entails education services (human capital formation).

Final output is produced using plain labor,  $L^Y$ , human capital (skilled labor),  $H^Y$ , and differentiated capital varieties as inputs:

$$Y_t = A_Y L_t^{Y\alpha_L} H_t^{Y\alpha_H} \sum_{i=0}^{A_t} k_t(i)^{\alpha_k} \quad (1)$$

with  $\alpha_L + \alpha_H + \alpha_k = 1.0$ . All differentiated capital varieties are of equal quantity and are valued equally. They are produced by symmetric firms (each capital variety is produced by a single oligopolist firm). That is,  $k_t(i) = k_t$  for all  $i = 1, \dots, A_t$ . Therefore, we have at any moment,  $\sum_{i=1}^{A_t} k_t(i)^{\alpha_k} = A_t k_t^{\alpha_k}$ .

Note that the  $Y$  -sector uses  $L_Y$ ,  $H_Y$ , and a series of inputs  $\{k_1 \dots k_A\}$ ; where  $\{A\}$  is the index of varieties of capital inputs available to this economy. As new research is conducted, the index set  $\{A\}$  expands. Following the idea in Funke and Strulik (2000) and Sequeira (2008) this is achieved in the R&D sector as follows:

$$A_{t+1} - A_t = \varphi H_t^A \quad (2)$$

New research is generated solely by human capital allocated to the production of new ideas (research personnel),  $H_A$  and excludes decreasing returns as well as the scale effects of  $A^8$ . The research productivity of each researcher is a factor  $\varphi > 0$ . In what follows, an additional driving source of this economy is the rate of human capital formulation:

$$H_{t+1} - H_t = \xi H_t^H + \gamma H_t^\epsilon A_t^{1-\epsilon} \quad (3)$$

In (3) human capital is a *non-market activity* and is thought to be “produced” via human capital allocated to education,  $H_H$ , and existing stock of ideas  $A$ . Past accumulation of human capital is also necessary to generate further human capital.

Generation of  $H$  is the end-result of schooling ( $\xi H^H$ ) where the parameter  $\xi$  acts as the productivity of schooling and sets the incentive to spend time in education. Sequeira (2008) refers the second term on the right hand side as “learning with varieties” since it is a composite of the stock of human capital and the existing knowledge (ideas) in the economy. This effect is driven by a productivity parameter,  $\gamma$ , which measures the relative importance of “learning with existing knowledge”. The elasticity parameter  $\epsilon$  measures the intensity of human capital to capture the existing knowledge.

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<sup>8</sup>Such a specification rather than the more general form  $A_{t+1} - A_t = \varphi H_t^A A_t$  as in Romer (1990), where the R&D production function admits positive externalities through past research, helps to ensure the steady state.

As human capital expands, research workers keep on producing new ideas at a constant speed. The growth rate of knowledge production,  $g^A$  becomes,

$$g_t^A = \frac{A_{t+1} - A_t}{A_t} = \varphi \frac{H_t^A}{A_t} \quad (4)$$

and remains constant under steady state when the share of human capital allocated to research,  $u_t^A = \frac{H_t^A}{H_t}$ , stabilizes. So, defining  $H_{t+1}/H_t = (1 + g_t^H)$ , growth rate of human capital becomes:

$$g_t^H = \xi \frac{H_t^H}{H_t} + \gamma \left( \frac{A_t}{H_t} \right)^{1-\epsilon} \quad (5)$$

At the balanced growth path,  $g_t^H$  is constant as long as the ratio of total available number of ideas to the stock human capital remain fixed. These formulations further necessitate that a steady state solution with a constant rate of growth requires a constant allocation of  $H_t$  along its components. This means that, under long run equilibrium, infinitely-lived people will dedicate in each period a constant amount of time-share between working and schooling.

The final good sector works under perfectly competitive conditions. The producer hires both types of labor and the capital varieties up to the point where the value of the marginal product of each factor is equated to its wage and rental costs, respectively. Therefore, labor is demanded according to

$$w_t^L = P_t^Y \frac{\partial Y_t}{\partial L_t^Y} \quad (6)$$

Human capital demand is similar

$$w_t^H = P_t^Y \frac{\partial Y_t}{\partial H_t^Y} \quad (7)$$

Capital varieties are demanded along the functions,

$$p_t^k(i) = P_t^Y \alpha_k L_t^{Y\alpha_L} H_t^{Y\alpha_H} k_t(i)^{\alpha_k-1} \quad i = 1, \dots, A_t \quad (8)$$

In the R&D sector, given public subsidies on R&D costs, human capital is demanded so as to satisfy its marginal productivity condition:<sup>9</sup>.

$$w_t^H = P_t^A \varphi \quad (9)$$

Note that, competitive conditions in factor markets necessitate that wage costs of human capital are equated across its uses in the R&D sector and in the final goods production sector. Thus,  $w_t^H = P_t^A \varphi = P_t^Y \alpha_H \frac{Y_t}{H_t^Y}$ .

### 3.2 The differentiated capital and investment decision

“Capital” is modeled here as a heterogenous input which accumulates by the varieties,  $k(i)$ . The intermediate firm purchases ‘*blueprints*’ (the technological knowledge generated in the R&D sector) and according to the instructions therein, produces a new capital variety. The number of new capital varieties produced at period  $t$  is equal to the number of new blueprints produced in the same period,  $A_t$ . Ignoring depreciation, the number of accumulated capital varieties in the economy at time period  $t$  is equal to the number of blueprints available in the economy. Each new capital input  $k(i)$  is produced by using real resources and other

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<sup>9</sup>In case of an R&D subsidy, the equation becomes:  $w_t^H = \frac{P_t^A \varphi}{(1-s^R)}$ , where  $s^R$  represents the subsidy rate to accumulate human capital in the R&D sector

inputs at a constant ratio,  $\eta$ , where  $\eta$  acts as the ‘*input-output coefficient*’ to produce one unit of  $k(i)$ . Costs of  $\eta$  is the rental price,  $r$  –the interest rate in this economy.

Now, observe that as the intermediate producer has purchased the R&D blueprints she had incurred the upfront fixed costs of research. These research costs totaling  $P_t^A$ , have to be borne up front by the intermediate capital variety firm. Thus, the expression  $P_t^A \Delta A_t$  becomes the fixed costs of production of  $k_t(i)$ , and leads to increasing returns in its production. Since the  $i$ -th firm has monopoly rights in the production of  $k_t(i)$ , it acts monopolistically in the capital goods market. Taking the demand function for  $k_t(i)$  from the final good producer (8) as given, each monopolist seeks to maximize the monopoly profits,

$$\max_{k_t(i)} \pi_t(i) = p_t^k(i) \cdot k_t(i) - \eta r_t k_t(i) - P_t^A \Delta A_t \quad (10)$$

In (10) the term  $\eta r_t k_t(i)$  is the variable costs of production. For each unit of  $k_i$  produced  $\eta$  units of other inputs are rented out at the interest rate  $r_t$ . The solution of (10) reveals that the profit maximizing price  $p_t^k(i)$  is given by a ‘*mark-up*’ over the marginal costs,  $\eta r_t$ . Using the demand for  $k_t(i)$  from the final good producer’s decision we have the following optimal pricing rule for the monopolist:

$$P_t^Y \alpha_k^2 L_t^{Y\alpha_L} H_t^{Y\alpha_H} k_t(i)^{\alpha_k-1} = \eta r_t$$

Therefore, optimal quantity of the capital variety is set via:

$$k_t(i) = \left[ \frac{P_t^Y \alpha_k^2 L_t^{Y\alpha_L} H_t^{Y\alpha_H}}{\eta r_t} \right]^{\frac{1}{1-\alpha_k}} \quad (11)$$

The size of the monopolistic mark-up is  $1/\alpha_k$ .

$$p_t^k(i) = \frac{P_t^Y \eta r_t}{\alpha_k} \quad (12)$$

Since all firms are symmetric and they all set the same price (12) to sell their respective capital varieties, we will take  $p_t^k(i) = p_t^k$  and  $k_t(i) = k_t, \forall i$ . Under these conditions the maximal profits are given by

$$\pi_t^{\max}(k_t) = p_t^k \cdot k_t - \eta r_t \cdot k_t = (p_k - \eta r_t)k_t \quad (13)$$

