Internal Migration, Structural Change, and Economic Growth*

Dürdane Şirin Saracoğlu
METU

Terry L. Roe
University of Minnesota

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Abstract

Migration of labor out of rural areas and agriculture is a prevalent feature of economic development, growth and structural transformation. Essentially, this process involves the transformation of a rural-agrarian economy to an industrial-service based economy, as labor migrates from agriculture, capital deepening occurs and labor saving technologies are introduced. Not recognized in previous models of this process is the households' choice, inter-temporally, of urban - rural residency which not only alters the demand for regionally specific goods (e.g., housing, education, health), and hence the cost of living, but also the stock of rural – urban labor which affects the competition for resources within and between the urban – rural economies, and hence the rate of growth and structural change. This is the main contribution of this paper. We investigate the relationship between GDP growth, regional imbalances, and ongoing rural-urban migration over time using a neoclassical multi-region-sector growth model. The household decision for migration is dependent on the cost-of living differentials implied by the relative changes in regional home goods prices across regions as capital deepening occurs and the capital stock within each region evolves. Results show that allowing for residency choice provides a much deeper explanation of the forces of structural transformation and growth.

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1 Introduction

Movement of population from rural to urban areas or the reallocation of labor from agriculture to manufacturing and services has been one of the most important features of development process, historically in developed countries, currently in developing countries. Essentially, this process involves the transformation of a rural-agrarian economy to an industrial-service based economy, as labor is released from agriculture, capital deepening occurs and labor saving technologies are introduced (Henderson, 2003). This reallocation process, or structural change, has been well documented, especially by Kuznets (1957). As per Syrquin (1988), change in the sectoral composition of GDP is the most important characteristic of structural transformation. Syrquin associates the change in sectoral composition of GDP and income growth with changes in demand patterns, trade and factor use. In fact, Acemoglu (2009) focuses on the preference-related ("demand-side") and technology-related ("supply-side") reasons why an economy may experience structural transformation as it gets richer. The preference-related ("demand-side") reasons for structural transformation as income rises in an economy are linked to the *Engel's law*, which posits that as households' income increase, the share of income devoted to agricultural goods (food) decreases. Extending the implications of Engel's Law one step further, for example Kongsamut et al (2001) use Stone-Geary preferences to show that as households' income increase, they not only devote a smaller share of their income on food, but also a larger share on services.[1] Technology-related ("supply-side") reasons for structural change, on the other hand, may simply be originating from technological changes/advances in different sectors (for example, Baumol (1967) states that different sectors might be growing at different rates, hence changing the sectoral composition of GDP, simply because they have different rates of technological progress), and/or changes in factor distribution and factor use across sectors.

Not recognized in previous models of this structural transformation process is the households' choice, inter-temporally, of urban - rural residency which not only alters the demand for regionally specific goods (e.g., housing, education, health), and hence the cost of living, but also the stock of rural – urban labor which affects the competition for resources within and between the urban – rural economies, and hence the rate of growth and structural change. This is the main contribution of this paper. We investigate the relationship between GDP

[1]Other related work includes Laitner (2000) and Echevarria (1997), notwithstanding using different preference schedules, but with similar outcomes satisfying Engel's Law.

growth, regional imbalances, and ongoing rural-urban migration over time using a neoclassical multi-region-multi-sector growth model. The theoretical foundation of the model is based on the Ramsey representative agent single sector economic growth model (Ramsey, 1928; Barro and Sala-i-Martin, 2004). The Ramsey single sector growth model is extended to include four production sectors (agriculture, manufacturing, and services in rural and urban regions) in two regions (rural and urban), with labor mobility across regions. Within the scope of this model, we are able to trace the evolution of each region's output and factors of production (labor and capital) along the path of transition with migration, as well as the interregional disparities in income and output. The household decision for migration (i.e. labor mobility across regions) is dependent on the cost-of living differentials implied by the relative changes in regional home goods prices across regions as capital deepening occurs and the capital stock within each region evolves. We also capture the Engel's Law effects of changes in income of rural and urban household via Stone-Geary preferences, which, in the long run, yields same preference patterns across both types of households, but varying preferences at the initial equilibrium and during transition. The model economy's equilibrium path is obtained using the Time Elimination Method with Eigenvalues-Eigenvector Approach. For the solution of the model, we use numerical methods. GDP, production, expenditure, savings, and rural-urban allocation of labor data come from a typical developing economy such as Turkey. Based on 2006 data from the Turkish economy (obtained from TurkSTAT), and some assumed parameter values, a two-region Social Accounting Matrix (SAM) is built to match the model's initial equilibrium. Given the numerical solutions from the model, we also conduct growth accounting (or, contributions to growth) exercises to account for the contribution of reallocation of labor form one region to the other (i.e. internal migration) as the economy evolves. Results show that allowing for residency choice provides a much deeper explanation of the forces of structural transformation and growth. In our model, structural change or the change in sectoral composition of GDP occurs due both to changes in demand and changes in factor allocation across sectors, including changes in labor allocation across regions due to migration. Hence, as in real economies, we see the combined impact of both demand-side and supply-side factors on structural transformation. In fact, during transition towards the steady state, these two factors play out together, and their combined effect is enhanced by the inclusion of labor migration across regions. That is, previous models which do not consider labor migration across regions may be underestimating the effect of labor reallocation on structural transformation. We believe that accounting for this effect is one of the main contributions of our study. Furthermore, we find a positive and fairly significant contribution of labor reallocation across regions on overall growth, which is overlooked in other models of structural change which do not account for migration of labor across regions.

In the next section, we present the Literature Review. In section 3, the theoretical model is introduced together with the model's data description and the numerical solution to the model. In section 4, we show the results from the numerical solution the the model, including the growth accounting results. Section 5 concludes the study.

2 Literature Review

Internal migration in essence is a change in the spatial distribution of population in a given country over time, and is a transitionary process. Change in the spatial distribution of population is highly correlated to the various stages of development in a country (Tabuchi and Thisse, 2002). While the share of urban population in total population has increased in all regions of the world over the last 50 years, there has been little change in urbanization rates among the developed countries since the late 1970's; in fact, by upper middle income ranges countries become "fully" urbanized with about 60 to 90 percent urban population on average (Henderson, 2005). Tables 1 and 2 show that developing countries have experienced relatively rapid rates of urban population growth or urbanization in the post-World War II period. But how much does the internal, or, rural-urban net migration contribute to the rapid increase in urban population in the developing countries? According to Lucas (1997), there are three main factors contributing to the change in urban population: natural population increase among urban dwellers; net migration into the urban areas; and reclassification of areas as urban zones. In particular, Chen, et al. (1996) report that internal migration accounted for 40.3 percent, 44.1 percent and 54.3 percent of urban population growth in the developing world during the 1960's, 1970's and 1980's, respectively. Similarly, Becker and Morrison (1999) estimate that for the 1980-1993 period, contribution of rural-urban migration to urban population growth in low income countries was about 53 percent, while in East Asian countries, including China, was about 55 percent.

Country groups	1960-69	1970-79	1980-89	1990-99	2000-09
World	34.5	37.2	40.8	44.6	48.5
High income	63.3	68.1	71.2	74.0	76.3
Upper middle income	50.1	57.9	64.4	69.4	73.3
Middle income	27.1	30.4	35.3	40.5	45.7
Lower middle income	20.3	22.6	27.0	32.6	38.3
Low & middle income	25.5	28.9	33.5	38.3	43.0
Low income	13.9	16.8	20.5	23.5	26.9

Source: World Development Indicators, World Bank

Table 1: Urban population (% of total)

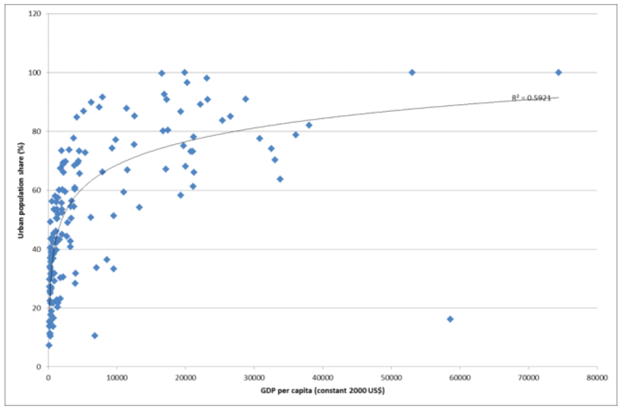
Country groups	1961-69	1970-79	1980-89	1990-99	2000-09
World	3.0	2.7	2.7	2.3	2.1
High income	1.9	1.5	1.1	1.1	1.0
Upper middle income	3.7	3.2	2.7	1.8	1.5
Middle income	3.6	3.4	3.5	2.8	2.3
Lower middle income	3.5	3.6	4.0	3.4	2.8
Low & middle income	3.7	3.5	3.5	2.9	2.5
Low income	5.7	5.0	4.3	3.8	3.7

Source: World Development Indicators, World Bank

Table 2: Urban population growth (annual %)

2.1 Internal Migration and Economic Growth

An examination of the share of rural population versus GDP per capita among 149 countries for the 1980-2010 averages shows that the share of urban population rapidly increases with increased income (Figure 1). From a careful examination of this graph, one can infer that economies with the highest GDP per capita have more than 70 percent urbanized population, while the countries with the lowest GDP per capita have about 25 percent urbanized population on average. However one striking feature of this figure is that as countries move from a low income to a lower middle or middle income category, share of urban population rises sharply, indicating that rapid urbanization is taking place along with economic growth. Based on cross-country data from 1991, Kojima (1996) divides the rate of urbanization across countries to four levels: rates below 20 percent, from 20 percent to 50 percent, from 50 percent to 70 percent, and rates above 70 percent. According to Kojima, genuine economic



Source: World Development Indicators, World Bank

Figure 1: Urban population share and GDP per capita

development in most of the countries with less than 20 percent urban population share has not yet begun, however in these countries, such as those in Africa and in the Middle East, the pace of urbanization is nevertheless very high, indicating that urbanization is taking place without industrialization. For the countries in East Asia, on the other hand, Kojima argues that for the most part urbanization has been progressing along with industrialization.

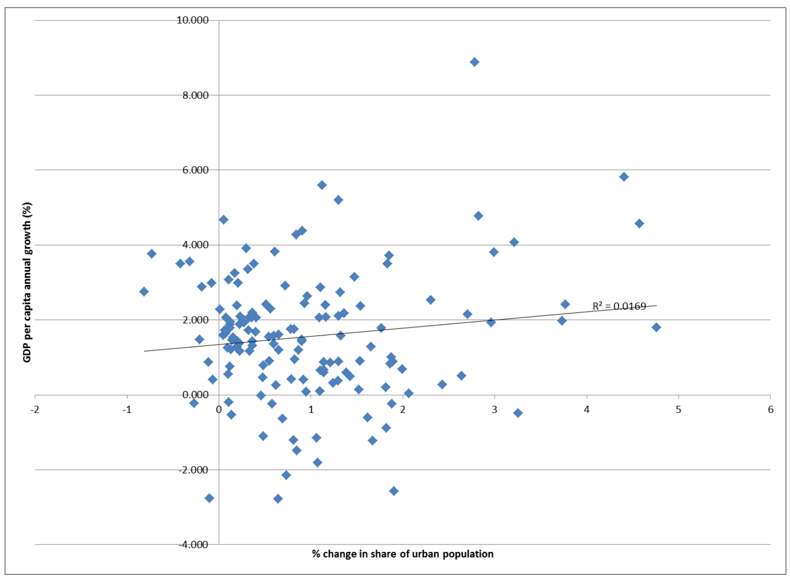
As can be inferred from the figure above, there is a positive correlation between the economic development (as measured by GDP per capita) of the country and its urbanization rate. As mentioned above, the change in the urban population can be decomposed into three factors, including rural to urban, or internal migration. Hence the next question we ask is whether there is a correlation between the change in urban population share approximated by internal migration rate and rate of economic growth, or not. Let u represent the share of urban population in total population, $u = U/L$, U is the urban population; L is the total population. Then, the change in the share of urban population u, is

$$\begin{aligned} \dot{u} \quad &= \quad \dot{\left(\frac{U}{L}\right)} = \frac{\dot{U}L - U\dot{L}}{L^2} = \frac{\dot{U}}{L} - \frac{U}{L}\frac{\dot{L}}{L} = \frac{U}{L}\left(\frac{\dot{U}}{U} - \frac{\dot{L}}{L}\right) \\ &\Longrightarrow \quad \frac{\dot{u}}{u} = \frac{\dot{U}}{U} - \frac{\dot{L}}{L} \end{aligned}$$

That is, the rate of change in the share of urban population is the difference between the rate of change in urban population and the rate of change in total population. Assuming that the natural change in urban population due to births net of deaths is the same as the general population, and assuming away the effect of reclassification of areas from rural to urban zones, $\frac{\dot{u}}{u}$ gives an approximate of the rate of internal migration in a country. Figure 2 depicts a slightly positive relationship between the change in the urban population share and annual GDP per capita growth for a large sample of 144 countries for the 1980-2010 averages, implying that countries with higher internal migration rates have a tendency to grow at a faster rate. Excluding from the sample the high income countries which are considered to be fully urbanized (and thus experience little or no change in urban population share), city states such as Singapore and Monaco which have 100 percent urban population, Sub-Saharan African countries which have very rapid change in urban population share without industrialization[2], and small island states which already have very small populations with little or no movement in population, we can find a stronger relationship between internal migration rate and economic growth rate (Figure 3). For the sample of 67 developing countries, this finding indicates that the countries on the transition path with relatively higher rates of internal migration experience higher growth in GDP per capita.

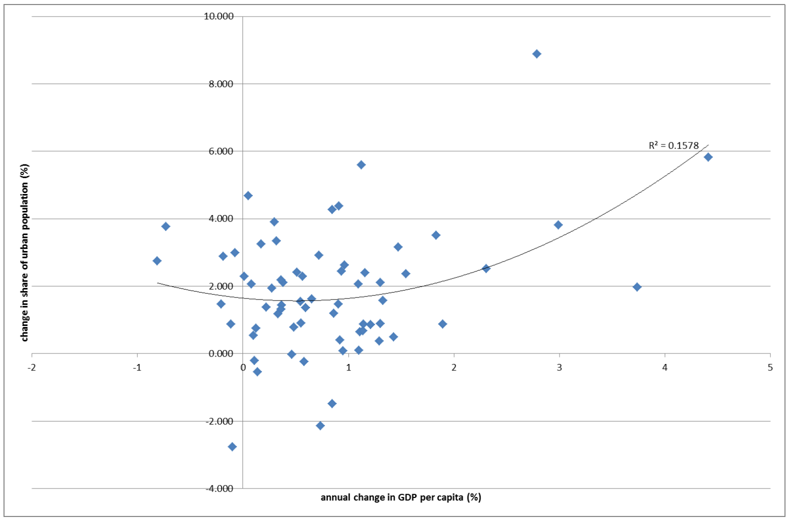
Breaking down the overall GDP into its main sectors, we see even a stronger relationship between the change in urban population share and the growth in manufacturing value added as well as the growth in services value added. This finding can be expected as both manufacturing and services are considered to be relatively more urban activities.

