

FORWARD-LOOKING ENERGY ELASTICITY PARAMETERS FOR NESTED CES PRODUCTION FUNCTION

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Long abstract

Elasticities of substitution are the key parameters in CGE modeling. However an estimation and validation of the parameters is not straightforward. One of the ways is to use a historical data. Technological shifts between types of fuels and capital and fuels, observed in the past, potentially could be used to calibrate or econometrically estimate the elasticity coefficients. However, historical trends do not describe all the possible investment options available now. Moreover such estimates will be based on investment decisions made in particular economic condition, policies, and available technological/investment options in the time of decision. Application of the parameters for evaluation of future policy options involves undesirable (and unavoidable) assumption that future technological options are equal or similar to those in the past. A variety of new technological options will be disregarded from the analysis.

A more natural way to model technological options is Bottom-Up technological models. The reference energy system modes have an extensive representation of energy sector, and take into account currently available and expected technological options, but consider only part of an economy and lack connectivity with other sectors, f.i. do not provide a demand respond. Therefore their application is usually limited to the energy sector. There are several attempts

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to connect the top-down (CGE/AGE) and bottom-up models known as “soft” or “hard link” (see f.i. Böhringer and Rutherford 2006, 2008). However both methodologies require significant reduction of the models’ scale or some compromise in connectivity between the models. The methodology proposed in the paper might be considered as another way of hybrid modeling where bottom-up model is used to calibrate parameters for a top-down model. It is expected, the energy nest of a CGE model should provide results similar to the bottom-up model.

Methodology

The methodology included two stages. On the first stage we generate a random sample of states of the world (SOW) and apply a multi-sector energy RU-TIMES models for electric power, and iron and steel sectors to simulate a sample of cost-efficient solutions for each of the randomly generated SOW. On the second stage we apply econometrics to approximate the simulated sample of cost-efficient solutions with a for-level nested CES production function. Comparing to historical data, where only one (supposedly optimal) solution is observed for particular economic conditions in the past, the randomly simulated sample represents a full set of optimal solutions for a number of possible combination of unknown variables (f.i. fuel prices). Therefore simulated data has an advantage over historical in two ways. First, it takes into account currently available and expected in the future technological options (in TIMES model). Second, simulated data accommodates a huge variety of economic conditions which are not observable in the real life, but theoretically possible.

Specification of numerical experiment

To generate a sample of optimal fuel mix structure under given economic conditions, we apply a bottom-up model and solve it for various SOWs. Here are the main specifications of the experiment:

1. Each economic sector is modeled separately (one bottom-up model for each sector).
2. Time horizon is from 2010 to 2030.
3. Prices are randomly assigned for each commodity (coal, gas, oil) and each SOW, and are constant for the all period of consideration.
4. Unlimited supply of each energy source under fixed price.
5. Final demand is growing with constant rate and equal for all SOW.

The only difference between SOWs is a set of energy prices (gas, oil, coal). We performed 2000 model runs to generate a sample of the same size for CES estimates.

Estimation of multi-level nested CES function

Estimation of CES function is not straightforward due to non-linearity. There are several methods exist for one level functions. However it is even more difficult to estimate multi-level, nested CES structure. In the paper we apply Bayesian econometrics to estimate CES functions. The applied method is similar to Tsurumi, Tsurumi (1976), but extended for multi-level cases. As a result, we estimate the system of following three equations for each level of CES function:

$$\log y = \log ad - \frac{1}{\rho} \cdot (d \cdot x_1^{-\rho} + (1 - d) \cdot x_2^{-\rho}) + \varepsilon_1$$

$$\log x_1 = \log y + \frac{1}{1 + \rho} \cdot (-\rho \cdot \log ad + \log d + \log p - \log w_1) + \varepsilon_2$$

$$\log x_2 = \log y + \frac{1}{1 + \rho} \cdot (-\rho \cdot \log ad + \log (1 - d) + \log p - \log w_2) + \varepsilon_3$$

where:

p — price of final product

y — final product

w_i - price of production factor for CES production function

x_i - production factor for CES production function

d — factor share parameter for CES production function

ad — productivity for CES production function

E — elasticity parameter for CES production function

ε — errors

ρ — elasticity parameter

$$E = \frac{1}{1 + \rho}$$

The structure of multi-level CES function is presented on Figure 1. The resulting estimates for two Russian industries – iron and steel, and electric power sector presented below.