Since  $r_t = \frac{\alpha_k p_t^k}{\eta}$  from above, we can express maximal profits of the monopolists as,

$$\pi_t^{\max}(k_t) = (1 - \alpha_k)p_t^k k_t \quad (14)$$

The monopoly firms have a forward-looking behavior. That is, they make investment decisions on developing new blueprints and producing new capital varieties so as to maximize the long-run expected returns from an infinite stream of monopoly profits. In particular, the expected returns from investment must be comparable with those from holding a “safe” asset such as bonds or bank deposits. Thus, asset market equilibrium requires, for any point in time, that the following non-arbitrage condition holds:

$$\pi_t + (P_t^A - P_{t-1}^A) = r_t P_{t-1}^A$$

where the term  $(P_t^A - P_{t-1}^A)$  denotes changes in the valorization of the  $i$ -th firm over time. In equilibrium, the value of the firm is equal to aggregate investment expenditures of this firm, which includes the cost of developing a new blueprint ( $P_t^A$ ), plus the material costs of investment goods. Imposition of the transversality condition to rule out speculative bubbles gives:

$$P_t^A = \sum_{t=0}^{\infty} R(t)\pi_t$$

that is, the value of the monopoly firm is equal to the discounted value of the stream of



monopoly profits, where  $R(t)$  is a discount factor defined according to

$$R(t) = \prod_{s=0}^t (1 + r_s)^{-1}$$

Note that, the above *no-arbitrage* condition can also be expressed more succinctly as,

$$(1 + r_t)P_{t-1}^A = \pi_t + P_t^A \quad (15)$$

Investment expenditures in this model, are used in generating new research and producing new capital varieties:

$$I_t^D = \eta[(A_{t+1} - A_t)k_t + (k_{t+1} - k_t)A_t] \quad (16)$$

### 3.3 Consumption and savings decisions

Households are endowed with human capital,  $H_t$  each period, and decide to allocate it among three uses, final good production, knowledge production and further human capital formation:

$$H_t = H_t^Y + H_t^H + H_t^A \quad (17)$$

where  $(H_t - H_t^Y)$  is associated with a wage rate  $w_t^H$  and  $H_t^H$  may be subsidized through  $s^H w_t^H$ , with  $s^H > 0$ . The representative household maximizes a utility function of the form:

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta} - 1}{1-\theta} \quad (18)$$

subject to

$$\begin{aligned}\sum_{t=0}^{\infty} R(t) P_t^C c_t &= TW_0 \\ H_{t+1} - H_t &= \xi H_t^H + \gamma H_t^\epsilon A_t^{1-\epsilon}\end{aligned}$$

with control variables  $c_t > 0$  and  $H_t^H \geq 0$ . Here,  $TW_0$  is the total wealth, which includes the present value of period-wise income.  $Y_t^H = (1 - t^Y)[w_t^H(H_t - H_t^H) + s^H w_t^H H_t^H + w_t^L L_t^Y + p_t^k k_t A_t]$  is the private household disposable income composed of returns to primary factors of production and the value of monopoly firms of capital variety.

For an interior solution, the F.O.C.'s associated with the maximization problem above are twofold:

$$\beta(1 + r_{t+1}) \left( \frac{c_{t+1}}{c_t} \right)^\theta = \frac{P_{t+1}^C}{P_t^C} \quad (19)$$

$$\frac{w_t^H}{w_{t+1}^H} = \frac{1}{1 + r_{t+1}} \left( \frac{\xi}{(1 - s^H)} + 1 + \gamma \epsilon \left( \frac{H_{t+1}}{A_{t+1}} \right)^{1-\epsilon} \right) \text{ with } H^H > 0 \quad (20)$$

The first condition above is the discrete version of the standard Ramsey rule. The second equation implies that the growth rate of wages must be sufficiently high enough compared to the interest rate to ensure positive investment in human capital.

Using  $\frac{w_t^H}{\varphi} = P_t^A$  from (9), we get,

$$\frac{w_{t+1}^H}{w_t^H} = \frac{P_{t+1}^A}{P_t^A}$$

The rate of growth of  $P_t^A$  above is narrated in the *no-arbitrage condition* (15).

Inserting in the equations for  $\pi_t$  and  $P_t^A$  and equating the two expressions for  $\frac{w_{t+1}^H}{w_t^H}$ , gives

us:

$$1 + \frac{1 - \alpha_k}{\alpha_H} \varphi \alpha_k \frac{u_{t+1}^Y H_{t+1}}{A_{t+1}} = \frac{\xi}{(1 - s^H)} + \left( 1 + \gamma \epsilon \left( \frac{H_{t+1}}{A_{t+1}} \right)^{(1-\epsilon)} \right)$$

Now assume that we denote the share of  $H_t$  allocated to final goods production,  $H_t^Y$  as  $u_t^Y$ .

The equation above should provide the value of  $u_{t+1}^Y$ , given  $H_{t+1}/A_{t+1}$  which is critical in terms of the allocation of human capital to different sectors of the economy. It also implies  $u_{t+1}^Y = u^Y$  at the steady state.

### 3.4 Export and import functions and balance of payments

The representative final good producer has the following production possibility boundary between exports,  $E_t$  and domestic sales,  $DC_t$  (the constant elasticity of transformation - CET frontier):

$$X_t = \bar{Z}_X \left( \nu E_t^{(1+\sigma)/\sigma} + (1 - \nu) DC_t^{(1+\sigma)/\sigma} \right)^{\sigma/(1+\sigma)} \quad (21)$$

In equilibrium, the ratio of exports to domestic good becomes:

$$\frac{E_t}{DC_t} = \left( \frac{P_t^E}{P_t^D} \right)^\sigma \left( \frac{1 - \nu}{\nu} \right)^\sigma \quad (22)$$

Import decisions are derived from the *Armingtonian composite commodity* specification, where imports  $M_t$ , and domestic good,  $DC_t$ , are regarded as imperfect substitutes in trade.

$$CC_t = \bar{Z}_{CC} \left( \kappa M_t^{(\psi-1)/\psi} + (1-\kappa) DC_t^{(\psi-1)/\psi} \right)^{\psi/(\psi-1)} \quad (23)$$

In equilibrium the ratio of imports to the domestic good becomes

$$\frac{M_t}{DC_t} = \left( \frac{P_t^D}{P_t^M} \right)^\psi \left( \frac{1-\kappa}{\kappa} \right)^\psi \quad (24)$$

where  $P_t^M = (1+t^m)P_t^{WM}$  and  $P_t^E = P_t^{WE}$  with  $t^m$  representing tariff rate at period  $t$ . The economy has balanced trade in each time period.

### 3.5 National Income Identities and Equilibrium Growth

Intra-temporal equilibrium requires that at each time period, (i) demand for primary factors  $(L_Y, H_A, H_Y)$  equal their respective supplies; (ii) Human capital allocation among Final Good Production,  $Y$ , R&D Production,  $\Delta A$ , and Education,  $\Delta H$  exhausts its total supply; (iii) domestic demand plus export demand for the output of each sector equal its supply; (iv) the output of R&D, that is the number of new blueprints, equal the number of new capital varieties invested; (v) household savings equal investment – costs of new blueprints plus costs of investment goods in capital variety production; (vi) the value of total exports equal the value of total imports; and (vii) the government budget is satisfied. These conditions imply that the commodity market is in equilibrium with

$$CC_t = C_t + G_t + I_t^D \quad (25)$$

Saving investment balance is maintained through:

$$S_t = P_t^C I_t^D + P_t^A \Delta A_t \quad (26)$$

Government's budget is in balance:

$$P_t^C G_t + s^H w_t^H H_t^H + s^R w_t^H H_t^A = GREV_t \quad (27)$$

with government revenues equal to total tax revenues<sup>10</sup>.