[2]According to Hardoy and Satterthwaite (1989), many sub-Saharan African countries experience rapid growth in urbanization simply because they had a relatively small urban base to start with in the early 1960s, when most of these countries gained political independence from colonial powers. The development of national government departments and ministries, judiciaries, armed forces, city and municipal governments, universities and other higher education institutes and other aspects of independent nation-states supported rapid urbanization since independence, even though national economies have not grown as rapidly. Sommers (2003), on the other hand, emphasizes the impact of civil wars in rapid growth in urban populations all across sub-Saharan African countries: from Angola to Sudan and the Democratic Republic of Congo to Sierra Lione, people displaced by civil wars have taken refuge in capital cities.



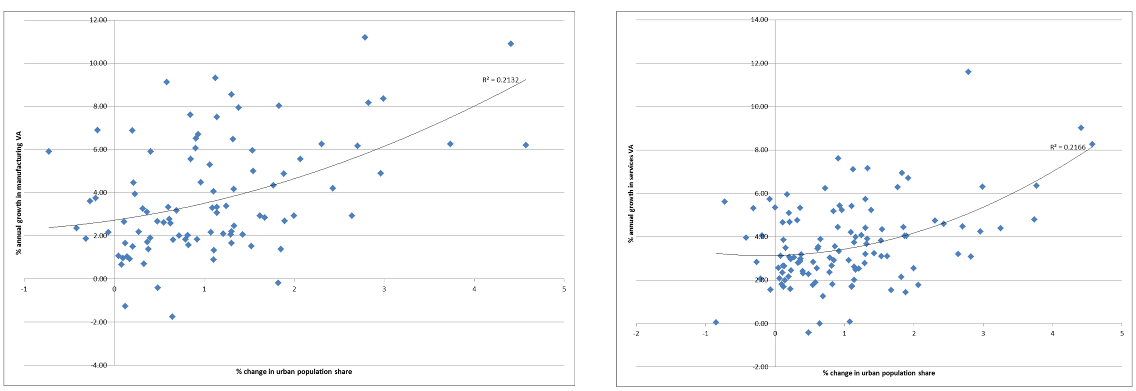
Source: World Development Indicators, World Bank

Figure 2: Change in urban population share and GDP per capita growth, 1980-2010 averages



Source: World Development Indicators, World Bank

Figure 3: Change in share of urban population and GDP per capita growth, developing countries excluding SSA and small island states



Source: World Development Indicators, World Bank

Figure 5: Growth in manufacturing and services value added versus internal migration

The interrelationship between internal migration and economic growth in a multi-sector economy with factor productivity differences across sectors is a complex one, and the causality between internal migration and economic growth may run in both ways (Yap, 1976). Surplus labor models as by Lewis (1954) and by Ranis and Fei (1961) focused on the transfer of labor from low-productivity agriculture to (presumably) high-productivity manufacturing creating efficiency gains by the reallocation of labor. From the point of view of these surplus labor models, urbanization unambigously leads to positive changes in national output (Morrison and Guo, 1996). Alhough the literature on internal migration has for the most part concentrated on the *determinants* of the individual household's decision to migrate and the motivations to migrate (e.g. Greenwood, 1987), relatively much less work exist on the *consequences* of migration, except for some of the studies on the link between urbanization and economic growth. Urbanization can be seen as a consequence of labor moving out of labor-intensive agricultural production to manufacturing and services, which are predominantly located in urban areas because of agglomeration economies (Krugman, 1991; Fu, 2004; Henderson, 2005). According to Henderson (2005), benefits from agglomeration, such as localized information and knowledge spillovers (Lucas, 1988), may be enhanced through urbanization, and thus efficient urbanization may induce economic growth. In fact, Gallup et. al (1999) suggest that urbanization may cause economic growth and argue that positive effects of migratory movements on economic growth are closely linked to the geographic position of the country or the region: in coastal areas where the transportation costs are low and division of labor is high, a rising population (represented by an increase in population density) may be linked to stable or even rising per capita incomes through increasing returns to scale in labor, whereas in the hinterland, output is predominantly characterized by decreasing returns to labor with limited supplies of land. Henderson (2003), on the other hand, suggests that increases in productivity growth are not necessarily due to to urbanization per se, but rather to the degree of urban concentration.

Although there are not many theoretical as well as empirical studies on the impact of internal migration on economic growth, Yap (1976) builds a theoretical model and uses simulation techniques to quantify the impact of internal migration on the growth of Brazilian GDP for the period 1950-1965. Simulations with and without migration show that internal migration has had a strong positive effect on Brazilian growth. Using non-stationary panel data techniques, McCoskey and Kao (1998) find a long-run relationship between the change in urbanization levels and growth in output per worker for 30 developing countries and 22 developed countries for the period 1965-1989, however they mention that the impact of urbanization on economic growth varies across countries. For the case of Lima, Peru for the period 1988-1993, Morrison and Guo (1996) estimate that through the net in-migration the largest impact on output was on the commerce and services sectors, which absorbed about 74 percent of all migrants to this city. In a growth accounting exercise for the period 1978-2003, Dekle and Vandenbroucke (2010) find that 26 percent of the labor productivity growth in China can be attributed to the reallocation of labor from agriculture to non-agricultural sectors, which can be regarded as representing rural-urban migration.

2.2 Cost of Living Differentials and Internal Migration

In the present study, we construct a two-region, multi-sector model of an economy to determine the impact of internal or rural to urban migration of labor on the growth of the economy as a whole, as well as the individual sectors and the regions. Although the main focus of the study is the impact of internal migration on growth of the economy, the migration decision from region to region is also endogenous in the model, i.e. it depends on household behavior, and is affected by the rate of growth and sectoral as well as regional distribution of output. In formulating the behavior of rural-urban migration of economic agents, much of the earlier literature has focused on the simple rural-urban wage gap (for example Lewis; 1954, Ranis and Fei, 1961; for the Brazilian case, Yap, 1976), or rural-urban differentials in expected incomes (Todaro, 1969; Harris and Todaro, 1970). In Todaro (1969) and Harris-Todaro (1970)-type probabilistic models, migrants are attracted to the cities with the expectation of a higher wage than they receive in agriculture, and are willing to accept the probability of urban unemployment, or lower wages and "underemployment" in the urban informal (traditional) sector. According to Todaro, the migrant is willing to accept urban unemployment or lower wages in the urban informal sector as long as he expects to "graduate" to the urban modern sector in the future. Extensions of these probabilistic migration models include Gupta (1988, 1993, 1997), Basu (2000), Chaudhuri (2000), and Bhattacharya (2002).

However, focusing solely on the rural-urban wage differentials, the Harris-Todaro-type migration models fail to take the cost-of-living differences across rural and urban regions into consideration in migration decision. Bell (1991) points out that in the presence of spatially non-mobile regional factors of production, there will be differences in regional household incomes. Further regional heterogeneity may arise due to the existence of regional non-traded goods, which exacerbates the differences in cost of living across regions. Along the lines of Heady (1981, 1988), Bell emphasizes that for an individual to be in equilibrium (i.e.

no migration), it must be the case that his expected utility derived from staying in the rural region is equal to the expected utility derived by moving to the urban region. Since the household's income and consumer prices in a region directly affect the consumption decision, they also affect the household's expected utility from staying or migrating.

Several empirical studies consider the role of regional cost-of living differentials in internal migration decision of households. For the United States, Renas and Kumar (1978) show that higher cost of living discourages in-migration and induces out-migration. They also show that a rapid rate of change in the cost of living in an area also discourages in-migration, however, it is not as statistically significant as the level of the cost of living. Nevertheless, excluding the rate of change in cost of living in migration regression leads to a misspecification problem. Liu (1975) considers the cost of living as a component in living conditions as part of quality of life index and finds that living conditions constitute the most important factor in non-whites' migration decision in the United States between 1960-1970. In an empirical study on Spain, Antolin and Bover (1997) approximate the regional cost of living differentials with house price differentials, and find that individuals have a higher probability of moving if they live in a region with a higher than average house prices. Cebula (1980, 1993, 2005), in empirical studies for the United States, find that in-migrants prefer areas where living costs are lower. Giannetti (2003) on the other hand, in an overlapping generations model, shows how heterogeneity in skill levels of the migrants and cost of living differentials across areas may explain some of the puzzles in internal migration and regional development: shortage of unskilled workers in high-technology areas such as Silicon Valley can be explained by the existence of skill complementarities that increase not only aggregate productivity but also the prices of non-traded goods (such as housing). As a result, workers who gain less from skill premia leave these areas. Giannetti's results also provide evidence to the regional divergences as mobility of labor may generate poverty traps in regions.

In the current study, we investigate the household's decision to migrate and the impact of migration on economic growth around these concepts. To this end, we construct a two region, multi-sector model of an economy in which regional wage disparities as well as differences in regional prices leading to regional cost of living differentials play a role in the migration decision. Unlike the previous studies cited, this study treats the migration decision in a dynamic general equilibrium setting.[3] The theoretical foundation of the model is based on

[3] Except for Yap (1976), which also considers a general equilibrium setting. However in Yap, there is no explicit household utility maximization rule, and household savings are simply determined by a linear function

the Ramsey representative agent single sector economic growth model (Ramsey, 1928; Barro and Sala-i-Martin, 2004). The Ramsey single sector growth model is extended to include four production sectors in two regions, with labor mobility across regions. Within the scope of this model, we are able to trace the evolution of each region's output and factors of production (labor and capital) along the path of transition with migration, as well as the interregional disparities in income and output.

3 The Theoretical Model

In this section, we introduce the theoretical set up and the model environment with assumptions. Then we present the solution strategy, as well as the method for growth accounting in an economy with internal migration.

3.1 Methodology and the Model Environment

The small open economy consists of two regions and is endowed with resources of labor, capital and land. The amount of land is assumed to be fixed. Households are the owners of factors of production in each respective region. Each region produces two goods, its own home good and a traded good, traded both internationally and across regions. Households in each region consume three goods: both of the tradable goods from both regions, and the home-good specific to the region in which they reside and work.

For simplicity, there is a single capital market, and a single market clearing interest rate; however labor markets are segmented across regions, in each region labor markets clear seperately at different wages at a given point in time. The two regional labor markets are "connected" in a sense that agents are allowed to move from one region to another so as to equate the indirect utility gained from consumption in each region, i.e. agents can choose residency and move across regions across time.

3.1.1 Production

In region i, $i = 1, 2$, production takes place in two sectors indexed j, $j = 1, 2$, utilizing constant returns to scale production technologies:

$$Y_{ij} = Y^{ij}(\mathbf{x})$$

of income.

where Y_{ij} indicates the aggregate output in sector j in region i, and $\mathbf{x}$ specifies the vector of inputs. More specifically, under constant returns to scale assumption, we can specify Cobb-Douglas production technologies in each sector as follows:

$$
\begin{aligned}
Y_{11} &= A_{11} L_{11}^{\beta_1} K_{11}^{\beta_2} T^{\beta_3} &(1)\\
Y_{12} &= A_{12} L_{12}^{\alpha} K_{12}^{1-\alpha} &(2)\\
Y_{21} &= A_{21} L_{21}^{\delta} K_{21}^{1-\delta} &(3)\\
Y_{22} &= A_{22} L_{22}^{\phi} K_{22}^{1-\phi} &(4)
\end{aligned}
$$

Here, L_{ij} and K_{ij} represent the labor and capital used in sector j of region i, respectively, and T is the amount of land in agricultural production in region 1. More specifically, Y_{11} is the output in the agricultural sector (traded good) and Y_{12} is the output in home-good sector (non-traded good) in region 1, whereas Y_{21} is the output in the manufacturing sector (traded good) and Y_{22} is the output of the home-good sector (non-traded good) in region 2.