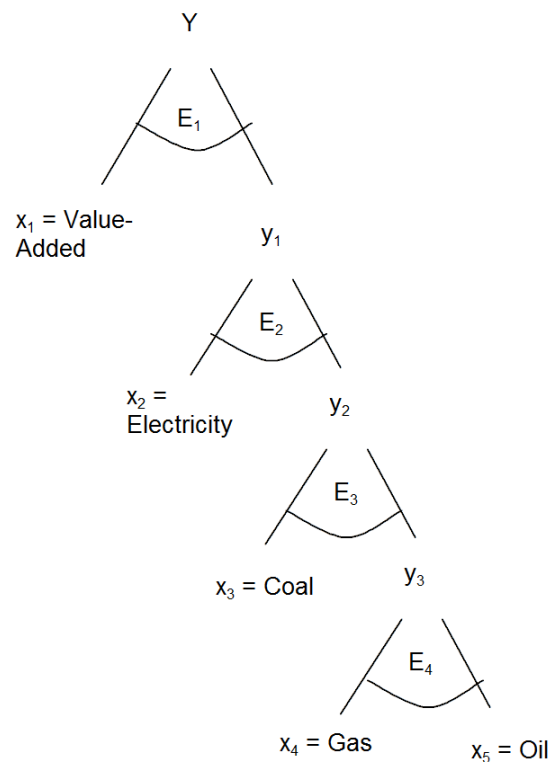


Figure 1. Nested CES production function structure.

Four-level elasticity parameters estimates for Russian industries

On the generated with RU-TIMES one-sector models data we estimate for level elasticity (E1-E4) and share (d1-d4) parameters, as well as scale or productivity factor (ad) for each year of the modeled horizon (2010-2030). The resulting estimates are presented in the Table 1 and on Figures 2 and 3. Some details on convergence of MCMC estimates for each parameter and kernel distributions of the estimates are presented in the Appendix (Figures 5-9).

Table 1. Estimation results of four-level CES functions for two industries.

Sector	Parameter	2010	2015	2020	2025	2030
Iron and Steel	ad	0.40	0.45	0.40	0.37	0.34
	E1	0.12	0.13	0.15	0.16	0.16
	E2	0.09	0.09	0.09	0.10	0.10
	E3	0.09	0.16	0.31	0.58	0.74
	E4	1.92	1.75	1.51	1.43	1.39
	d1	0.99	0.99	0.99	0.99	0.99
	d2	0.02	0.29	0.85	0.89	0.89
	d3	0.99	0.92	0.53	0.31	0.23
	d4	0.99	0.99	0.99	0.99	0.99
Electric Power Industry	ad	1.61	1.66	1.59	1.60	1.63
	E1	0.18	0.18	0.19	0.18	0.18
	E2	0.19	0.19	0.19	0.19	0.18
	E3	0.19	0.19	0.19	0.19	0.19
	d1	0.52	0.50	0.50	0.50	0.51
	d2	0.50	0.50	0.48	0.48	0.48
	d3	0.48	0.49	0.49	0.50	0.49