Gross domestic product (GDP) at factor cost (exclusive of production taxes) is the sum of value added of the final good, human capital expenditures, and the R&D sectors:

$$GDP_t = P_t^Y Y_t + P_t^A \Delta A_t \quad (28)$$

$$= w_t^L L_t^Y + w_t^H (H_t^Y + H_t^A) + \sum_{i=1}^{A_t} p_t^k(i) k_t(i) \quad (29)$$

Using  $\sum_{i=1}^{A_t} p_t^k(i) k_t(i) = p_t^k A_t k_t$ , which in turn will be equal to  $\alpha_k P_t^Y Y_t$ , the identity in

(29) can also be written as

$$p_t^k A_t \cdot k_t = \alpha_k (GDP_t - P_t^A \Delta A_t)$$

or, using (29),

$$\begin{aligned} [GDP_t - w_t^L L_t^Y - w_t^H (H_t^Y + H_t^A)] &= p_t^k A_t k_t \\ &= \alpha_k P_t^Y Y_t \end{aligned} \quad (30)$$

Furthermore, using the definition of profits from (14), the GDP identity can also be written as:

$$P_t^Y Y_t + P_t^A \Delta A_t = w_t^L L_t^Y + w_t^H (H_t^Y + H_t^A) + A_t \frac{\pi}{(1 - \alpha_k)} \quad (31)$$

In the steady state equilibrium all quantity variables grow at a constant rate which is proportional to the growth rate of human capital formation. All prices, including prices

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<sup>10</sup>Total government tax revenues compose of income tax, consumption tax, production tax and tariff revenues.

for final goods produced and consumed domestically, the unit cost of the R&D output, differential capital varieties, and the interest rate grow at a constant rate in the steady state. Also, the allocation of  $H_t$  among its uses will be constant; hence, given  $H_t^Y = u^Y H_t$ ,  $H_t^A = u^A H_t$  and  $H_t^H = u^H H_t$ , with  $u^A + u^H + u^Y = 1$ .

Based on these specifications, and the growth rates of  $H$ ,  $g_t^H$  and  $A$ ,  $g_t^A$  implies that at steady state  $H_t/A_t$  is constant. Combining the definitions of  $g_t^H$  and  $g_t^A$ , we have  $g^H = g^A$  at the steady state.

We know that  $w_t^H$  in R&D sector is the same as the  $w_t^H$  in the final goods sector. Equating the two, we have:

$$\begin{aligned} P_t^A \varphi &= \alpha_H L_t^{Y^{\alpha_L}} H_t^{Y^{\alpha_H}-1} A_t k_i^\alpha \\ P_t^A &= \frac{\alpha_H L_t^{Y^{\alpha_L}} H_t^{Y^{\alpha_H}-1} A_t k_i^\alpha}{\varphi} \end{aligned}$$

Similarly,

$$\pi_t(i) = (1 - \alpha_k) \alpha_k L_t^{Y^{\alpha_L}} H_t^{Y^{\alpha_H}} k_i^{\alpha_k}$$

Therefore:

$$(1 + g^{PA}) = r_t - \frac{(1 - \alpha_k)}{\alpha_H} \alpha_k \frac{H_t^Y}{A_t} \varphi \quad (32)$$

Since we now have the solution for  $P_t^A$  above, we can also derive the growth rate  $P_t^A$  at the steady state as:

$$(1 + g^{PA}) = (1 + g^H)^{\frac{\alpha_H}{1 - \alpha_k}} \quad (33)$$

Finally, since  $Y_t = A_Y L_t^{Y\alpha_L} H_t^{Y\alpha_H} \sum_{i=0}^{A_t} k_t(i)^{\alpha_k}$ , we have

$$\frac{Y_{t+1}}{Y_t} = \left( \frac{L_{t+1}^Y}{L_t^Y} \right)^{\alpha_L} \left( \frac{H_{t+1}^Y}{H_t^Y} \right)^{\alpha_H} \frac{A_{t+1}}{A_t} \left( \frac{k_{t+1}}{k_t} \right)^{\alpha_k}$$

So at steady state:

$$(1 + g^Y) = (1 + g^H)^{\frac{1 + \alpha_H - \alpha_k}{1 - \alpha_k}}$$

## 4 Policy Analysis: Dynamic Effects of the Selected Public Policies

Now we turn to an analysis of the basic mechanisms of growth-generating dynamics of the model incorporating both accumulation of R&D and accumulation of human capital. Since the framework employed here takes into account the complementarity between human capital and the R&D activities, and the externalities associated with the accumulation of both, we first explore the basic mechanisms of “correcting” the “market failures” toward superior outcomes. To this end, we investigate two policy instruments, each of which promotes the accumulation of factors that are most needed in the production of the final good in the economy. Taking into account the “partial excludability” property of “knowledge” in this model leads, in the absence of public intervention, to under-investment in the provision and acquisition of new technologies, we lastly analyze and report the effects of subsidy to the production of differentiated capital in the economy.

In order to set up an environment to “suitably” compare and contrast both the short-run and the long-run effects of alternative subsidy schemes, we fix the total subsidy financing to an amount that would correspond to 0.5% of benchmark GDP every period. For each subsidy type, we calculate the corresponding subsidy rate and document the averages in the results below. The burden of the cost of each targeted policy is born as lump-sum from the government budget, affecting public consumption under the current model closure.

### *Effects of Human Capital and R&D Promoting Policies*

Here, we focus on the basic mechanisms of growth generating dynamics of the model



by investigating two policy instruments. Each instrument is designed to enhance growth via stimulating the accumulation process of factors affecting the growth rate of the economy each period. Specifically, we study subsidization of education (subsidy on the buildup of human capital through skill-accumulation function via  $s^H$ ) and contrast it with subsidization of the R&D activities (subsidy on the input costs to R&D via  $s^R$ ). The first policy experiment is designed to analyze the households' response to allocate human capital among different sectors and activities in the economy under the conditions of increased reward to education activities. Since the instrument,  $s^H$ , enters into representative household's intertemporal maximization problem, we shall observe the effects on the derivation of the future wages both in the final goods and the R&D sectors of the economy and the trade-offs embedded. The other policy instrument analyzed at this stage is designed to promote R&D activities through a demand stimuli. It is implemented through the addition of an ad valorem subsidy to the input cost of the production of new R&D.

Table 4 documents both the short-run and the long-run comparisons for a chosen set of variables under alternative scenarios. The Figures 1-7, in addition, display the transition dynamics for selected variables. At a first glance, Table 4 reveals a general observation that education subsidy provides more favorable results with respect to steady-state growth rates, both in terms of output and the knowledge stock of the economy. Such a result, of course is related to the changes in the ratio of total human capital to total R&D ( $H/A$ ) and therefore, allocation of human capital among different sectors and activities in the economy. A subsidy to education, bidding more human capital to skill-accumulation activities, leads to a higher stock of human capital and lower stock of R&D, compared to both the benchmark and the R&D subsidy scenario. The long-run equilibrium under this instrument is achieved

at a  $H/A$  ratio 3.0% higher than the benchmark and 14.3% higher than the R&D subsidy cases. Likewise, under the human capital subsidy scheme, share of education in the allocation of human capital is 19.0%; under R&D subsidy scheme it drops to 11.1%. Concomitantly, the share of R&D sector in the allocation of total human capital is 28.9% under education subsidy and it increases to 32.4% under R&D subsidy.

On the other hand, taking into account the transition dynamics, one could observe interesting trade-offs of the adjustment processes. First, we see that the education subsidy induces relatively large re-allocations of the primary resources, the major adjustment of which mostly occur in the initial periods. Immediately, the human capital allocated to education increases by 61.2%. After this initial swing, the adjustment dynamics reveal an average increase of 12.0% for this variable, over the base-run. Thus, although education subsidy displays higher growth rates with respect to benchmark and R&D subsidy scenarios at the steady state, it initially creates a large negative effect on the growth path of the economy. Such observation is basically due to the allocation of human capital away from marketed activities, and seems to take quite long time to be recovered. The education subsidy scheme, although promises a higher long-run growth rate, its transition path displays notably negative effects for the current generations as also revealed by the variables corresponding to welfare (consumption, saving utility index) in Table 4.

The education subsidy in the model is represented by a direct transfer of income from the government budget to the human capital accumulation activity. An announcement of subsidy to human capital accumulation activity basically drives resources away from the R&D activity, leaving the amount allocated to final goods sector only slightly lower. As a

result, the accumulation of human capital in the economy continues at a higher pace than the accumulation of R&D (Figures 1, 2). The output growth, which is dependent on both the accumulation of R&D and the human capital allocated to final goods sector is adversely affected. Although the rate of growth of GDP quickly bounces back, the immediate negative effect of bidding resources away from the other sectors of the economy is felt during a long transition period.

The announcement of an R&D subsidy as reflected in the reduction of cost of input (wage of human capital,  $w^H$ ) employed by the producers of R&D, on the other hand, encourages them to pull primary resources away from other sectors. Under such an instrument, the demand for R&D activities is increased to a higher steady-state level (3.8%), compared to the benchmark and the education subsidy scenario. On the other hand, total human capital built up is lower (6.6%) with respect to the benchmark. Since both the R&D, therefore, human capital employed in R&D activities and the human capital allocated to education are effective in the production of new human capital in the economy, total human capital produced is 9.6% lower with respect to the education subsidy scenario. As more human capital is devoted to R&D activities through subsidization, less is devoted to education, leading to an adjustment toward education activities in the following period. Such effects on total R&D and total human capital stock of the economy are visible in Figures 1, 2.