Firms in agricultural sector in region 1 hire labor from region 1 labor market and capital from economywide capital market and they take the factor prices as given, and choose the combinations of labor L_{11} and capital K_{11} so as to minimize variable factor costs. Given the cost-minimizing combinations of L_{11}, K_{11}, factor prices, and the world price of agricultural good, p_1, and the fixed amount of land T, firms in agricultural sector in region 1 maximize profits, for all t,

$$\pi\left(w_1, r, p_1, T\right) \equiv \max_{Y_{11}} \left\{\left(p_1 Y_{11} - w_1 L_{11} - r K_{11}\right) \mid T' \leq T\right\}$$

Since output $Y_{11} = A_{11} L_{11}^{\beta_1} K_{11}^{\beta_2} T^{\beta_3}$ is homogenous of degree one, then $\pi\left(w_1, r, p_1, T\right)$ is linear in T :

$$\pi\left(w_1, r, p_1, T\right) = \pi^a(w_1, r, p_1) T$$

where $\pi^a(w_1, r, p_1)$ can be interpreted as a shadow price of a unit of land. Profits from agriculture accrue as land rents to the landowner household. Profit maximization in agriculture implies the following under Cobb-Douglas production function for agriculture as given above:

$$\pi(w_1, r, p_1, T) = (a_{11} p_1)^{\frac{1}{\beta_3}} w_1^{-\frac{\beta_1}{\beta_3}} r^{-\frac{\beta_2}{\beta_3}} T \tag{5}$$

Firms in the remaining sectors (region 1 and region 2 home good, and region 2 manufacturing) make normal profits, unlike the firms in the agricultural sector. All firms have access to labor in respective regional labor markets at regional wages, and to capital in the economywide capital market at rate r. The profit maximization conditions for these firms are given as

$$MC_{ij}(w_{i,}r) - p_{ij} = 0$$

where $MC_{ij}(w_i, r)$ denote the marginal cost of the firm in sector j in region i, except for the agricultural sector. Under profit maximization,

$$\frac{1}{a_{12}} w_1^{\alpha} r^{1-\alpha} - p_{12} = 0 \tag{6}$$

$$\frac{1}{a_{21}} w_2^{\delta} r^{1-\delta} = p_{21} = 1 \tag{7}$$

$$\frac{1}{a_{22}} w_2^{\phi} r^{1-\phi} - p_{22} = 0 \tag{8}$$

One characterization of intra-temporal equilibrium concerning the production side of the economy is to use the zero profit conditions to express p_{12}, p_{22} and w_2 as functions of w_1 and r (with $p_{21} = 1$ as the numeraire):

$$\begin{aligned}
p_{12}(w_1, r) &= \frac{1}{a_{12}} w_1^{\alpha} r^{1-\alpha} \\
w_2(r) &= \left[\frac{a_{21}}{r^{1-\delta}}\right]^{1/\delta} \\
p_{22}(r) &= \frac{1}{a_{22}} a_{21}^{\phi/\delta} r^{\frac{\delta-\phi}{\delta}}
\end{aligned}$$

3.1.2 Households

Households in each region i, $i = 1, 2$, are endowed with L_i units of labor, and K_i units of capital. Household in region i derives income from renting labor and capital services to the firms in the region in which he/she resides in. For simplicity, we assume that land is equally distributed among residents in region 1 and region 2, hence households in both regions receive equally distributed rents from land. Under these assumptions, the aggregate household income in region i is given by

$$\mathcal{I}_i = w_i L_i + r K_i + \pi L_i T$$

and in per regional worker terms, the household income can be expressed as

$$\begin{aligned}
\frac{\mathcal{I}_i}{L_i} &= w_i + r\frac{K_i}{L_i} + \pi T \\
&= w_i + r\bar{k}_i + \pi T
\end{aligned}$$

Each household in both regions wishes to maximize the present value of intertemporal utility $\mathcal{U}_i$ and choose consumption and saving paths through time, for $i = 1, 2$:

$$\max_{(\bar{q}_i, \bar{k}_i) > 0} \int_0^{\infty} \frac{\bar{q}_i(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{9}$$

subject to the intertemporal budget constraint

$$\frac{\dot{K}_i}{L_i} \leq w_i + r\bar{k}_i + \pi T - E_i(\bar{q}_i, p_i)$$

the transversality constraint

$$\lim_{t \to \infty} \bar{k}_i(t) e^{-\int_0^{\nu} r(\nu) d\nu} = 0$$

and the non-negativity constraints for capital and consumption. Above, $\bar{q}_i$ is an index of per regional worker composite consumption, p_i is a vector of regional prices, $1/\theta$ is the intertemporal elasticity of substitution, ρ is the time preference rate, and $E_i(\bar{q}_i, p_i)$ is expenditures on consumption per regional worker in region i. In addition to the intertemporal decision of savings and aggregate consumption, households also have to make the intra-temporal decision concerning the allocation of expenditures among the consumption of different goods at each point in time, given their prices. At each point in time, each household consumes three types of goods, each indexed m, $m = 1, 2, 3$, a tradable good from region 1, a tradable good from region 2, and the home good in his/her respective region. The intra-temporal consumption composite per regional worker[4] in region i, is

$$\bar{q}_i = B_i(c_{i1} - \gamma_i)^{\lambda_1} c_{i2}^{\lambda_2} c_{i3}^{\lambda_3}$$

Note that the parameter $0 < \lambda_m < 1$, $m = 1, 2, 3$ denotes the expenditure share of consumption on good m in total expenditures. Households have Stone Geary (non-homothetic) preferences so as to account for differences in income across households in different regions, but otherwise the preferences across households are the same. The household in region i faces Stone-Geary preferences with subsistence level of consumption of γ_i in agricultural good. Households in both regions devote a smaller fraction of their incomes on agricultural good as their incomes increase over time; effectively, the income elasticities of demand for

[4] In other words, $\bar{q}_i$ is the intra-temporal felicity of the household from consumption in region i.

agricultural good for both households is less than one. Given the Stone-Geary preferences, the per regional worker expenditures of household-i are

$$\bar{\epsilon}_i = p_1\gamma_i + p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} \bar{q}_i$$

where p_1 is the price of the agricultural good (common in both regions), p_2 is the price of the manufactured good (common in both regions), and p_{i2} is the price of the home good in region i, $i = 1, 2$. As can be seen from the equation above, the per regional worker expenditure is composed of two components: spending on subsistence agricultural good required to meet the basic needs, and spending on remaning consumption goods. We refer to the former as subsistence expenditure which is identical for all households in all regions, and latter the supernumerary expenditures, which vary across regions. Denote the per regional worker supernumerary expenditures as $\bar{\mu}_i$ for household in region i :

$$\bar{\mu}_i = p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} \bar{q}_i$$

The aggregate flow budget constraint for household in region i is given by

$$\dot{K}_i = w_i L_i + r K_i + \pi L_i T - E_i$$

Now divide by L_i for the flow budget constraint expressed in per regional worker terms:

$$\frac{\dot{K}_i}{L_i} = [w_i L_i + r K_i + \pi L_i T - E_i] \div L_i$$

$$\dot{\bar{k}}_i = w_i + \bar{k}_i \left(r - \frac{\dot{L}_i}{L_i} \right) + \pi T - \bar{\epsilon}_i$$

Since

$$\ell_i = \frac{L_i}{L} \Rightarrow \frac{\dot{\ell}_i}{\ell_i} = \frac{\dot{L}_i}{L_i} - \overbrace{\frac{\dot{L}}{L}}^{=0} \Rightarrow \frac{\dot{\ell}_i}{\ell_i} = \frac{\dot{L}_i}{L_i}$$

we have for $i = 1, 2$:

$$\dot{\bar{k}}_i = w_i + \bar{k}_i \left(r - \frac{\dot{\ell}_i}{\ell_i} \right) + \pi T - \bar{\epsilon}_i$$

The household in region i has the following utility maximization problem:

$$\max_{(\bar{q}_i, \bar{k}_i)} \int_0^\infty \frac{\bar{q}_i(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

$$\text{subject to } \dot{\bar{k}}_i = w_i + \bar{k}_i \left(r - \frac{\dot{\ell}_i}{\ell_i} \right) + \pi T - \left(p_1\gamma_i + p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} \bar{q}_i \right)$$

$$\lim_{t\to\infty} \bar{k}_i(t) e^{-\int_0^{\nu} r(\nu)d\nu} = 0$$

and

$$\begin{aligned} \bar{q}_i &\geq 0 \text{ for all } t \\ \bar{k}_i(0) &\leq \bar{k}_{i,0} \end{aligned}$$

Since p_1 and p_2 are given world prices for the agricultural and manufacturing goods, respectively,

$$\frac{\dot{p}_1}{p_1} = \frac{\dot{p}_2}{p_2} = 0$$

then, the Euler equation, or the Ramsey rule for optimal saving for the household in region i can be written as

$$\frac{\dot{\bar{q}}_i}{\bar{q}_i} = \frac{1}{\theta}[r - \frac{\dot{\ell}_i}{\ell_i} - \rho - (\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}})]$$

Now note that expenditures per regional worker in region i are:

$$\begin{aligned} \bar{\epsilon}_i &= p_1\gamma + p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} \bar{q}_i \\ &= p_1\gamma_i + \bar{\mu}_i \end{aligned}$$

First component of the expenditures is the same across households. Total differentiating the supernumerary expenditures $\bar{\mu}_i$ with respect to time we get:

$$\dot{\bar{\mu}}_i = \lambda_3 \frac{\dot{p}_{i2}}{p_{12}} \bar{\mu} + \bar{\mu} \frac{\dot{\bar{q}}_i}{\bar{q}_i}$$

Divide by $\bar{\mu}_i$:

$$\begin{aligned} \frac{\dot{\bar{\mu}}_i}{\bar{\mu}_i} &= \lambda_3 \frac{\dot{p}_{i2}}{p_{i2}} + \frac{\dot{\bar{q}}_i}{\bar{q}_i} \\ &= \lambda_3 \frac{\dot{p}_{i2}}{p_{i2}} + \frac{1}{\theta}[r - \frac{\dot{\ell}_i}{\ell_i} - \rho - (\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}})] \\ &= \frac{1}{\theta}\left(r - \frac{\dot{\ell}_i}{\ell_i} - \rho\right) + (1 - \frac{1}{\theta})\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}} \\ &= \frac{1}{\theta}\left[\left(r - \frac{\dot{\ell}_i}{\ell_i} - \rho\right) - (1-\theta)\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}}\right] \end{aligned}$$

Note that per economywide labor, the supernumerary expenditures of the household in region i become:

$$\mu_i = \bar{\mu}_i \ell_i$$

Differentiate with respect to time and divide each side by per economywide labor supernumerary expenditures, $\mu_i = \bar{\mu}_i \ell_i$:

$$\frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{\bar{\mu}}_i}{\bar{\mu}_i} + \frac{\dot{\ell}_i}{\ell_i}$$

Then for each of the households in each region, $i = 1, 2$:

$$\begin{aligned} \frac{\dot{\bar{\mu}}_i}{\bar{\mu}_i} + \frac{\dot{\ell}_i}{\ell_i} &= \frac{1}{\theta}\left[\left(r - \frac{\dot{\ell}_i}{\ell_i} - \rho\right) - (1-\theta)\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}}\right] + \frac{\dot{\ell}_i}{\ell_i} \\ &= \frac{1}{\theta}(r-\rho) - \left(\frac{1-\theta}{\theta}\right)\lambda_3 \frac{\dot{p}_{i2}}{p_{i2}} + \left(\frac{1-\theta}{\theta}\right)\frac{\dot{\ell}_i}{\ell_i} \end{aligned}$$

If $\theta = 1$ (i.e. preferences are in log utility form, implying a desire to perfectly smooth consumption over time), then

$$\frac{\dot{\mu}_i}{\mu_i} = r - \rho$$

that is, the per economywide labor supernumerary expenditures of households in each region grow at the same rate, during transition and at the steady state i.e., they remain proportional at some constant rate $\bar{a} > 0$,

$$\frac{\mu_1(t)}{\mu_2(t)} = \frac{\mu_1(0).e^{[r(t)-\rho]t}}{\mu_2(0).e^{[r(t)-\rho]t}} = \frac{\mu_1(0)}{\mu_2(0)} = \bar{a}$$

3.1.3 Households' Residency Choice

The residency choice condition is an equilibrium condition where the household's instantaneous utility (intra-temporal felicity from consumption) in region 1 and the instantaneous utility in region 2 are equalized, that is, the household is indifferent between residing in region 1 or in region 2 at any given point in time. We can also call this the "migration equilibrium" condition. What this condition implies is that whenever there are discrepancies between the utility gained in regions, the household will be on the move from one region to the other. Then, migration in fact is a disequilibrium phenomenon. When the instantaneous utility across regions is equalized, migration ceases to occur.

Note that we represent the instantaneous utility of the household in region i as $\bar{q}_i$, then we require

$$\bar{q}_1 = \bar{q}_2$$

where the composite consumption per regional worker in region i is

$$\bar{q}_i = \frac{Q_i}{L_i}, i = 1, 2$$

The composite consumption per economywide worker in region i is:

$$q_i = \frac{Q_i}{L}$$

Then,

$$q_i L = Q_i = \bar{q}_i L_i$$

which implies that

$$\bar{q}_i = q_i \frac{L}{L_i} = \frac{q_i}{\ell_i}$$

Let $\ell_1 = \ell$ for simplicity of notation, and since $\ell_1 + \ell_2 = 1$, thus $\ell_2 = 1 - \ell$, hence the migration equilibrium condition $\bar{q}_1 = \bar{q}_2$ is equivalent to

$$\begin{aligned}
\frac{q_1}{\ell} &= \frac{q_2}{1-\ell} \\
&\Longrightarrow q_1(1-\ell) = \ell q_2 \\
&\Longrightarrow q_1 - q_1\ell = q_2\ell \\
&\Longrightarrow q_1 = \ell(q_1 + q_2) \\
&\Longrightarrow \ell = \frac{q_1}{q_1 + q_2}
\end{aligned}$$

From household's intra-temporal solution, we have

$$\begin{aligned}
q_1 &= \frac{\mu_1}{p_{12}^{\lambda_3}} \\
q_2 &= \frac{\mu_2}{p_{22}^{\lambda_3}}
\end{aligned}$$

where

$$\begin{aligned}
\mu_i &= p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} q_i \\
\text{with } q_i &= \frac{Q_i}{L}
\end{aligned}$$

Then,

$$\ell = \frac{\mu_1 p_{22}^{\lambda_3}}{\mu_1 p_{22}^{\lambda_3} + \mu_2 p_{12}^{\lambda_3}}$$

Intially, at $t = 0$, with $p_{12} = p_{22} = 1$, and given $\mu_1/\mu_2 = \bar{a}$;

$$\ell = \frac{\bar{a}}{1 + \bar{a}}$$

In transition; with $\mu_1/\mu_2 = \bar{a}$, we can write

$$\ell(w_1, r) = \frac{\bar{a} p_{22}(r)^{\lambda_3}}{\bar{a} p_{22}(r)^{\lambda_3} + p_{12}(w_1, r)^{\lambda_3}}$$

Note further from the migration equilibrium condition $\frac{q_1}{\ell} = \frac{q_2}{1-\ell}$ that

$$\begin{aligned}
\ell &= (1-\ell)\frac{q_1}{q_2} \\
&= (1-\ell)\frac{\frac{\mu_1}{p_{12}^{\lambda_3}}}{\frac{\mu_2}{p_{22}^{\lambda_3}}} \\
&= (1-\ell)\bar{a}\frac{p_{22}^{\lambda_3}}{p_{12}^{\lambda_3}}
\end{aligned}$$

Take log's on both sides of the equation, and take time derivatives,

$$\begin{aligned}
\ln[\ell] &= \ln(1-\ell) + \ln[\bar{a}] + \lambda_3 \ln[p_{22}] - \lambda_3 \ln[p_{12}] \\
\frac{\dot{\ell}}{\ell} &= \frac{-\dot{\ell}}{1-\ell} + \lambda_3 \frac{\dot{p}_{22}}{p_{22}} - \lambda_3 \frac{\dot{p}_{12}}{p_{12}} \\
&\implies \frac{\dot{\ell}}{\ell} = (1-\ell)\,\lambda_3 \left(\frac{\dot{p}_{22}}{p_{22}} - \frac{\dot{p}_{12}}{p_{12}}\right)
\end{aligned} \tag{10}$$

From the perspective of the change in wages, we have, (using profit maximization conditions)[5]:

$$\frac{\dot{\ell}}{\ell} = (1-\ell)\underbrace{\frac{1}{\varepsilon_{p_{12}}^{w_1}}}_{(+)}\lambda_3 \left(\underbrace{\frac{\varepsilon_{p_{12}}^{w_1} + \varepsilon_{p_{22}}^{w_1}}{\varepsilon_{p_{22}}^{w_2}}}_{\in(0,1]}\frac{\dot{w}_2}{w_2} - \frac{\dot{w}_1}{w_1}\right) \tag{11}$$

[5] See Appendix I for the derivation.

with

$$\frac{\varepsilon_{p12}^{w_1} + \varepsilon_{p22}^{w_1}}{\varepsilon_{p22}^{w_2}} = \frac{\frac{\phi}{a} - \delta}{1 - \delta}$$

Note that for the case when $\alpha \geq \phi$, i.e. when the rural region home good production is relatively more labor intensive than the urban region home good production,

$$\frac{\varepsilon_{p12}^{w_1} + \varepsilon_{p22}^{w_1}}{\varepsilon_{p22}^{w_2}} = \frac{\frac{\phi}{\alpha} - \delta}{1 - \delta} \leq 1$$

Then, as long as the rural wages grow at a faster rate than urban wages, i.e. $\frac{\dot{w}_2}{w_2} \leq \frac{\dot{w}_1}{w_1}$, migration out of the rural region, $\frac{\dot{\ell}}{\ell} < 0$ is satisfied:As can be seen from the results, as the rural wages rise at a faster rate than the urban wages, what is crucial in determining the direction of migration is the relative changes in the home good prices, which are directly affected by changes in wages, a cost factor in production. Hence as long as rural wages rise at a faster rate than urban wages, then it is expected that rural home good prices rise at a faster rate than urban home good prices, affecting the equilibrium utility of the household, and thus determining the direction of migration.