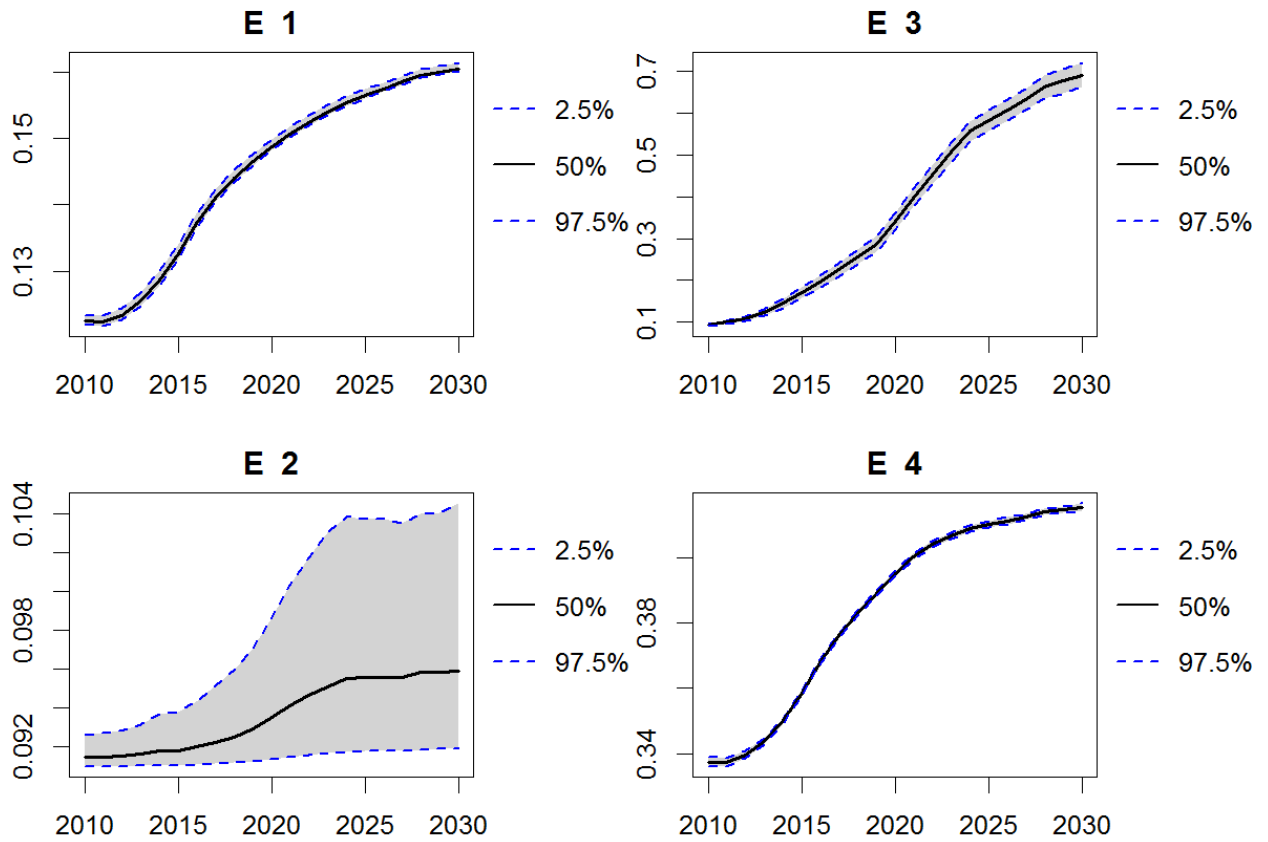


Figure 2 Estimated elasticity parameters with confidence intervals for IIS industry.

As follows from the figure, the elasticity parameters have a tendency to grow over time. This fact has an intuitive understanding that the possibility for technological/fuel switch is higher when we have longer time.

Also, we will have more possibility for switching between fuels if we expect higher economic growth, higher final demand for our products. To meet growing demand one should extend producing capacities. New investments can be done in any technology, based on coal or gas. Therefore the elasticity should be higher. A comparison of estimates for 1% and 3% growth is shown on Figure 3.