As the R&D production cost is reduced by the subsidy, the stream of monopoly rents, acquired from the property rights of the blueprint increases. Such an increase stimulates further incentives for the production of capital, as new firms are attracted by increased

profits. So, the subsidy to the cost of R&D production begins to encourage an upward shift in the demand for differentiated capital (new information technologies) production sector, leading to higher investment and higher capital accumulation in the economy, both during later stages of transition and at the steady state (See Table 4 and Figure 4). It is partially due to this stimulation of the activity in the final goods sector that keeps the wage rate of human capital higher under this scenario.

Figure 3 displays real GDP under alternative subsidy schemes. The initial negative effect of the education subsidy on the productive sectors keeps such a subsidy plan at a lower path compared to base-run and the path under R&D subsidy. Although the growth rate recovers in the long-run, the GDP and consumption paths under education subsidy are much less favorable for the current generations. On the other hand, the R&D subsidy scheme creates a more direct effect in terms of the allocation of resources in the economy, leading to a higher average growth rate during transition toward the new steady state. It is because of the differences in the growth dynamics of the economy under different scenarios that leads to differentiated burden of the same amount of subsidy as a ratio to GDP.

### *Subsidizing Capital Accumulation*

Observing the importance of capital accumulation in both the generation of transition dynamics and the steady state path of the economy, the last scenario we analyze corresponds to promoting growth through a subsidy to employers of differentiated capital. With this experiment, the cost of investment in terms of final goods allocated to transforming R&D to a productive technology is lower. Such a subsidy would provide incentives to increase the

production of differentiated capital, which in turn would have a stimulation effect on the accumulation of capital stock and the knowledge stock in the economy.

Note that in such an environment the mechanism of re-allocation of resources in the economy is different than either the education subsidy or the R&D subsidy scenarios. In case of education or R&D subsidy, the subsidized sector pulls the resources away from the other activities in the economy. In case of subsidy to investment, on the other hand, the direct beneficiary is the final goods sector. The demand for factors of production in this sector increases, leading to a much favorable environment in terms of their returns and the private household factor income. Since both profits generated in the production of differentiated capital and the wages are also part of household income, direct subsidy to allocation of resources in the final goods production breeds a favorable environment in terms of the steady state dynamics. Higher saving supports higher investment, allowing for increased consumption. Growth is achieved through accumulation of physical capital, however does not affect the steady state growth rate of the economy.

### *Dynamics of Adjustment*

The working of different adjustment mechanisms through the transition path, under alternative policy scenarios are most visible in the price of human capital as a factor of production ( $w^H$ ) and the price of R&D ( $P^A$ ). Figures 5, 6 display the transition dynamics of these two variables over periods 1-30. Figure 7, to present the background for the calculations of the welfare index, displays the private consumption dynamics over the same period.

As the education subsidy bids the human capital toward the skill accumulation

activity, the market price of the human capital as well as the price of R&D are initially highest for this scenario. As human capital supply increases, it turns out that the returns to human capital as well as returns to R&D take the lowest values. As investment subsidy creates higher demand for R&D and subsidization of the human capital in the R&D sector creates higher demand for human capital in R&D, the price of R&D gets the highest under the first and the price of human capital gets the highest under the second scenarios, throughout the transition (see Table 4).

We analyze the social welfare consequences of the policy simulations by computing a measure of utility through discounted sum of utilities both during transition and after the economy reaches to the new steady state. We calculate the utility of the households for the benchmark case and for each subsidy scheme and document the percentage deviations of this index from the benchmark in Table 1. Figure 7, in addition, illustrates the path of private consumption under different scenarios.

Accordingly, we find that under education subsidy scenario, the welfare index changes sign during the course of adjustment toward steady state. The negative effect of significantly lower production, income and consumption throughout the transition period becomes more pronounced as the economy moves toward the new steady state maintaining a lower level of consumption (Figure 7). On the other hand, the R&D subsidy scheme provides a higher level of consumption right away, during transition and is able to provide 4.0% higher level of consumption relative to the benchmark reaching to the new steady state. We also find that, under the capital accumulation subsidy scheme, the immediate negative impact of increased investment/saving is quickly reversed, letting the economy to a high-consumption path.

## 5 Conclusion

In this paper we investigated alternative public policy intervention schemes within the context of an applied endogenous growth model with human and knowledge capital accumulation. Utilizing data from the Canadian economy, we first studied subsidization of the buildup of human capital through skill accumulation against subsidization of the R&D activities. Such instruments are basically designed to enhance growth via stimulating the accumulation process of factors influencing the growth rate of the economy. Subsidy to education affects the household's response to allocate human capital among different sectors/activities. Subsidy to R&D, on the other hand, drives the primary resources toward the R&D sector, by offering a higher (subsidized) return.

Under alternative subsidy schemes, we observe that the model reveals various trade offs and contrasting dynamics under transition to the new steady state. This observation led us to design a hybrid scenario. Under this compound subsidy scheme we simultaneously utilized alternative subsidy instruments (education subsidy and R&D subsidy) that operate through different dynamics. We found that this hybrid instrument presents better results in terms of growth and welfare. Such an outcome emphasizes the importance of designing comprehensive policies toward accumulation of human capital and accumulation of knowledge in the Canadian context.

In order to understand the importance of capital accumulation both for the transition and the steady-state path of the economy, we also studied the effects of investment subsidy. We observe that under this subsidy scheme, the mechanism of re-allocation of resources operates differently than either the education subsidy or the R&D subsidy scenarios.

Investment subsidy affects the dynamics of the economy through the final goods sector whereas the mechanism under education or the R&D subsidy works through re-allocation of the primary factors of production among different activities in the economy.



## Appendix: The data and the calibration strategy

### Calibration steps

The data related to the initial period's equilibrium are drawn primarily from the HRSD-Canada data set for the year 2003. As the HRSDC data are originally in the form of annual flow values and primarily compiled for the purpose of static general equilibrium analyses, they need to be further augmented by information associated with the Canadian growth path, namely, capital stock, technological knowledge stock, R&D expenditures, growth rate(s), interest rate, and the discount rate in the intertemporal utility function.

The intertemporal elasticity of substitution,  $1/\sigma$ , in the household utility function is chosen in the range estimated by Hall (1988). The rate of time preference,  $\rho$ , is taken from Lucas (1988). The average long run growth rate of the economy,  $g^Y$ , taken to be 3.0%, matches the data on the long-run growth path of the Canadian economy. The initial steady state growth rate for human capital, hence for R&D thereby, is solved in accordance with the steady state rate of growth of the economy and the shares of human and physical capital in the value added of the final output,  $\alpha^H$  and  $\alpha^k$ . The initial interest rate,  $r_0$  then has to be calculated in a way consistent with the choices of  $\sigma$ ,  $\rho$ , and  $g_A(0)$ <sup>11</sup>. We further assume that the depreciation rate of capital varieties is zero.

Detailed data on Canadian Labor Force of the Statistics Canada, together with the data on total R&D personnel<sup>12</sup> is compiled to adjust the original HRSDC data for different

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<sup>11</sup>As in static applied GE models, where calibration is based on the assumption that data reflect an economy in equilibrium, we assume that the benchmark data depict an initial steady state growth path. This steady-state assumption for the benchmark data is widely used in applied intertemporal general equilibrium models. For example, Goulder and Summers (1989), Go (1994), Mercenier and Yeldan (1997), and Diao, Roe and Yeldan (1999).

<sup>12</sup>Science Statistics, May 2008.

labor input categories. Accordingly, R&D personnel in Higher Education and Government sectors with MS and PhD degrees in Science and Engineering and Social Science together with the MS and PhD students in Science and Engineering are assumed to form the basis for the share of human capital in education,  $u^H$  (17.6%). The rest of the R&D personnel in Industry/Business and Government are considered to be at the basis for the estimation of the share of human capital in R&D production,  $u^A$  (29.2%). The rest of the (employed) Science and Engineering and Social Science graduates with MS degrees and higher are assumed to be employed in the final goods sector,  $u^Y$  (53.2%). Given the total employment figure for the Canadian economy for 2003, we assume that the rest of the labor force is employed under the category of plain labor,  $L$  in the model.<sup>13</sup>

Data on total returns to capital in the benchmark are provided by the HRSDC database. We distinguish the returns to the differentiated capital from the returns to labor resource based on these data. The task is accomplished under the calibration restrictions imposed by the model. For the purpose of calibration, we normalize the ratio of (index of ) initial stock of human capital to the (index of) initial stock of R&D,  $H/A$  to 1.