Although there is not enough empirical evidence due to data restrictions, as given in Figures 6, 7 and 8, a simple observation of the relationship between integral migration behavior and the regional home-good (health, education, housing and utilities) 2009-1010 price changes for Turkey show that areas with higher price changes have higher rates of net out-migration, and areas with relatively lower rates of price changes have net in-migration or lower rates of net out-migration. [6]:

[6] This observation is based on TURKSTAT data on 26 NUTS Level 2 regions of Turkey.

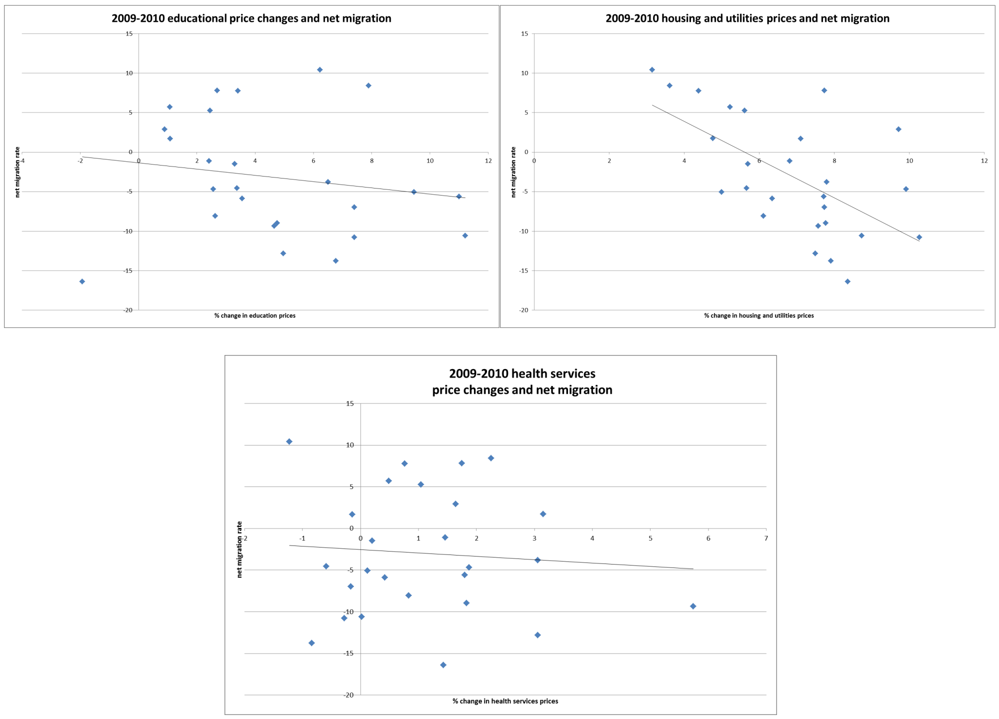


Figure 6: 2009-2010 changes in services prices and net migration

3.2 Characterization of equilibrium

Intra-temporal Equilibrium For equilibrium characterization, we require the intra-temporal equilibrium conditions from the production side:

$$MC_{21}\left(w_2, r\right) - p_{21} = 0 \quad (12)$$

$$MC_{12}\left(w_1, r\right) - p_{12} = 0 \quad (13)$$

$$MC_{22}(w_2, r) - p_{22} = 0 \quad (14)$$

At every point in time, labor markets in region 1 and region 2 clear independently, and the share of population residing at any given point in time ar the rural region (region 1) is ℓ, and at the urban region (region 2) is $1 - \ell$. Within a given period, firms in each region hire labor amongst the residents of their respective regions. That is, demand for labor by firms in region 1 must be equal to the fraction of households choosing to reside in region 1, and demand for labor in region 2 must be equal to the fraction of households choosing to reside in region 2:

$$\partial_{w_1} MC_{12}(w_1, r)\, y_{12} - \partial_{w_1} \pi(w_1, r)\, T = \ell \tag{15}$$

$$\partial_{w_2} MC_{21}(w_2, r)\, y_{21} + \partial_{w_2} MC_{22}(w_2, r)\, y_{22} = (1 - \ell) \tag{16}$$

The economy's capital market, on the other hand, is not segmented, and there is a single capital market for both regions:

$$-\partial_r \pi(w_1, r) + \partial_r MC_{12}(w_1, r)\, y_{12} + \partial_r MC_{21}(w_2, r)\, y_{21} + \partial_r MC_{22}(w_2, r)\, y_{22} = k \tag{17}$$

Home goods markets in both regions clear within respective region, and home goods market clearing condition in each region is given by $c_{i3} = y_{i2}$; that is, for $i = 1, 2$,

$$c_{i3} = \frac{\partial \epsilon_i}{\partial p_{i2}} = \frac{\partial \left(p_1 \gamma_i \ell_i + p_1^{\lambda_1} p_2^{\lambda_2} p_{i2}^{\lambda_3} \bar{q}_i \right)}{\partial p_{i2}} = \frac{\lambda_3}{p_{i2}} \mu_i = y_{i2},$$

or,

$$\begin{aligned} \frac{\lambda_3}{p_{12}(w_1, r)} \mu_1 &= y_{12} \\ \frac{\lambda_3}{p_{22}(w_2, r)} \mu_2 &= y_{22} \end{aligned}$$

Note that all variables here are per economywide labor. Given the per economywide labor expenditures ϵ_i of household in region i, the conditional demand function for each of the goods can be found as

$$\begin{aligned} c_{i1} &= \frac{\lambda_1 \left[\epsilon_i - p_1 \gamma_i \ell_i \right]}{p_1} + \gamma_i \ell_i \\ c_{i2} &= \frac{\lambda_2 \left[\epsilon_i - p_1 \gamma_i \ell_i \right]}{p_2} \\ c_{i3} &= \frac{\lambda_3 \left[\epsilon_i - p_1 \gamma_i \ell_i \right]}{p_{i2}} \end{aligned}$$

Finding the value for γ_i For finding the value for the subsistence level of agricultural good consumption, we need to make the assumption for what the expenditure shares would be at the steady state when the expenditure on subsistence consumption is negligable in total expenditures; in other words, when the preferences converge to Cobb-Douglas preferences.

Thus, we need to make an assumption on λ_1, λ_2 and λ_3, based on what they would be at the steady state. Here, we can take the (average) expenditure shares of developed economies, which are assumed to be at their steady states. Given λ_1, λ_2 and λ_3, and using the base year information from data,

$$
\begin{aligned}
\frac{\lambda_1(\epsilon_{i,0} - p_1\gamma_i\ell_i) + p_1\gamma_i\ell_i}{\epsilon_{i,0}} &= \frac{p_1 c_{i1,0}}{\epsilon_{i,0}} = \text{ initial share on agricultural good given in data} \\
\frac{\lambda_2(\epsilon_{i,0} - p_1\gamma_i\ell_i)}{\epsilon_{i,0}} &= \frac{p_2 c_{i2,0}}{\epsilon_{i,0}} = \text{ initial share on manufactured good given in data} \\
\frac{\lambda_3(\epsilon_{i,0} - p_1\gamma_i\ell_i)}{\epsilon_{i,0}} &= \frac{p_{i2} c_{i3,0}}{\epsilon_{i,0}} = \text{ initial share on services given in data}
\end{aligned}
$$

Inter-temporal Equilibrium and Transition Dynamics We obtain the economy-wide budget constraint by combining the budget for both regions for $i = 1, 2$:

$$\dot{k} = w_1\ell + w_2\,(1-\ell) + rk + \pi T - (\epsilon_1 + \epsilon_2)$$

With

$$
\begin{aligned}
\epsilon_i &= \mu_i + p_1\gamma_i\ell_i \\
\epsilon_1 + \epsilon_2 &= \mu_1 + \mu_2 + p_1\gamma_1\ell + p_1\gamma_2(1-\ell)
\end{aligned}
$$

we have

$$
\begin{aligned}
\dot{k} &= \mathtt{k}\,(w_1, r, k) \equiv && (18) \\
& w_1\;\ell(w_1, r) + w_2(r)(1 - \ell(w_1, r)) + rk && (19) \\
& +\pi(w_1, r)T - (\mu_1 + \mu_2 + p_1\gamma_1\ell(w_1, r) + p_1\gamma_2(1 - \ell(w_1, r)))
\end{aligned}
$$

where $k = k_1 + k_2$.

We now derive the differential equations for $\dot{w}_1$, $\dot{r}$ and $\dot{k}$. The law of motion for economy-wide capital per capita is:

$$
\begin{aligned}
\dot{k} &= w_1\ell(w_1, r) + w_2(r)(1 - \ell(w_1, r)) + rk + \pi(w_1, r)T \\
&\quad -(\mu_1 + \mu_2 + p_1\gamma_1\ell(w_1, r) + p_1\gamma_2(1 - \ell(w_1, r)))
\end{aligned}
$$

where

$$\begin{aligned}\frac{\mu_1}{\mu_2} &= \bar{a};\\ \mu_1 &= \bar{a}\mu_2\end{aligned}$$

$$\begin{aligned}\dot{k} &= w_1\ell(w_1,r) + w_2(r)(1-\ell(w_1,r)) + rk + \pi(w_1,r)T\\ &\quad -[(1+\bar{a})\mu_2 + p_1\gamma_1\ell(w_1,r) + p_1\gamma_2(1-\ell(w_1,r))]\\ &= w_1\ell(w_1,r) + w_2(r)(1-\ell(w_1,r)) + rk + \pi(w_1,r)T\\ &\quad -(1+\bar{a})\frac{p_{22}(r)y_{22}(w_1,r,k)}{\lambda_3} - (p_1\gamma_1\ell(w_1,r) + p_1\gamma_2(1-\ell(w_1,r))\end{aligned}$$

Now derive the law of motion equations for w_1 and r :

Total differentiate with respect to time the home goods market clearing conditions for each region:

$$\begin{aligned}\lambda_3\mu_1 &= p_{12}(w_1,r)y_{12}(w_1,r)\\ \lambda_3\mu_2 &= p_{22}(w_1,r)y_{22}(w_1,r,k)\end{aligned}$$

then solve the resulting two equations for $\dot{w}_1$ and $\dot{r}$:

$$\dot{w}_1 = f_1(w_1,r,k)$$

$$\dot{r} = f_2(w_1,r,k)$$

Note that $\dot{k}$ appears in both. We have a 3-differential-equation-system:

$$\begin{aligned}\dot{w}_1 &= f_1(w_1,r,k)\\ \dot{r} &= f_2(w_1,r,k)\\ \dot{k} &= f_3(w_1,r,k)\end{aligned}$$

to solve for the three endogenous variables, (w_1,r,k).

3.2.1 Solution strategy

Steady state In the long-run equilibrium in this economy, all endogenous variables are constant for all t, under the assumption that $\dot{k}=0$, in particular. Such an equilibrium is called the steady state equilibrium:

Definition 1 *A steady state is an equilibrium such that it satisfies all equilibrium conditions above, and given p_1, p_2, and k_0, all endogenous variables $(p_{12}(t), p_{22}(t))$, $(q_i(t))_{i=1,2}$, $(\omega_i(t),\ r(t))_{i=1,2}$, $(y_{ij}(t), l_{ij}(t), k_{ij}(t))_{i=1,2;j=1,2,3}$ and $\ell(t)$ are constant for all periods of time, t.*

By definition, at the steady state, it must be the case that

$$\dot{k} = 0 \tag{20}$$

$$\dot{r} = 0 \tag{21}$$

$$\dot{w}_i = 0 \tag{22}$$

$$q_i = 0 \tag{23}$$

$$\dot{\ell} = 0 \tag{24}$$

Then, from the Euler conditions, at the steady state,

$$r^{ss} = \rho \tag{25}$$

From the profit maximization conditions, the value w_2^{ss} can be found directly, which allows us to find the value p_{22}^{ss} directly, as well. On the other hand, without knowing the steady state value of w_1, we cannot calculate the steady state value for p_{12}. To find the steady state values of the remaining variables, we construct the system of six equations of two labor market clearing conditions, one capital market clearing condition, two home-good market clearing conditions, and one economywide budget constraint in six unknowns of $w_1, y_{12}, y_{21}, y_{22}, k$ and μ_2 : from the residency choice condition,

$$\ell^{ss} = \frac{(p_{22}^{ss})^{\lambda_3}\,\bar{a}}{(p_{22}^{ss})^{\lambda_3}\,\bar{a} + (p_{12}^{ss})^{\lambda_3}}$$

where

$$p_{22}^{ss} = \frac{a_{21}^{\frac{\phi}{\delta}}}{a_{22}}(r^{ss})^{\frac{\delta-\phi}{\delta}}$$

and the profit maximization condition for firm in sector-2 of Region-1,

$$p_{12}^{ss}(w_1) = \frac{1}{a_{12}} w_1^{\alpha}(r^{ss})^{1-\alpha}$$

Hence, at the steady state,

$$\ell^{ss}(w_1) = \frac{(p_{22}^{ss})^{\lambda_3}\,\bar{a}}{(p_{22}^{ss})^{\lambda_3}\,\bar{a} + (p_{12}^{ss}(w_1))^{\lambda_3}}$$

where

$$\frac{\mu_1}{\mu_2} = \bar{a}$$

Then we use $\mu_1 = \bar{a}\mu_2$ throughout.