To ensure the existence of a balanced growth path, we calibrate  $\alpha^k$ , price of R&D,  $P^A$  and the growth rate of R&D,  $g^A(0) = \Delta A_0/A_0$  simultaneously, depending on equations (26), (30), (31) and (32):

Under the steady state we know that  $r_{SS} = g^{PA} + \frac{\Pi_{SS}}{P^A}$ . Now, recall the GDP identity

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<sup>13</sup>Such an assumption, of course, classifies for instance, BA graduates under plain labor category,  $L$  in the model. Therefore, the share of plain labor in the final good value added,  $\alpha^L$  is estimated to be around 30.0%, a value higher than what is usually set for the Canadian economy (Sturgill, 2008).

from (31):

$$P^Y Y + P^A \Delta A = w^L L^Y + w^H (H^Y + H^A) + \frac{\Pi}{(1 - \alpha_k)}$$

Using the the no-arbitrage equation we have  $(1 + g^{PA}(t)) = \frac{1+r_{t+1}}{1+\frac{H_{t+1}}{P^A_{t+1}}}$ . Thus, under the steady state equilibrium the national income identity satisfies,

$$GDP = w^L L^Y + w^H (H^Y + H^A) + \frac{P^A}{(1 - \alpha_k)} \left( \frac{1 + r}{1 + g^{PA}} - 1 \right)$$

or equivalently,

$$(1 - \alpha_k) [GDP - w^L L^Y - w^H (H^Y + H^A)] = P^A \left( \frac{1 + r}{1 + g^{PA}} - 1 \right) \quad (34)$$

Using (30) and (34) together,

$$(1 - \alpha_k) \alpha_k [GDP - P^A \Delta A] = P^A \left( \frac{1 + r}{1 + g^{PA}} - 1 \right) \quad (35)$$

Using the fact that  $g^{PA}$  and  $g^A$  are related via equation (33) in discrete time as  $(1 + g^{PA}) = (1 + g^A)^{\frac{\alpha_H}{1 - \alpha_k}}$ , and recalling that we have  $\Delta A_0 / A_0 = g_A$ , this information will allow us to utilize the following relationship for calibrating  $P_A$  and  $\alpha_k$ :

$$(1 - \alpha_k) \alpha_k [GDP - g^A P^A] = P^A \left( \frac{1 + r}{1 + g^{PA}} - 1 \right) \quad (36)$$

The second simultaneous relationship between  $P_A$  and  $\alpha_k$  is obtained from the savings - investment equilibrium condition (26). Using (16) and (26) together, and making note of the fact that  $\Delta A / A = g_A$ , we have

$$SAV = r \cdot \eta \cdot g^A \cdot k + r \cdot \eta \cdot \Delta k + P^A g^A$$

Since, from the optimal pricing rule of the monopolist (equation 12)  $r = \frac{\alpha_k p_k}{\eta}$ , the saving - investment equilibrium can be re-written as

$$SAV = \frac{\alpha_k(g^A + g^{PA})}{r(1 - \alpha_k)} \cdot P^A \left( \frac{1 + r}{1 + g^{PA}} - 1 \right) + P^A g^A \quad (37)$$

Table A-1 presents the initial levels of selected variables and parameters obtained from sources other than the main data base or from this calibration process.

| <b>Table A-1. Pre-assumed and Calibrated Values of Structural Parameters</b>          |        |
|---------------------------------------------------------------------------------------|--------|
| Share of human capital in final good value added, $\alpha_H$                          | 0.232  |
| Share of plain labor in final good value added, $\alpha_L$                            | 0.300  |
| Share of rental value of differentiated capital in final good value added, $\alpha_K$ | 0.468  |
| R&D Production productivity parameter, $\varphi$                                      | 0.0695 |
| Productivity of schooling in human capital formation, $\xi$                           | 0.0296 |
| Productivity of learning via knowledge and varieties, $\gamma$                        | 0.0157 |
| Share of past human capital in human capital formation, $\epsilon$                    | 0.6956 |
| Value of input output coefficient to produce unit capital variety, $\eta$             | 9.3619 |
| Share of human capital allocated to final good production, $\frac{H_Y}{H} (u^Y)$      | 0.5317 |
| Share of human capital allocated to R&D production, $\frac{H_A}{H} (u^A)$             | 0.2922 |
| Share of human capital allocated to human capital formation, $\frac{H_H}{H} (u^H)$    | 0.1761 |
| Value of R&D sector as a ratio to GDP, $\frac{P_A \Delta A}{GDP}$                     | 0.109  |
| Ratio of aggregate savings to GDP, $\frac{SAV}{GDP}$                                  | 0.229  |
| Share of oligopolistic profits in GDP, $\frac{\Pi}{GDP}$                              | 0.214  |
| Inverse of intertemporal elasticity of substitution for the consumer, $\sigma$        | 1.0001 |
| Subjective discount rate, $\rho$                                                      | 0.023  |
| Income tax rate, $t_y$                                                                | 0.092  |
| Sales tax rate on consumption good, $t_c$                                             | 0.046  |
| Armingtonian elasticity of substitution between $M$ and $DC$ , $\varepsilon_{CC}$     | 3.000  |
| CET elasticity of transformation between $E$ and $DC$ , $\varepsilon_{CET}$           | 3.000  |

We read the values of  $SAV$ , wages paid to human capital in the production of the final good, and the value added of  $Y$  from data. For, calibration, we set the initial values of  $u^A$ ,  $u^H$ , and  $u^Y$  in accordance with the data. Using the F.O.C. in the R&D production,

with  $w^H$  given, we have  $\varphi = w^H/P_A$ . Equations (5), (3.3) together with the information that at the steady state,  $g^A = g^H$ , help one to solve for  $\xi$ ,  $\gamma$  and  $\epsilon$ .

Finally, using discrete time, condition for equilibrium growth of consumption is

$$\left( \frac{1 + r_{SS}}{1 + \rho} \right)^{1/\sigma} = 1 + g^c \quad (38)$$

Thus growth in final good becomes  $g^Y = g^c$ . The rest of the system is calibrated using standard methods of applied general equilibrium.

## Parameters, Exogenous and Endogenous Variables

### *Parameters*

|                |                                                                               |
|----------------|-------------------------------------------------------------------------------|
| $s^H$          | subsidy to education                                                          |
| $s^R$          | subsidy to R&D                                                                |
| $\xi$          | productivity of education                                                     |
| $\gamma$       | productivity of learning with R&D                                             |
| $\epsilon$     | weight of human capital in learning with R&D                                  |
| $\theta$       | CRRA utility parameter                                                        |
| $\beta$        | private household discount rate                                               |
| $\varphi$      | productivity of R&D human capital                                             |
| $A_Y$          | final goods production function shift parameter                               |
| $\alpha_L$     | share parameter for plain labor in final goods production function            |
| $\alpha_H$     | share parameter for human capital in final goods production function          |
| $\alpha_k$     | share parameter for differentiated capital in final goods production function |
| $\eta$         | marginal cost of a unit of capital variety in terms of final good             |
| $\bar{Z}_X$    | shift parameter in CET function of foreign trade                              |
| $\bar{Z}_{CC}$ | shift parameter in Armington function of foreign trade                        |
| $\nu$          | share parameter in CET function                                               |
| $\kappa$       | share parameter in Armington function                                         |
| $\sigma$       | elasticity of substitution in CET function                                    |
| $\psi$         | elasticity of substitution in Armington function                              |
| $t^Y$          | income tax rate                                                               |
| $t^m$          | tariff rate                                                                   |
| $t^X$          | production tax rate                                                           |
| $t^C$          | consumption tax rate                                                          |

### *Exogenous Variables*

|          |                    |
|----------|--------------------|
| $L$      | plain labor supply |
| $P^{WM}$ | world import price |
| $P^{WE}$ | world export price |