Then, the system of six equations in six unknowns is

$$\begin{aligned}
-\frac{\partial \pi^a(w_1, r, p_1)}{\partial w_1}T + \frac{\partial MC_{12}(w_1, r)}{\partial w_1}y_{12} - \ell(w_1, r) &= 0 \\
\frac{\partial MC_{21}(w_2, r)}{\partial w_2}y_{21} + \frac{\partial MC_{22}(w_2, r)}{\partial w_2}y_{22} - (1 - \ell(w_1, r)) &= 0 \\
-\frac{\partial \pi^a(w_1, r, p_{11})}{\partial r}T + \frac{\partial MC_{12}(w_1, r)}{\partial r}y_{12} + \frac{\partial MC_{21}(w_2, r)}{\partial r}y_{21} + \frac{\partial MC_{22}(w_2, r)}{\partial r}y_{22} - k &= 0 \\
\frac{p_{12}y_{12}}{\lambda_3} - \bar{a}\mu_2 &= 0 \\
\frac{p_{22}y_{22}}{\lambda_3} - \mu_2 &= 0 \\
w_1\ell + w_2(1 - \ell) + rk + \pi T - (1 + \bar{a})\mu_2 - (p_1\gamma_1\ell(w_1, r) + p_1\gamma_2(1 - \ell(w_1, r))) &= 0
\end{aligned}$$

which is solved for the steady state values of $w_1^{ss}, \mu_2^{ss}, y_{12}^{ss}, y_{21}^{ss}, y_{22}^{ss}$, and k^{ss} given $r = r^{ss}$.

Transition path equilibrium Given the steady state values and initial conditions, the Time-Elimination Method by Mulligan and Sala-i-Martin (1991) and by Barro and Sala-i-Martin (2004) is utilized to numerically solve for the transition path equilibrium using the system of differential equations:

$$\begin{aligned}
\dot{w}_1(t) &= f_1(w_1(t), r(t), k(t)) \\
\dot{r}(t) &= f_2(w_1(t), r(t), k(t)) \\
\dot{k}(t) &= f_3(w_1(t), r(t), k(t))
\end{aligned}$$

3.3 Data and parameters of the model

For the solution of the model, we use numerical methods. GDP, production, expenditure, savings, and rural-urban allocation of labor data come from a typical developing economy such as Turkey. Based on 2006 data from the Turkish economy (obtained from TurkSTAT) and as explained below, some assumed parameter values, a two-region Social Accounting Matrix (SAM) is built to match the model's initial equilibrium. In 2006, 34 percent of total labor in Turkey was allocated in the rural region. Using the initial rural labor share of 34 percent, we find the constant ratio of supernumerary expenditures $\bar{a} = \mu_1/\mu_2 = 0.51$, while the ratios of rural and urban expenditures in economywide expenditures are 38 percent and 62 percent, respectively. The economywide saving rate in Turkey for 2006 is 16.6 percent. We find that (by way of calibrating the model's initial equilibrium to data) 33 percent of GDP is due to rural production (agriculture and rural services), and the remaining 67 percent is due to urban production (manufacturing and urban services). Foreign trade is assumed to be balanced at all times. Initial consumption and expenditure patterns of rural and urban households in Turkey in 2006, and estimated long-run consumption patterns (similar to developed country expenditure patterns, same for both rural and urban household types), are given as the following:

Parameter	**Symbol**	**Value**
Rural household		
Initial expenditure share on agricultural good		0.36
Initial expenditure share on manufacturing good		0.13
Initial expenditure share on services		0.51
Urban household		
Initial expenditure share on agricultural good		0.22
Initial expenditure share on manufacturing good		0.16
Initial expenditure share on services		0.62
Common parameters		
Long-run expenditure share on agricultural good	λ_1	0.12
Long-run expenditure share on manufacturing good	λ_2	0.18
Long-run expenditure share on services	λ_3	0.7
Elasticity of intertemporal substitution	$1/\theta$	1
Time preference rate	ρ	0.042

Table 3: Model's parameters

As for the factor elasticities, we use the following labor, capital and land shares in each sector to match the predictions of the model for rural to urban migration: urban manufacturing sector is relatively more capital intensive than both rural and urban services sector, and rural services sector is relatively more labor intensive than urban services sector. To match the data from Turkish economy, agricultural sector is taken as the least labor intensive sector (i.e. with the lowest share of labor payments in total factor payments among all sectors).

Factor shares	Labor	Capital	Land
Rural region			
Agriculture	0.25	0.6	0.15
Services	0.75	0.25	
Urban region			
Manufacturing	0.55	0.45	
Services	0.6	0.4	

Table 4: Factor elasticities in production

4 Results

We now present the model's results in Table 5 with the help of Rybczynski and Stolper-Samuelson Theorems. In Table 5, first column presents the initial values from data, second and third column from model results with migration, fourth and fifth columns present results from the model with no migration. *Rybczynski Theorem*: if the endowment of a factor increases, then the industry which uses that factor relatively intensively will (i) expand; (ii) expand more than proportionately to the increase in endowment; the other industry will contract. *Stopler-Samuelson Theorem*: increase in the price of good-j will lead to an increase in the rental rate of the factor used intensively in its production.

4.1 Results from the models with and without migration

		Initial value	Initial value from Model solution (with migration)	Steady state value (With migration)	Initial value from Model solution (No migration)	Steady state value (No migration)
Prices						
Interest rate	r	0.09	0.09	0.042	0.09	0.042
Rural wage	w1	278.66	246.4	1183.83	250.9	862.38
Urban wage	w2	335.52	332.2	625.94	331.80	625.94
Rural prices	p12	1	0.9	2.44	0.93	1.93
Urban prices	p22	1	1	1.07	1	1.07
Output						
GDP	GDP	576.32	567.43	1305.22	569.7	1295.64
Agricultural output	y11	95.64	111.8	180.97	107.41	306.84
Rural services output	y12	93.99	85.9	119.48	83.60	148.96
Manufacturing otuput	y21	203.24	223.6	261.71	233.50	140.87
Urban services output	y22	183.44	153.6	532.15	151.50	523.13
Agr output share	p11y11/GDP	0.166	0.197	0.139	0.190	0.237
Rural serv share	p12y12/GDP	0.163	0.138	0.224	0.140	0.222
Manuf. output share	p21y21/GDP	0.353	0.394	0.201	0.410	0.109
Urban serv. share	p22y22/GDP	0.318	0.270	0.437	0.260	0.433
Factor allocation						
Rural residency	L1	0.339	0.352	0.223	0.339	0.339
Urban residency	L2	0.661	0.647	0.777	0.661	0.661
Agricultural labor	L11	0.086	0.113	0.038	0.110	0.089
Rural services labor	L12	0.253	0.239	0.185	0.230	0.250
Manufacturing labor	L21	0.333	0.370	0.230	0.390	0.124
Urban services labor	L22	0.328	0.270	0.547	0.270	0.537
Agricultural capital	K11	637.63	736.6	2585.21	705.5	4383.46
Rural services capital	K12	261.10	215.7	1739.38	212.1	1709.91
Manufacturing capital	K21	1016.22	1104.4	2804.02	1150.3	1509.36
Urban services capital	K22	815.31	673.6	5431.68	662.4	5339.65
Consumption shares						
Rural household						
Agricultural good		0.36	0.390	0.12	0.39	0.12
Manufacturing		0.13	0.124	0.18	0.12	0.18
Services		0.51	0.470	0.7	0.48	0.7
Urban household						
Agricultural good		0.22	0.23	0.12	0.23	0.12
Manufacturing		0.16	0.16	0.18	0.16	0.18
Services		0.62	0.61	0.7	0.61	0.7

Table 5: Results from migration model

In the model, households are encouraged to save as long as the interest rate remains above the time preference rate, $r > \rho$, during the transition to steady state. This saving behavior allows them to increase per period consumption and utility, albeit at a decreasing rate, at the equilibrium. As capital deepening occurs along with household saving (since at equilibrium all household saving leads to equivalent increases in physical capital), according to the *Rybczynski Theorem* rural traded good supply increases, as the rural traded good is relatively more capital intensive than the rural home good production. Holding everything constant, including the regional allocation of labor, *Rybczynski Theorem* predicts that rural labor will be pulled into the rural traded sector out of the rural home good sector, and consequently rural home good supply decreases. However as incomes increase in the rural region (with a more efficient use of labor, wages increase and capital income increases with capital deepening), demand for all types of goods increases, and since the home good market must clear, price of home good must increase in order for the home good sector to compete for capital. Increase in rural wages is also simultaneously a result of the *Stopler-Samuelson Theorem.* Since rural traded good prices are fixed at world prices, rural home good price rises relatively compared to rural traded good price.

Similarly, in the urban region, as capital accumulates, since manufacturing production is relatively more capital intensive, manufacturing output tends to expand, holding all else constant, including the regional allocation of labor, urban home good output must contract. But as urban incomes rise with more efficient use of labor and capital deepening, household demand for home good rises, and with the urban home good market clearing condition, urban home good price rises. Since urban home good production is labor intensive, urban wages rise. As in the rural region, in the urban region, home good price rises relatively compared to traded good price, which is fixed at world price.

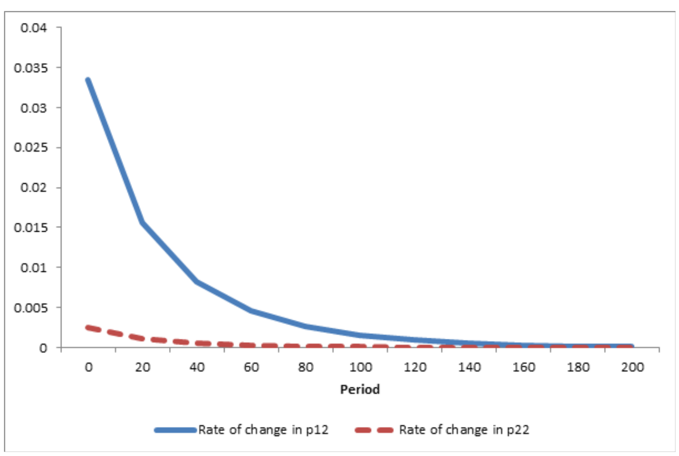


Figure 9: Rate of change in regional prices

1. *Rural wages rise proportionately more than the urban wages:* Comparing the two traded goods, the traded good in the rural region is relatively more capital intensive than the traded good in the urban region. Everyting else being constant, the capital intensive

sector in the rural region will increase capital demand proportionately more than the capital intensive sector in the urban region, which implies that the increase in the production of the capital intensive sector in rural region is proportionately more than that of the urban region. Then, as per the *Rybczynski Theorem,* the contraction required in the labor intensive good supply in the rural region is higher than the contraction required in the labor intensive good supply in the urban region, implying that the required increase in the price of home good in rural region is proportionately higher than the required increase in the price of home good in the urban region. This in turn implies that the rural wages must rise proportionately more than the urban wages. As shown in (11) in equilibrium, if the rural wages rise proportionately more than the urban wages, according to the household's indifference condition, there will be a decrease in the rural labor, increase in the urban labor.

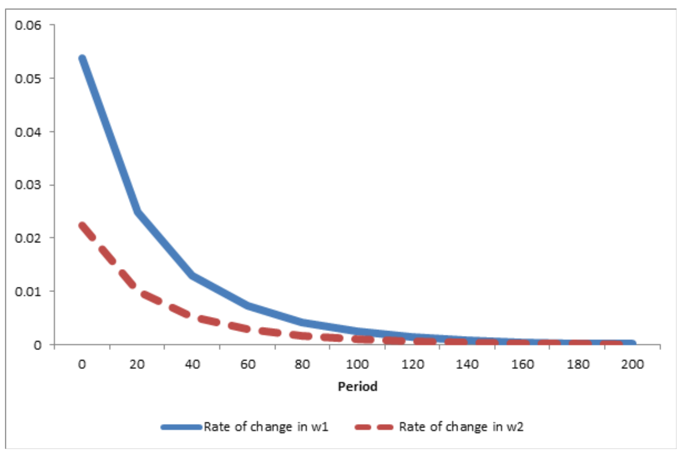


Figure 10: Rate of change in regional wages

2. *Rural labor migrates to urban region:* Hence, as capital deepening occurs, with the given factor elasticities, home good prices and wages in the rural region are rising proportionately at a higher rate than the home good prices and wages in the urban region, leading to a loss of labor in the rural region, since the per regional household utility at any given point in time is equalized across regions.

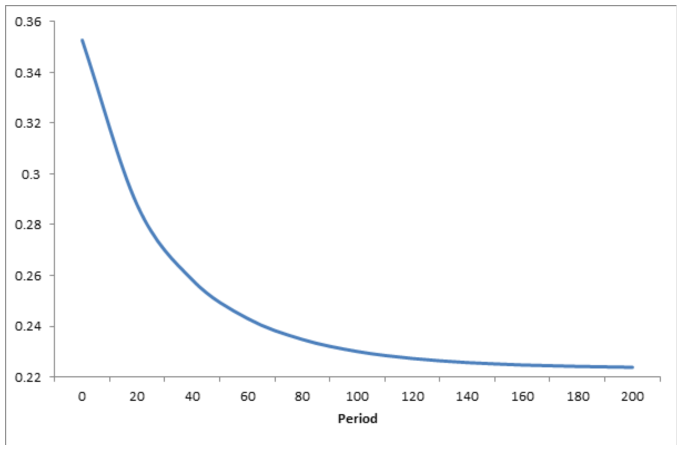


Figure 11: Residency choice

3. *Sectoral reallocation of labor*: As inter-regional migration continues, rural region sectors both lose labor to the urban region, on the other hand in the urban region, there is a drop in manufacturing labor while there is an increase in the urban services labor. This implies that urban services not only pulls labor from the rural region, it also pulls labor from the manufacturing sector as capital deepening occurs. BUT, the drop in the manufacturing labor would have been much more significant if no inter-regional migration were allowed, hence we can say that allowing migration across regions helps slow down the loss of labor out of manufacturing to urban services; without allowing migration across regions, all labor to support the increase in the urban services production must be pulled out of manufacturing; allowing migration slows down this reallocation of labor from manufacturing to urban services. The reason that urban services sector pulls labor from all other sectors is because as incomes rise, and as labor relocates to urban region, the demand for urban services is ever increasing. Since services markets must clear at all times, increase in demand for urban home good must be supported by increased production. Higher amounts of labor also requires higher amounts of capital, hence capital use in the urban services sector rises at a faster rate than in the other urban sector, manufacturing.

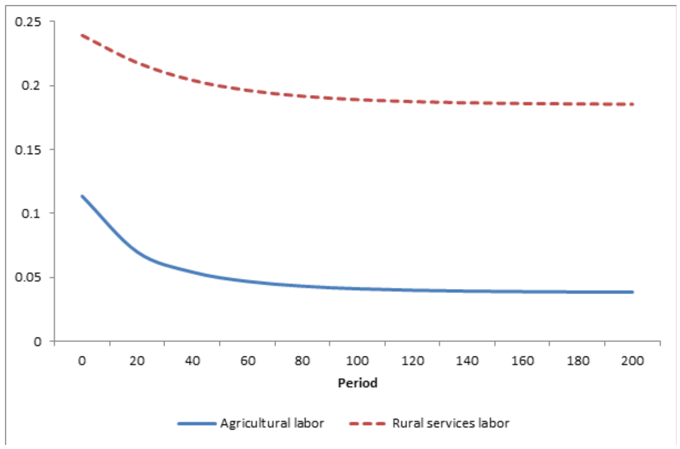


Figure 12: Rural region labor allocation

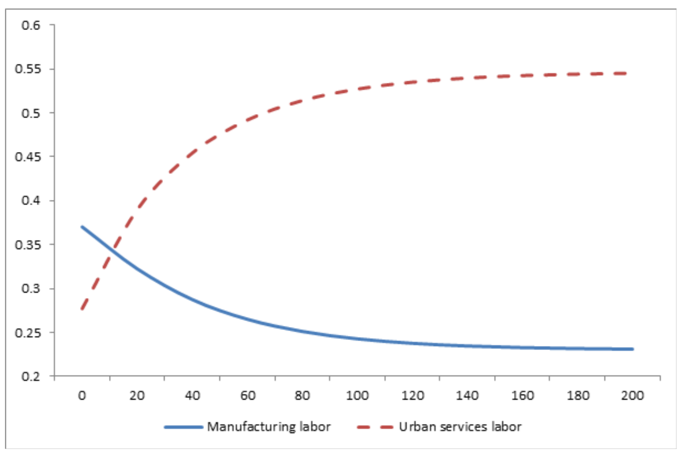


Figure 13: Urban region labor allocation

In fact, without migration, there is little or no reallocation of labor within the rural region. As the relatively capital intensive sector, rural traded good sector tends to pull labor out of the relatively labor intensive home good sector. But as incomes rise and as the demand for home good increases because of the changing demand pattern (the share of expenditure on home good increases with income), price of the home good increases and this keeps labor from flowing into the rural traded good sector. Similarly in the urban sector, holding all else constant, the relatively more capital intensive manufacturing sector tends to pull labor out of the relatively more labor intensive home good sector. But with the increase in incomes and increase in the expenditure share of home good in total expenditures of the households, demand for urban home good rises relatively more than the demand for urban traded good, the relative price of home good rises and home good sector pulls labor out of the manufacturing sector. Note here that the increase in urban home good labor is supported only by the drop in urban manufacturing labor; in the case with migration, it was also supported by the inflow of labor from rural region; hence in the case without migration, the loss of labor in the manufacturing sector is more pronounced.

Now we can ask the question why don't we see an increase in *rural* services labor, since with increase in income, demand for rural services must alo rise? Because as rural labor exits rural region, the demand for services output in rural region slows down. There are no longer as many people to increase the demand for services output, although the remaining people have an increased consumption demand for services. As these people move to urban region, the create *new* demand for urban services, hence the demand for urban services rises not only because of the change in incomes and demand patterns, but also because of new people moving into the region. Hence, increase in urban services production is at a higher rate than it would be without migration.

4. *Change in regional output shares:* Figures 14 and 15 present the evolution of sectoral output shares in GDP in the model with migration and the model without migration. Results from both the model with migration and the model wthout migration show that over time, the contribution of the urban and rural services production to GDP increase, while that of the manufacturing production decreases. In the model with migration, there is a distinctive decrease in the share of agricultural production, while in the model with no migration (where the labor is not allowed to move to the urban region), the share of agriculture in GDP is rather stable, and even increases somewhat. What must be noted in comparing the results from the two models is that the fall in the share of manufacturing is much more pronounced in the no-migration model than in the migration model. In the migration model, part of the labor to support the increase in the urban services comes from the rural region, thus the pressure on the manufacturing sector to reallocate labor to the urban services sector is dampened. On the other hand in the model where no migration is allowed, all labor needed to support the increase in the urban services sector must be through reallocation from the manufacturing sector. As manufacturing loses labor to the urban services, and as this sector cannot compete for capital as much as the services sector (as manufacturing price is fixed at the world prices), its share in total production decreases as well.

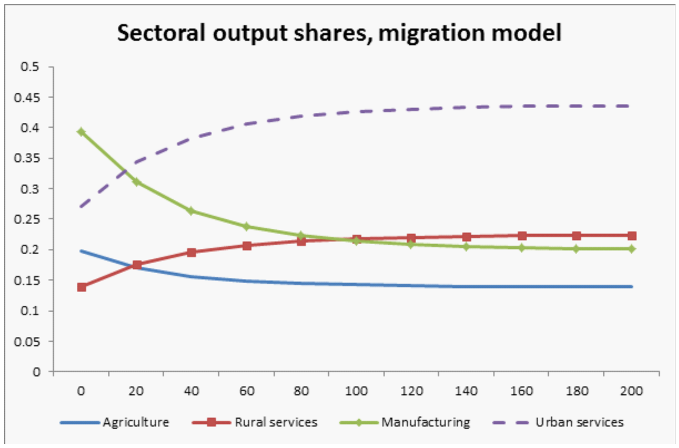


Figure 14: Sectoral output shares, migration model

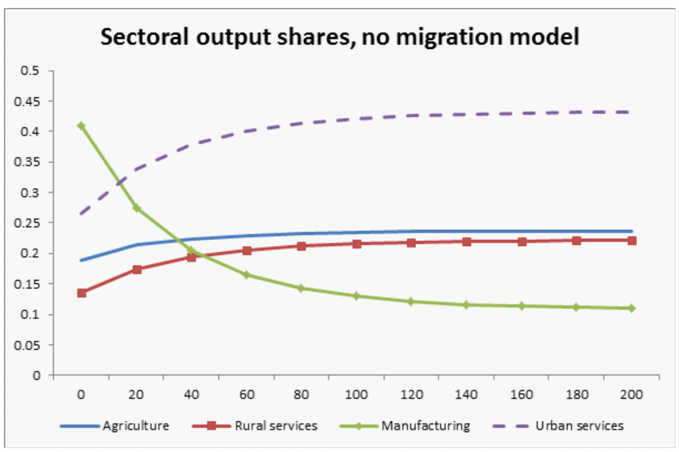


Figure 15: Sectoral output shares, no-migration model

5. *Change in regional incomes and utility:* Figure 16 depicts the evolution of the share of rural income in real GDP (excluding the effect of changing relative prices). With migration of rural labor to the urban region, the share of rural income in real GDP drops from 35 percent to a little less than 20 percent. Without migration, this share remains around 35 percent.

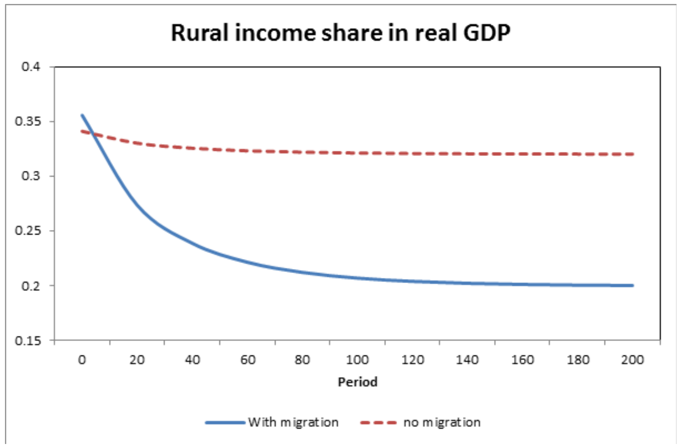


Figure 16: Rural income share in Real GDP

When we examine the income of the rural household with and without migration, we observe that the level of income per rural household is higher in the case with migration than in the case without migration. First of all, in the case without migration, the share of rural labor does not change and the rural wages rise at a slower rate than the case with migration. Also as can be seen from Figure 17.2, the utility of the rural household is less than what it would be with migration. On the other hand, the urban household is worse off with migration of rural labor compared to the case without migration. Note that under the migration case, the utility per rural household and the utility per urban household are identical to allow for the migration equilibrium. Basically we can say that the rural household benefits from migration in terms of relatively higher wages and higher utility at household level.

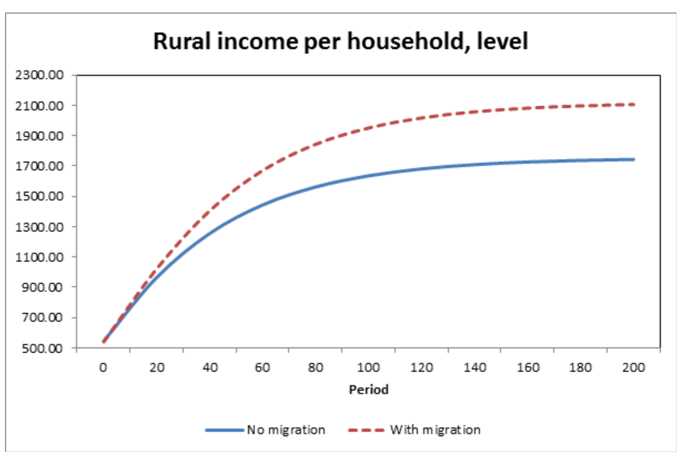


Figure 17.1: Rural income level per household

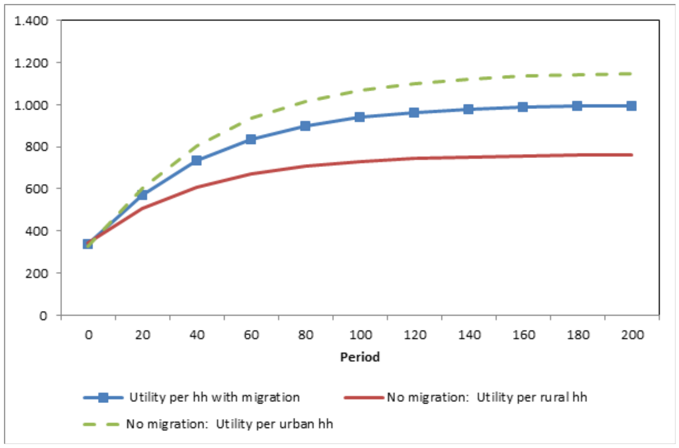


Figure 17.2: Utility per household, with and without migration

4.2 Analysis of contributions to growth in real GDP per capita

As we have shown in the previous section, the growth equation for real GDP per capita can be formulated as[7]

$$
\begin{aligned}
\frac{\dot{y}}{y} &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}} + \frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}} + \frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}} + \frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}} \\
&\quad + \dot{\ell}_2\left(\frac{y_{21}-y_{12}}{y}\right) + \dot{\ell}_{11}\left(\frac{y_{11}-y_{12}}{y}\right) + \dot{\ell}_{22}\left(\frac{y_{22}-y_{21}}{y}\right)
\end{aligned}
$$

According to this expression, the growth in aggregate output per economywide labor can be decomposed into: contributions of each sector weighted by their sectoral shares in aggregate output; the reallocation of labor from rural to urban region $\dot{\ell}_2\left(\frac{y_{21}-y_{12}}{y}\right)$; the reallocation of labor within rural region, $\dot{\ell}_{11}\left(\frac{y_{11}-y_{12}}{y}\right)$; and the reallocation of labor within the urban region $\dot{\ell}_{22}\left(\frac{y_{22}-y_{21}}{y}\right)$. We noted that the reallocation of labor from rural to urban is weighted by the change in productivity from rural services to manufacturing; the reallocation of labor within rural region is weighted by the change in productivity from agriculture to services, and similarly the rellocation of labor within urban region is weighted by the change

[7]Full derivation is in Appendix II.

in productivity from manufacturing to services. In the following subsections, we elaborate on the evolution of contribution of each sector and labor reallocation within and across regions[8].

4.2.1 Contributions of sectoral output

The evolution of percentage contributions of each sector's output to real GDP in the model with migration and without migration are depicted in Figures 18.1 and 18.2. What is similar in both cases is that the contribution of urban services output to growth in real GDP is the highest, increasing from 30 percent contribution to growth of real GDP up to about 60 percent. What is strikingly different beween the two models is the behavior of the contribution of the manufacturing output. The contribution of manufacturing output to real GDP growth in decreases from 45 percent to 27 percent in the migration model, and to 16 percent in the no-migration model, which implies that in the urban region, since the urban services contribution remains more or less the same, the migration of rural labor to urban region benefits the manufacturing sector; without migration, the contribution of manufacturing to the overall economy becomes less important, as manufacturing sector loses labor to the services sector. Another striking difference between the mideol with migration and without migration is the behavior of the contribution of agricultural output to the growth of real GDP. In the model with migration, we see a distinct decrease in the contribution of agriculture as labor exits this sector and as it loses competitiveness to the rural and urban services sectors in attracting capital. However in the model without migration, there is not a significant movement of labor within the rural region between agricultural and rural servives sectors, and hence the agricultural sector does not have to compete with the rural services sector for capital as much as it does as in the case with migration, and as a result does not lose its contribution to real GDP growth as much as it does in the model with migration. Similarly for the rural services, since there is not much reallocation of labor across sectors within the rural region, the contribution of the rural services output to real GDP growth remains the same throughout the transition.

[8] A detailed account of the contributions is given in Appendix III.