### *Endogenous Variables*

|       |                                                  |
|-------|--------------------------------------------------|
| $H$   | total human capital                              |
| $H^Y$ | human capital allocated to final good production |
| $H^H$ | human capital allocated to education             |
| $H^A$ | human capital allocated to R&D activities        |
| $Y^H$ | household income                                 |
| $C$   | private consumption                              |
| $S$   | household saving                                 |
| $w^H$ | wage paid to human capital                       |
| $w^L$ | wage paid to plain labor                         |
| $r$   | interest rate                                    |
| $A$   | the R&D stock                                    |
| $P^A$ | price of blueprints                              |

|        |                                                     |
|--------|-----------------------------------------------------|
| $k(i)$ | a single capital variety                            |
| $p(i)$ | price of each capital variety                       |
| $\pi$  | monopoly profit of a capital variety producing firm |
| $g^A$  | growth rate of R&D                                  |
| $g^H$  | growth rate of human capital                        |
| $g^Y$  | growth rate of final output                         |
| $g^k$  | growth rate of capital varieties                    |
| $Y$    | final output                                        |
| $P^Y$  | price of final output                               |
| $ID$   | investment demand                                   |
| $P^C$  | price of consumption                                |
| $P^D$  | domestic price                                      |
| $P^E$  | export price                                        |
| $P^M$  | import price                                        |
| $CC$   | composite commodity                                 |
| $D$    | domestic commodity                                  |
| $M$    | imports                                             |
| $E$    | exports                                             |

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| <b>Table 1. Selected Research and Development Statistics (2003-2009* Avg.)</b> |               |            |                  |              |
|--------------------------------------------------------------------------------|---------------|------------|------------------|--------------|
|                                                                                | <b>Canada</b> | <b>USA</b> | <b>OECD Avg.</b> | <b>EU-15</b> |
| <b>Research and Development - Expenditures</b>                                 |               |            |                  |              |
| R&D Expenditures/GDP (%)                                                       | 1.99          | 2.63       | 2.24             | 1.92         |
| R&D Expenditures by Gov./ GDP (%)                                              | 0.64          | 0.77       | 0.65             | 0.65         |
| % R&D Financed by Industry                                                     | 49.55         | 65.18      | 63.28            | 54.70        |
| % R&D Financed by Government                                                   | 32.12         | 29.30      | 29.04            | 34.13        |
| <b>Research and Development - Production</b>                                   |               |            |                  |              |
| % R&D Performed by Industry/Businesses                                         | 55.21         | 70.77      | 68.55            | 63.29        |
| % R&D Performed by Government                                                  | 9.68          | 11.54      | 11.57            | 12.77        |
| % R&D Performed by Higher Educ.                                                | 34.61         | 13.64      | 17.42            | 22.83        |
| <b>Research and Development - Employment</b>                                   |               |            |                  |              |
| % Researchers working in Businesses                                            | 61.95         | 80.24      | 64.14            | 48.66        |
| % Researchers working in Government                                            | 6.26          |            | 8.31             | 12.02        |
| % Researchers working in Higher Educ.                                          | 31.79         |            | 27.54            | 39.32        |
| * : Depending on data availability                                             |               |            |                  |              |
| Source: OECD Main Science and Technology Indicators                            |               |            |                  |              |

**Table 2. Canadian Labor Force (2001)**

|                                                      | Number of<br>People | (%)          |
|------------------------------------------------------|---------------------|--------------|
| <b>Total</b>                                         | <b>23,901,360</b>   | <b>100</b>   |
| <b>University with BA or Higher</b>                  | <b>3,687,645</b>    | <b>15.43</b> |
| Earned Doctorate                                     | 128,625             | 0.54         |
| Master's                                             | 642,055             | 2.69         |
| Bachelor's                                           | 2,411,475           | 10.09        |
| Degree in Medicine+Vet.                              | 122,535             | 0.51         |
| Univ. Certificate or Diplom. Above                   |                     |              |
| Bachelor's                                           | 382,955             | 1.60         |
| <b>Univ. Certificate or Diplom. Below Bachelor's</b> | <b>601,425</b>      | <b>2.52</b>  |
| <b>College Certificate or Diplom.</b>                | <b>3,578,400</b>    | <b>14.97</b> |
| <b>Below College</b>                                 | <b>16,033,890</b>   | <b>67.08</b> |

Source: Statistics Canada

**Table 3. Spending and Performance of Research and Development, Canada, 2008/2009 (million of current dollars)**

| Sources of Funds                          | Performance of R&D     |              |                  |           | TOTAL         |
|-------------------------------------------|------------------------|--------------|------------------|-----------|---------------|
|                                           | Industry/business ent. | Government   | Higher education | Other     |               |
| Industry/business ent.                    | 11,652                 | 82           | 679              | 14        | 12,427        |
| Government                                | 369                    | 2,278        | 5,083            | 32        | 7,762         |
| Higher education                          | -                      | -            | 1,706            | -         | 1,706         |
| Other<br>(Priv. non-profit, Foreign etc.) | 2,073                  | -            | 675              | 38        | 2,786         |
| <b>TOTAL</b>                              | <b>14,095</b>          | <b>2,361</b> | <b>8,143</b>     | <b>92</b> | <b>24,681</b> |

Source: Statistics Canada-Science Statistics (2010), OECD Main Science and Technology Indicators

**TABLE 4**

**Effects of Different Subsidy Schemes in the Short-Run and in the Long-Run**

| Steady-State Implications                     | (Average 6.67 % points) |                        | (Average 4.25 % points) |                    | (Average 4.19 % points) |          |
|-----------------------------------------------|-------------------------|------------------------|-------------------------|--------------------|-------------------------|----------|
|                                               | Benchmark               | Subsidizing Human Cap. | Subsidizing R&D         | Investment Subsidy |                         |          |
| gy (%)                                        | 3.00                    | 3.05                   | 3.00                    | 3.00               |                         |          |
| gA (%)                                        | 2.08                    | 2.12                   | 2.07                    | 2.08               |                         |          |
| H/A                                           | 1.02                    | 1.05                   | 0.92                    | 1.02               |                         |          |
| uy (%)                                        | 53.2                    | 52.1                   | 56.5                    | 56.5               |                         |          |
| uA (%)                                        | 29.2                    | 28.9                   | 32.4                    | 29.2               |                         |          |
| uE (%)                                        | 17.6                    | 19.0                   | 11.1                    | 11.1               |                         |          |
| Human Capital Index (H)                       | 4.14                    | 4.27                   | 3.86                    | 4.12               |                         |          |
| R&D Index (A)                                 | 4.14                    | 4.15                   | 4.29                    | 4.12               |                         |          |
| Total Differentiated Capital (K)              | 3,422,959.0             | 3,390,146.6            | 3,536,341.7             | 3,695,151.6        |                         |          |
| Price of R&D                                  | 1.86                    | 1.84                   | 1.85                    | 1.93               |                         |          |
| Wealth and Welfare Implications               | Subsidizing Human Cap.  |                        | Subsidizing R&D         |                    | Investment Subsidy      |          |
|                                               | Immediate               | Long-Run               | Immediate               | Long-Run           | Immediate               | Long-Run |
| Ratio w.r.t. Benchmark                        |                         |                        |                         |                    |                         |          |
| GDP                                           | -6.873                  | 0.091                  | 2.100                   | 3.843              | 3.723                   | 3.360    |
| Private Income                                | -6.646                  | 0.612                  | 2.196                   | 3.860              | 3.865                   | 3.360    |
| Private Consumption                           | 0.121                   | 0.586                  | 1.004                   | 4.022              | 0.228                   | 4.100    |
| Private Saving                                | -14.248                 | 0.693                  | 5.641                   | 3.379              | 6.251                   | 1.158    |
| Total R&D (A)                                 | -1.434                  | 0.328                  | 0.948                   | 3.742              | 0.700                   | -0.349   |
| Total Human Cap. (H)                          | 0.495                   | 3.274                  | -0.290                  | -6.596             | -0.248                  | -0.379   |
| Human Capital Alloc. To Educ. ( $H^H$ )       | 61.178                  | 11.598                 | -31.323                 | -41.015            | -21.133                 | -0.375   |
| Human Capital Alloc. To R&D ( $H^A$ )         | -33.584                 | 2.059                  | 20.173                  | 3.489              | 10.619                  | -0.434   |
| Human Capital Alloc. To Final Goods ( $H^Y$ ) | 0.451                   | 1.184                  | -1.485                  | -0.739             | -0.026                  | -0.350   |
| Government Revenues                           | -2.088                  | 0.249                  | 0.610                   | 3.566              | 1.490                   | 3.366    |
| Government Consumption                        | -4.426                  | -2.089                 | -1.728                  | 1.229              | -0.848                  | 1.028    |
| Utility (Welfare Index)                       | 0.003                   | -0.028                 | 0.076                   | 0.204              | 0.033                   | 0.211    |