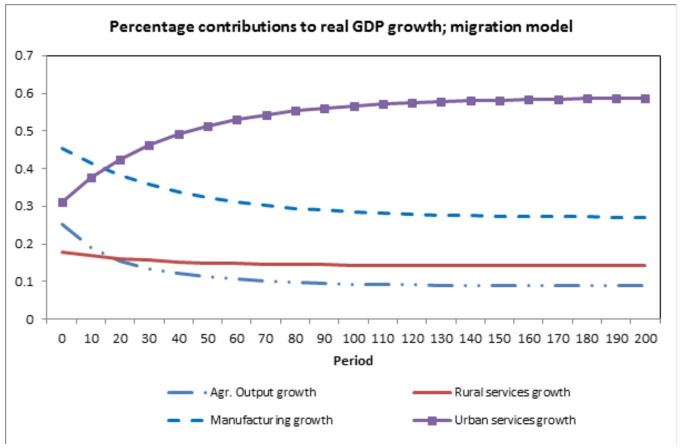


Figure 18.1: Contributions to real GDP growth; migration model

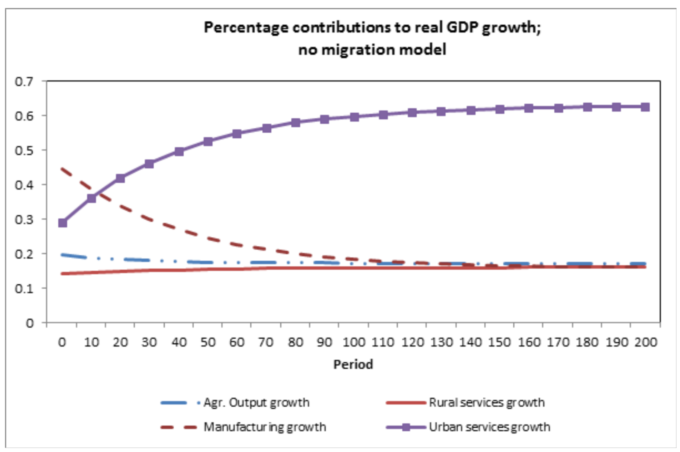


Figure 18.2: Contributions to real GDP growth; no migration model

4.2.2 Contributions of labor reallocation

Firstly, in the migration model, the contribution of the reallocation of labor from the rural region to the urban region is represented by the term $\dot{\ell}_2\left(\frac{y_{21}-y_{12}}{y}\right)$, which is the change in urban labor multiplied by the change in labor productivity from rural to urban region relative to total productivity. Initially the contribution of labor reallocation from rural to urban is 12 percent of total real GDP growth, and over time it drops to 9.5 percent, and it remains a positive contribution throughout the transition. Since $\dot{\ell}_2$ is positive with rural labor moving to the urban region, we can deduce that $\left(\frac{y_{21}-y_{12}}{y}\right)$ is also positive throughout the transition, as $\dot{\ell}_2 \left(\frac{y_{21}-y_{12}}{y}\right)$ is positive but diminishing. Basically, there is a gain in labor productivity as it is a positive contributor to overall real GDP growth in labor migration, but this contribution slightly declines towards the steady state, as the productivity gain cannot keep up with the movement of labor from rural to urban region.

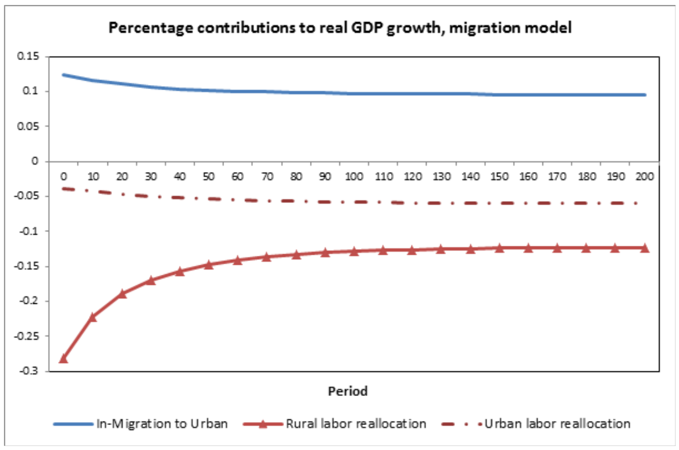


Figure 19.1: Contributions to real GDP growth; migration model

When we look at the contributions of labor reallocation within the regions, we see a different pattern. Mainly, labor reallocation across sectors within both regions contribute negatively to overall real GDP growth. In the rural region, within region labor reallocation contribution is represented by the term $\dot{\ell}_{11} \left(\frac{y_{11}-y_{12}}{y}\right)$, and in the urban region it is $\dot{\ell}_{22} \left(\frac{y_{22}-y_{21}}{y}\right)$. In both regions, throughout transition,

$$\begin{aligned} \dot{\ell}_{11} \left(\frac{y_{11}-y_{12}}{y}\right) &< 0, \\ \dot{\ell}_{22} \left(\frac{y_{22}-y_{21}}{y}\right) &< 0 \end{aligned}$$

Since $\dot{\ell}_{11} < 0$ throughout the transition, the negative contribution of the rural region labor reallocation is due to the actual movement of labor from agriculture to services. In the urban region, $\dot{\ell}_{22} > 0$, hence the negative contribution is due to the decrease in labor productivity as labor moves from manufacturing to urban services. Regardless of the reason for negative contribution, reallocation of labor *within* both regions prove to be detrimental to overall real GDP growth, while reallocation or labor *across* regions is beneficial for real GDP growth.

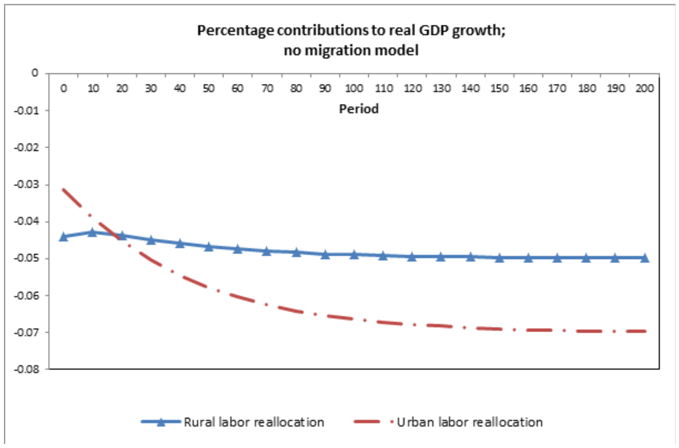


Figure 19.2: Contributions to real GDP growth; no migration model

When we examine the case without migration, and consider only the contributions of labor reallocation *within* the regions, we still see that contributions to real GDP growth are negative; but in this case, since there is not a very significant movement of labor within the rural region, the negative contribution remains more of less the same throughout the transition. For the urban region, on the other hand, we see an even larger within region movement of labor compared to the migration case, and as labor continues to move from manufacturing sector to urban services sector, the loss of productivity increases further, leading to an increasingly negative contribution of urban labor reallocation over time.

5 Conclusions and Comments

In our model, structural change or the change in sectoral composition of GDP occurs due both to changes in demand and changes in factor allocation across sectors, including changes in labor allocation across regions due to migration. Hence, as in real economies, we see the combined impact of both demand-side and supply-side factors on structural transformation. In fact, during transition towards the steady state, these two factors play out together, and their combined effect is enhanced by the inclusion of labor migration across regions. That is, previous models which do not consider labor migration across regions may be underestimating the effect of labor reallocation on structural transformation. We believe that accounting for this effect is one of the main contributions of our model. Furthermore, we find a positive and fairly significant contribution of labor reallocation across regions on overall growth, which is overlooked in other models of structural change which do not account for migration of labor across regions. During the transition towards the steady state, the contribution of rural to urban migration of labor to overall real GDP growth accounts for from 12 percent initially to 9.6 percent near the steady state.

One of the most significant results from our model appear when we compare the sectoral output distributions for the cases with and without rural-urban migration of labor. In both cases, we see a distinctive increase in the share of services and a decline in the share of manufacturing, while a distinctive decrease in the share of agriculture in the model with migration, and a slight increase in the share of agriculture in the model with no migration. What must be noted in comparing the results from the two models is that the fall in the share of manufacturing is much more pronounced in the no-migration model than in the migration model. In the migration model, part of the labor to support the increase in the urban services comes from the rural region, thus the pressure on the manufacturing sector to reallocate labor to the urban services sector is dampened. On the other hand in the model where no migration is allowed, all labor needed to support the increase in the urban services sector must be through reallocation from the manufacturing sector. We can safely conclude from these results that rural-urban migration of labor works to support urban manufacturing, and without rural-urban migration of labor, the share of manufacturing in total GDP would be much smaller than otherwise[9].

Another notable result arises when we compare the per household utilities of rural and urban households under the cases with and without migration. For instance, utility per rural household is higher under the case with migration compared to the case with migration; on the other hand, utility per urban household is higher under the case without migration compared to the case with migration[10]. Essentially, we can say that rural household (those stay behind in the rural region) benefits from migration in terms of relatively higher wages and higher utility at household level.

In the model with migration, time to double to real GDP is 25.3 periods, and the half-life of adjustment to the steady state equilibrium is 27.4 periods, while in the model with no migration, 26.2 periods and 29.3 peridods, respectively. This indicates to us that the transition to the steady state in the model with no migration is slower than that in the

[9] Although the share of manufacturing is declining in both cases.

[10] Here, we must recall that under migration, utility per urban household and utility per rural household are the same (implying equilibrium).

model with migration. Although the aforementioned periods are fairly close to eachother, nevertheless we can still conclude that transition, thus growth, is faster in the model with migration, internal migration speeds up the transition process. One explanation may be that labor is leaving agriculture fast in the model with migration, while in the model without migration, there is little or no change in the labor allocation within the rural region away from agriculture. When labor is allowed to migrate from rural to urban, it moves to support manufacturing, which is relatively more productive than agriculture.

The basic model introduced in this study can be broadened to include transaction costs, both in terms of the movement of goods across regions, and the movement of labor across regions. Inclusion of both types of transaction costs are expected to affect the differences in cost-of-living across regions, thus have an effect on the migration decision of the households. Another possible extension for studying rural-urban migration would be to consider household's utility from migrating to urban areas, where the households take advantage, or derive direct utility from the urban amenities, or public goods. This pattern of preferences is expected to have an impact on the price of urban services, and thus urban cost-of-living, affecting the residency choice of households.

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6 Appendix I

From profit maximization conditions, we have

$$
\begin{aligned}
w_1 &= \frac{p_{12}^{1/\alpha} a_{12}^{1/\alpha} (p_{21}a_{21})^{\frac{\phi}{\delta-\phi}(\frac{1-\alpha}{\alpha})}}{(p_{22}a_{22})^{\frac{\delta}{\delta-\phi}(\frac{1-\alpha}{\alpha})}} \\
w_2 &= \frac{(p_{21}a_{21})^{\frac{1-\phi}{\delta-\phi}}}{(p_{22}a_{22})^{\frac{1-\delta}{\delta-\phi}}};
\end{aligned}
$$

where labor shares are: for rural home good production $0<\alpha<1$, for manufacturing $0<\delta<1$, and for urban home good production $0<\phi<1$. Here, if we assume that $\delta<\alpha,\phi$ (manufacturing is relatively more capital intensive than both services sectors)

$$
\begin{aligned}
\frac{\dot{w}_2}{w_2} &= -\left(\frac{1-\delta}{\delta-\phi}\right)\frac{\dot{p}_{22}}{p_{22}} = \varepsilon_{p_{22}}^{w_2}\frac{\dot{p}_{22}}{p_{22}}; \\
\varepsilon_{p_{22}}^{w_2} &= \frac{\partial w_2}{\partial p_{22}}\frac{p_{22}}{w_2} = -\left(\frac{1-\delta}{\delta-\phi}\right) > 0 \\
\frac{\dot{w}_1}{w_1} &= \frac{1}{\alpha}\frac{\dot{p}_{12}}{p_{12}} - \frac{\delta}{\delta-\phi}\left(\frac{1-\alpha}{\alpha}\right)\frac{\dot{p}_{22}}{p_{22}} \\
&= \varepsilon_{p_{12}}^{w_1}\frac{\dot{p}_{12}}{p_{12}} + \varepsilon_{p_{22}}^{w_1}\frac{\dot{p}_{22}}{p_{22}}; \quad \varepsilon_{p_{12}}^{w_1} > 0, \varepsilon_{p_{22}}^{w_1} > 0
\end{aligned}
$$

Solve for price changes as functions of changes in w_1 and w_2 and substitute into (10)

$$
\frac{\dot{p}_{22}}{p_{22}} = \frac{\dot{w}_2}{w_2}\frac{1}{\varepsilon_{p_{22}}^{w_2}}
$$

$$
\begin{aligned}
\frac{\dot{w}_1}{w_1} &= \varepsilon_{p_{12}}^{w_1}\frac{\dot{p}_{12}}{p_{12}} + \frac{\varepsilon_{p_{22}}^{w_1}}{\varepsilon_{p_{22}}^{w_2}}\frac{\dot{w}_2}{w_2} \\
\varepsilon_{p_{22}}^{w_2}\frac{\dot{w}_1}{w_1} - \varepsilon_{p_{22}}^{w_1}\frac{\dot{w}_2}{w_2} &= \varepsilon_{p_{22}}^{w_2}\varepsilon_{p_{12}}^{w_1}\frac{\dot{p}_{12}}{p_{12}} \\
\frac{\dot{p}_{12}}{p_{12}} &= \frac{1}{\varepsilon_{p_{12}}^{w_1}\varepsilon_{p_{22}}^{w_2}}\left(\frac{\dot{w}_1}{w_1}\varepsilon_{p_{22}}^{w_2} - \varepsilon_{p_{22}}^{w_1}\frac{\dot{w}_2}{w_2}\right)
\end{aligned}
$$

Thus, using $\frac{\dot{\ell}}{\ell} = (1-\ell)\,\lambda_3 \left(\frac{\dot{p}_{22}}{p_{22}} - \frac{\dot{p}_{12}}{p_{12}}\right)$;

$$\begin{aligned}
\frac{\dot{\ell}}{\ell} &= (1-\ell)\,\lambda_3 \left(\frac{\dot{w}_2}{w_2}\frac{1}{\varepsilon_{p_{22}}^{w_2}} - \frac{1}{\varepsilon_{p_{12}}^{w_1}\varepsilon_{p_{22}}^{w_2}}\left(\frac{\dot{w}_1}{w_1}\varepsilon_{p_{22}}^{w_2} - \varepsilon_{p_{22}}^{w_1}\frac{\dot{w}_2}{w_2}\right)\right) \\
&= (1-\ell)\,\lambda_3 \left(\frac{\dot{w}_2}{w_2}\frac{\varepsilon_{p_{12}}^{w_1}}{\varepsilon_{p_{12}}^{w_1}\varepsilon_{p_{s2}}^{w_2}} - \frac{1}{\varepsilon_{p_{12}}^{w_1}\varepsilon_{p_{22}}^{w_2}}\left(\frac{\dot{w}_1}{w_1}\varepsilon_{p_{22}}^{w_2} - \varepsilon_{p_{22}}^{w_1}\frac{\dot{w}_2}{w_2}\right)\right) \\
&= (1-\ell)\,\frac{1}{\varepsilon_{p_{12}}^{w_1}}\lambda_3 \left(\frac{\varepsilon_{p_{12}}^{w_1} + \varepsilon_{p_{22}}^{w_1}}{\varepsilon_{p_{22}}^{w_2}}\frac{\dot{w}_2}{w_2} - \frac{\dot{w}_1}{w_1}\right)
\end{aligned}$$

7 Appendix II: Accounting for Growth in real GDP in the Regional Model with Internal Migration

In our model economy, the total (overall) GDP or output value of the economy is Y :

$$Y = p_{11}Y_{11} + p_{12}Y_{12} + p_{21}Y_{21} + p_{22}Y_{22}$$

Since we have changing relative prices in both regions, p_{12} and p_{22} respectively, we redefine the real GDP in rural and urban regions as

$$\begin{aligned}
\tilde{Y}_{Rural} &= \frac{p_{11}Y_{11} + p_{12}Y_{12}}{p_{12}} \\
&= \tilde{Y}_{11} + Y_{12} \\
\tilde{Y}_{Urban} &= \frac{p_{21}Y_{21} + p_{22}Y_{22}}{p_{22}} \\
&= \tilde{Y}_{21} + Y_{22}
\end{aligned}$$

We rewrite economywide real GDP $\tilde{Y}$ in per capita terms (per economywide labor L) :

$$\begin{aligned}
\frac{\tilde{Y}}{L} &= \frac{\tilde{Y}_{11}}{L} + \frac{Y_{12}}{L} + \frac{\tilde{Y}_{21}}{L} + \frac{Y_{22}}{L} \\
&= \frac{\tilde{Y}_{11}}{L_{11}} \times \frac{L_{11}}{L} + \frac{Y_{12}}{L_{12}} \times \frac{L_{12}}{L} + \frac{\tilde{Y}_{21}}{L_{21}} \times \frac{L_{21}}{L} + \frac{Y_{22}}{L_{22}} \times \frac{L_{22}}{L} \\
&= y_{11} \times \ell_{11} + y_{12} \times \ell_{12} + y_{21} \times \ell_{21} + y_{22} \times \ell_{22}
\end{aligned}$$

Take total differentials with respect to time on both sides of the equation[11]:

$$\dot{y} = \dot{y}_{11}\ell_{11} + y_{11}\dot{\ell}_{11} + \dot{y}_{12}\ell_{12} + y_{12}\dot{\ell}_{12} + \dot{y}_{21}\ell_{21} + y_{21}\dot{\ell}_{21} + \dot{y}_{22}\ell_{22} + y_{22}\dot{\ell}_{22}$$

Divide both sides by y :

$$\begin{aligned}
\frac{\dot{y}}{y} &= \dot{y}_{11}\ell_{11}\frac{1}{y} + y_{11}\dot{\ell}_{11}\frac{1}{y} + \dot{y}_{12}\ell_{12}\frac{1}{y} + y_{12}\dot{\ell}_{12}\frac{1}{y} + \dot{y}_{21}\ell_{21}\frac{1}{y} + y_{21}\dot{\ell}_{21}\frac{1}{y} \\
&\quad + \dot{y}_{22}\ell_{22}\frac{1}{y} + y_{22}\dot{\ell}_{22}\frac{1}{y} \\
&= \dot{y}_{11}\ell_{11}\frac{1}{y} + \dot{y}_{12}\ell_{12}\frac{1}{y} + \dot{y}_{21}\ell_{21}\frac{1}{y} + \dot{y}_{22}\ell_{22}\frac{1}{y} \\
&\quad + \frac{1}{y}(y_{11}\dot{\ell}_{11} + y_{12}\dot{\ell}_{12} + y_{21}\dot{\ell}_{21} + y_{22}\dot{\ell}_{22})
\end{aligned}$$

[11] We follow the same strategy as in Dekle and Vandenboroucke (2010).

which yields the per capita output growth in our model economy. Note that:

$$\ell_{ij}\frac{1}{y} = \frac{L_{ij}}{L}\frac{L}{Y} = \frac{L_{ij}}{Y} = \frac{L_{ij}}{Y_{ij}}\frac{Y_{ij}}{Y} = \frac{1}{y_{ij}}\frac{Y_{ij}}{Y}$$

Then,

$$\begin{aligned}
\frac{\dot{y}}{y} &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}} + \frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}} + \frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}} + \frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}} \\
&\quad + \frac{1}{y}(y_{11}\dot{\ell}_{11} + y_{12}\dot{\ell}_{12} + y_{21}\dot{\ell}_{21} + y_{22}\dot{\ell}_{22})
\end{aligned}$$

Now, from the labor markets we know that

$$\begin{aligned}
L_{11} + L_{12} &= L_1 \\
L_{21} + L_{22} &= L_2 \\
\ell_{11} + \ell_{12} &= \ell_1 \\
\ell_{21} + \ell_{22} &= \ell_2 = 1 - \ell_1 \\
\dot{\ell}_{11} + \dot{\ell}_{12} &= -(\dot{\ell}_{21} + \dot{\ell}_{22})
\end{aligned}$$

Also denote the regional changes in urban and rural labor as:

$$\begin{aligned}
\dot{\ell}_1 &= \dot{\ell}_{11} + \dot{\ell}_{12} \\
\dot{\ell}_2 &= \dot{\ell}_{21} + \dot{\ell}_{22}
\end{aligned}$$

So that

$$\dot{\ell}_1 = -\dot{\ell}_2$$

Now rearrange the growth equation as follows:

$$\begin{aligned}
\frac{\dot{y}}{y} &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}} + \frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}} + \frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}} + \frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}} \\
&\quad + \frac{1}{y}[y_{11}\dot{\ell}_{11} + y_{12}(\dot{\ell}_1 - \dot{\ell}_{11}) + y_{21}(\dot{\ell}_2 - \dot{\ell}_{22}) + y_{22}\dot{\ell}_{22}] \\
&= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}} + \frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}} + \frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}} + \frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}} \\
&\quad + \frac{1}{y}[y_{11}\dot{\ell}_{11} + y_{12}\dot{\ell}_1 - y_{12}\dot{\ell}_{11} + y_{21}\dot{\ell}_2 - y_{21}\dot{\ell}_{22} + y_{22}\dot{\ell}_{22}] \\
&= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}} + \frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}} + \frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}} + \frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}} \\
&\quad + \dot{\ell}_2\left(\frac{y_{21} - y_{12}}{y}\right) + \dot{\ell}_{11}\left(\frac{y_{11} - y_{12}}{y}\right) + \dot{\ell}_{22}\left(\frac{y_{22} - y_{21}}{y}\right)
\end{aligned}$$

According to this expression, the growth in aggregate output per economywide labor can be decomposed into: contributions of each sector weighted by their sectoral shares in aggregate output; the reallocation of labor from rural to urban region $\dot{\ell}_2\left(\frac{y_{21}-y_{12}}{y}\right)$; the reallocation of labor within rural region, $\dot{\ell}_{11}\left(\frac{y_{11}-y_{12}}{y}\right)$; and the reallocation of labor within the urban region $\dot{\ell}_{22}\left(\frac{y_{22}-y_{21}}{y}\right)$. Note that the reallocaton of labor from rural to urban is weighted by the change in productivity from rural services to manufacturing; the reallocation of labor within rural region is weighted by the change in productivity from agriculture to services, and similarly the rellocation of labor within urban region is weighted by the change in productivity from manufacturing to services.

In the case *without* migration, there is no change in the regional total labor, although there is movement across sectors, hence we have

$$\begin{aligned}\dot{\ell}_{11}+\dot{\ell}_{12} &= 0\\ \dot{\ell}_{21}+\dot{\ell}_{22} &= 0\end{aligned}$$

Thus we can write

$$\begin{aligned}\dot{\ell}_{11} &= -\dot{\ell}_{12}\\ \dot{\ell}_{21} &= -\dot{\ell}_{22}\end{aligned}$$

Our growth equation becomes

$$\begin{aligned}\frac{\dot{y}}{y} &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}}+\frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}}+\frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}}+\frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}}\\ &\quad+\frac{1}{y}(y_{11}\dot{\ell}_{11}+y_{12}\dot{\ell}_{12}+y_{21}\dot{\ell}_{21}+y_{22}\dot{\ell}_{22})\\ &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}}+\frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}}+\frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}}+\frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}}\\ &\quad+\frac{1}{y}(-y_{11}\dot{\ell}_{12}+y_{12}\dot{\ell}_{12}-y_{21}\dot{\ell}_{22}+y_{22}\dot{\ell}_{22})\\ &= \frac{\dot{y}_{11}}{y_{11}}\frac{\tilde{Y}_{11}}{\tilde{Y}}+\frac{\dot{y}_{12}}{y_{12}}\frac{Y_{12}}{\tilde{Y}}+\frac{\dot{y}_{21}}{y_{21}}\frac{\tilde{Y}_{21}}{\tilde{Y}}+\frac{\dot{y}_{22}}{y_{22}}\frac{Y_{22}}{\tilde{Y}}\\ &\quad+\dot{\ell}_{11}\left(\frac{y_{11}-y_{12}}{y}\right)+\dot{\ell}_{22}\left(\frac{y_{22}-y_{21}}{y}\right)\end{aligned}$$

Notice what's different from the case with migration is the term $\dot{\ell}_2\left(\frac{y_{21}-y_{12}}{y}\right)$ does not appear here as $\dot{\ell}_2=0$. We only have intra-regional movement of labor across regional sectors.

8 Appendix III

Period	Agr. Output growth	Rural services growth	Manufacturing growth	Urban services growth	In-Migration to Urban	Rural labor reallocation	Urban labor reallocation
0	0.25275	0.17764	0.45398	0.31150	0.12392	-0.28171	-0.03807
10	0.18912	0.16803	0.41503	0.37616	0.11597	-0.22171	-0.04260
20	0.15501	0.16105	0.38381	0.42528	0.11060	-0.18940	-0.04634
30	0.13444	0.15601	0.35893	0.46276	0.10680	-0.16961	-0.04933
40	0.12112	0.15234	0.33926	0.49147	0.10402	-0.15655	-0.05167
50	0.11207	0.14963	0.32383	0.51354	0.10197	-0.14754	-0.05351
60	0.10571	0.14762	0.31176	0.53053	0.10043	-0.14112	-0.05494
70	0.10114	0.14610	0.30236	0.54363	0.09927	-0.13645	-0.05605
80	0.09779	0.14496	0.29505	0.55374	0.09839	-0.13300	-0.05692
90	0.09530	0.14409	0.28937	0.56154	0.09772	-0.13043	-0.05759
100	0.09344	0.14342	0.28496	0.56757	0.09721	-0.12849	-0.05811
110	0.09203	0.14291	0.28155	0.57222	0.09681	-0.12702	-0.05851
120	0.09096	0.14253	0.27890	0.57582	0.09651	-0.12590	-0.05882
130	0.09015	0.14223	0.27686	0.57860	0.09628	-0.12505	-0.05906
140	0.08953	0.14199	0.27527	0.58075	0.09610	-0.12439	-0.05925
150	0.08905	0.14182	0.27404	0.58241	0.09596	-0.12389	-0.05940
160	0.08868	0.14168	0.27309	0.58370	0.09586	-0.12350	-0.05951
170	0.08840	0.14157	0.27236	0.58469	0.09577	-0.12321	-0.05960
180	0.08819	0.14149	0.27179	0.58546	0.09571	-0.12298	-0.05966
190	0.08802	0.14143	0.27135	0.58605	0.09566	-0.12280	-0.05971
200	0.08789	0.14138	0.27101	0.58651	0.09562	-0.12266	-0.05975

Table A1: Percentage contributions to real GDP growth; migration model

Period	GDP growth	Agr. Output growth	Rural services growth	Manufacturing growth	Urban services growth	Rural labor reallocation	Urban labor reallocation
0	1	0.19654	0.14181	0.44723	0.28972	-0.04403	-0.03127
10	1	0.18779	0.14572	0.38649	0.36172	-0.04285	-0.03887
20	1	0.18290	0.14887	0.33845	0.41863	-0.04366	-0.04519
30	1	0.17979	0.15138	0.30032	0.46362	-0.04478	-0.05032
40	1	0.17766	0.15336	0.27008	0.49916	-0.04581	-0.05445
50	1	0.17616	0.15493	0.24615	0.52718	-0.04667	-0.05775
60	1	0.17505	0.15616	0.22727	0.54924	-0.04736	-0.06036
70	1	0.17424	0.15713	0.21239	0.56659	-0.04791	-0.06243
80	1	0.17362	0.15789	0.20068	0.58022	-0.04835	-0.06406
90	1	0.17315	0.15848	0.19149	0.59092	-0.04869	-0.06535
100	1	0.17280	0.15895	0.18427	0.59930	-0.04896	-0.06636
110	1	0.17252	0.15931	0.17861	0.60588	-0.04917	-0.06715
120	1	0.17231	0.15960	0.17417	0.61102	-0.04933	-0.06777
130	1	0.17214	0.15982	0.17070	0.61506	-0.04946	-0.06826
140	1	0.17202	0.16000	0.16797	0.61821	-0.04956	-0.06864
150	1	0.17192	0.16013	0.16584	0.62068	-0.04964	-0.06894
160	1	0.17184	0.16024	0.16418	0.62262	-0.04970	-0.06918
170	1	0.17178	0.16032	0.16287	0.62413	-0.04975	-0.06936
180	1	0.17173	0.16039	0.16185	0.62531	-0.04979	-0.06950
190	1	0.17170	0.16044	0.16105	0.62624	-0.04981	-0.06961
200	1	0.17167	0.16048	0.16043	0.62696	-0.04984	-0.06970

Table A2: Percentage contributions to real GDP growth; no migration model