Figure 1. Total Human Capital Under Different Subsidy Schemes (w.r.t. Base Run)

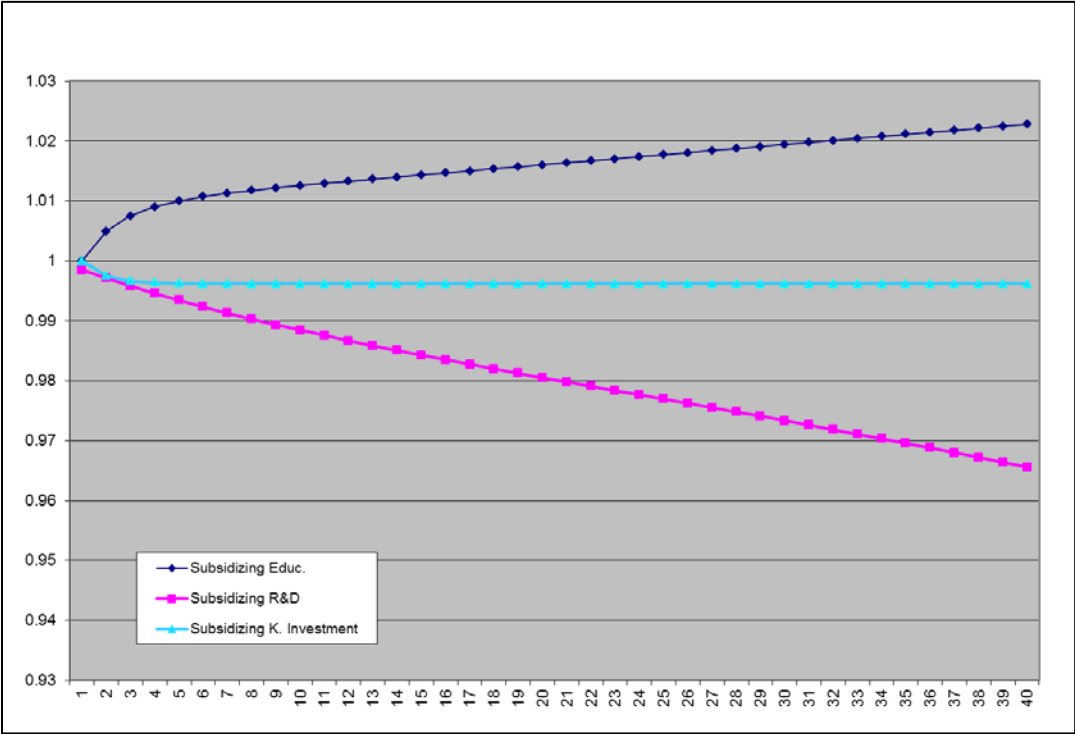


Figure 2. Total R&D Under Different Subsidy Schemes (w.r.t. Base Run)

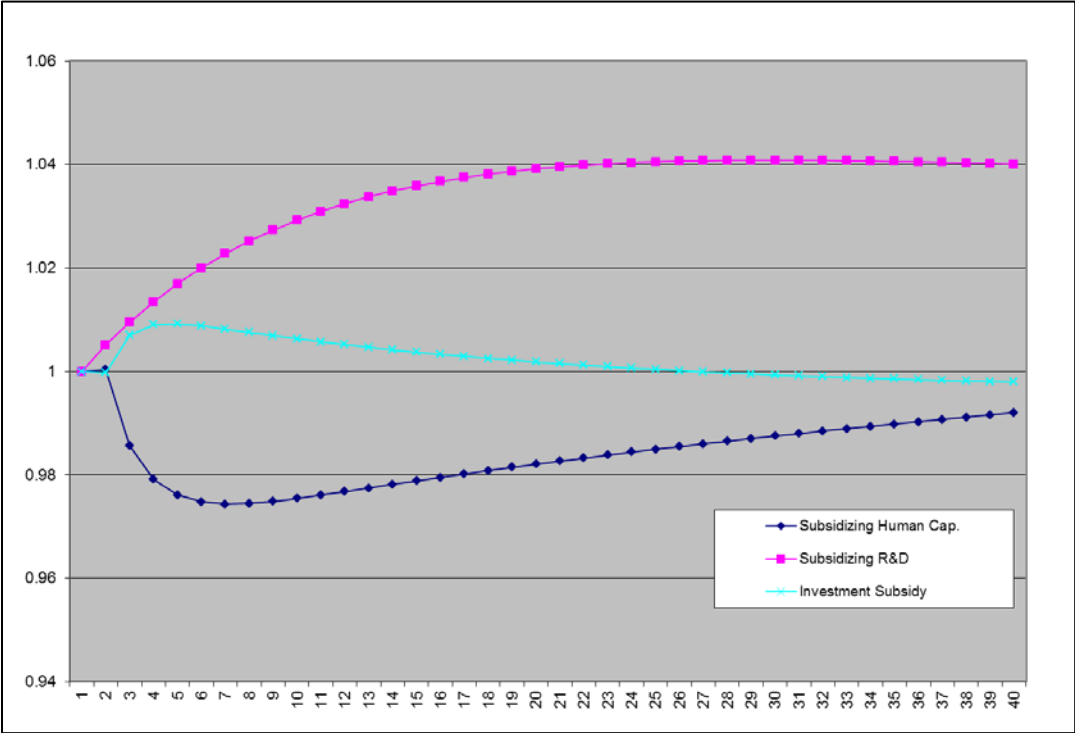


Figure 3. GDP Under Different Subsidy Schemes (w.r.t Base Run)

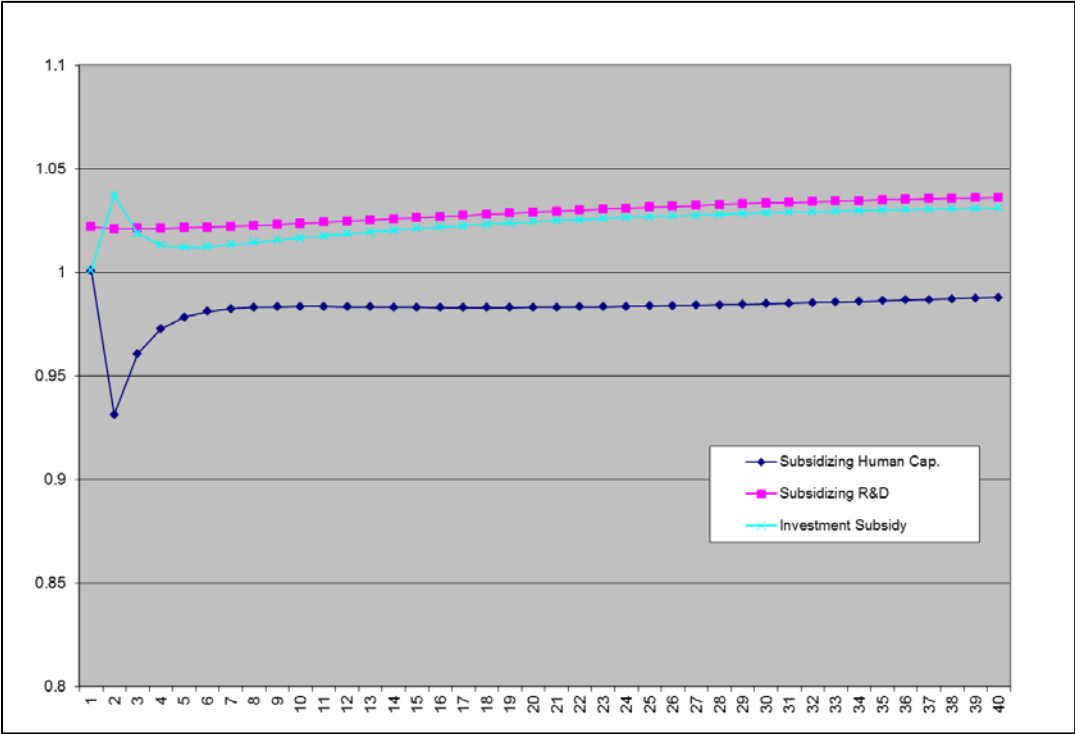
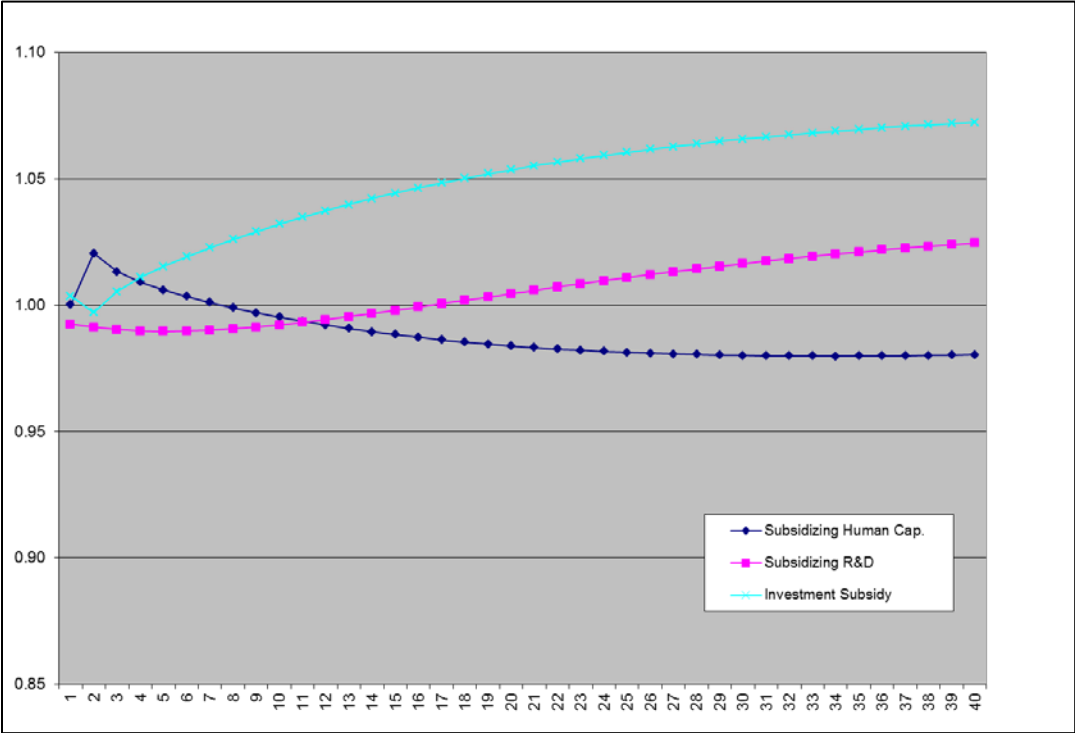
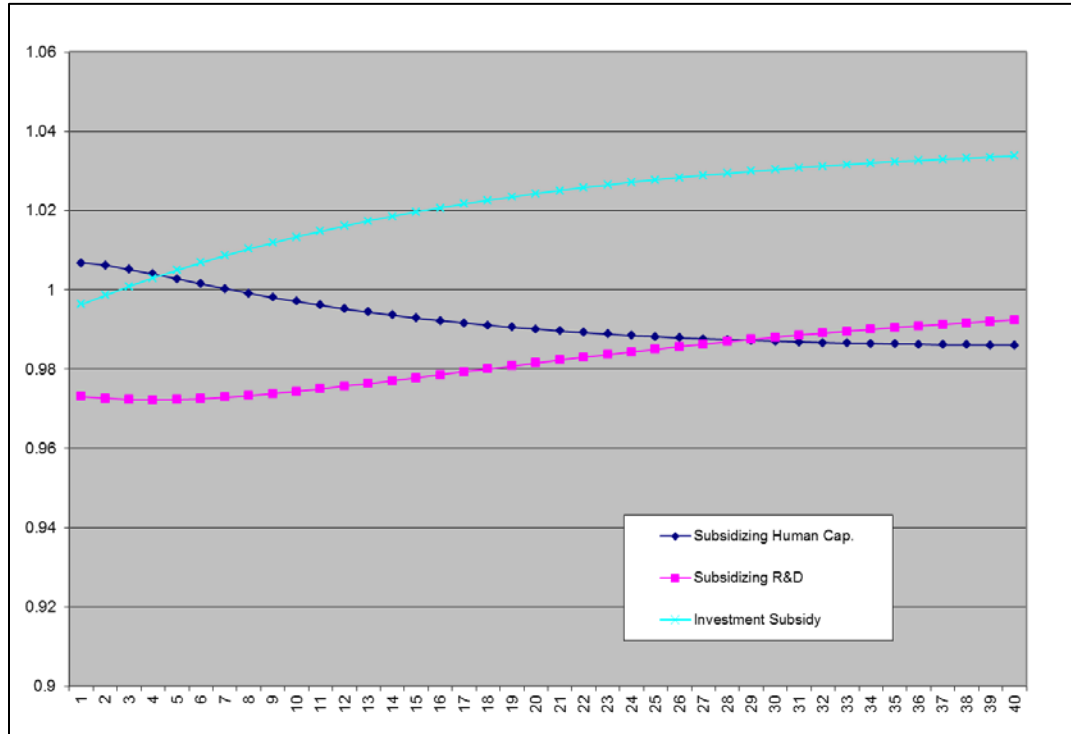


Figure 4. Total Differentiated Capital Under Different Subsidy Schemes (w.r.t. Base Run)



**Figure 5. Price of R&D under Different Subsidy Schemes (w.r.t. Base Run)**



**Figure 6. Wage Rate (of Human Capital ) under Different Subsidy Schemes (w.r.t. Base Run)**

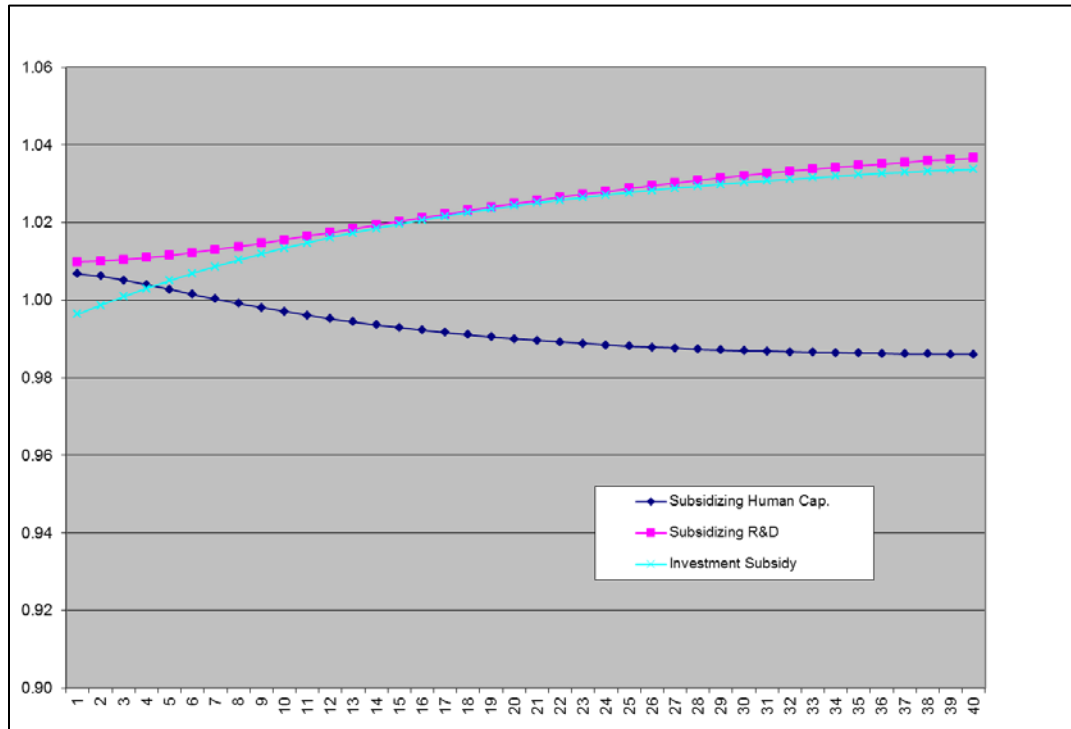


Figure 7. Private Consumption Under Different Subsidy Schemes (w.r.t Base Run)