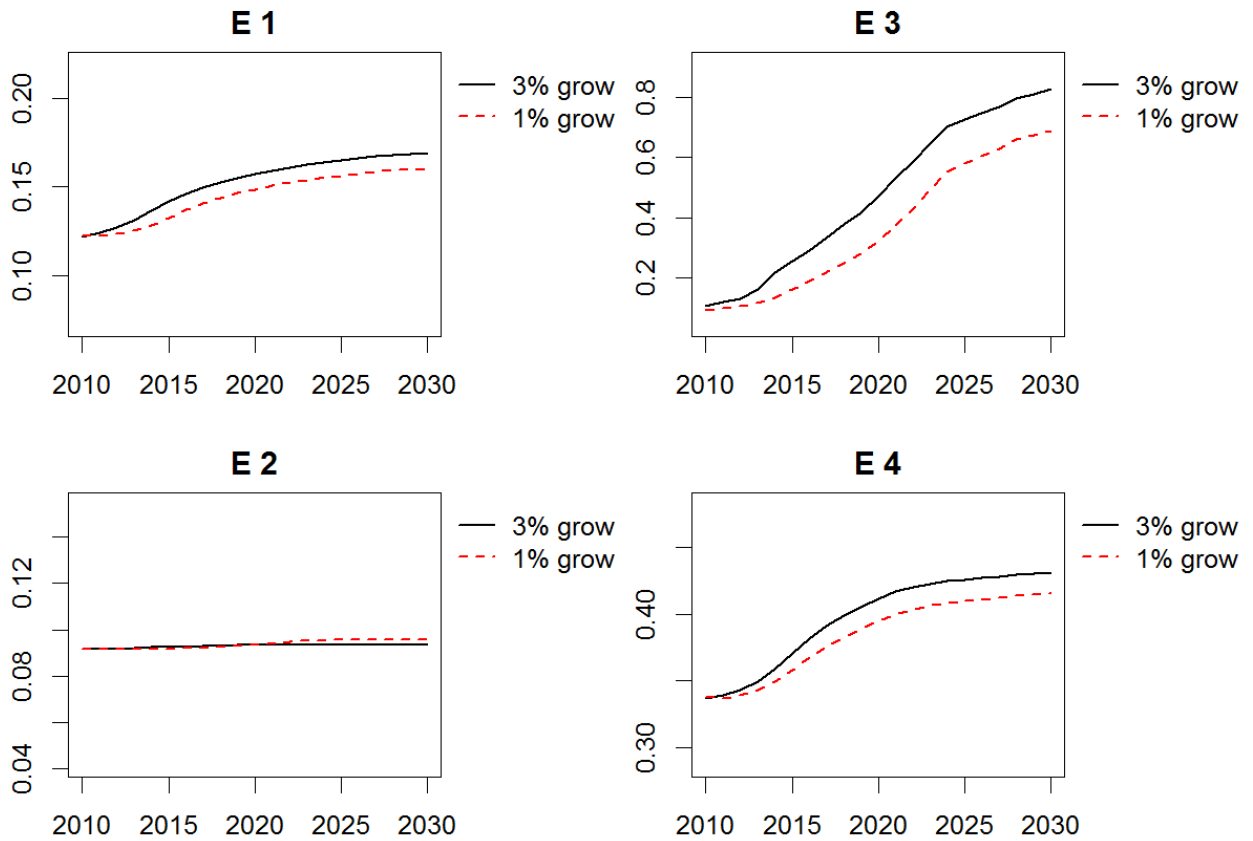


Figure 3. Result estimation elasticities for IIS with different growth path.

Some conclusions

The developed methodology applies bottom-up energy models to estimate nested CES elasticity parameters for top-down (CGE) models. The resulting estimates have several advantages over historical elasticity parameters estimates. First, they take into account all available now and in the future technological options (based on bottom-up model specification). Second, they take into account all possible set of economic variables, instead of only one observed in the past. Therefore the methodology is much better in approximation of technological switching.

There are several key observations should be mentioned regarding energy elasticity parameters for the CGE modeling:

- Elasticity parameters depend on horizon of planning (experiment). Longer horizon of planning usually lead to higher potential of switching between fuels and technologies, i.e. elasticity parameters are higher.
- Assumption of higher economic growth should result in higher elasticity of substitution. Currently existing capacities limit an opportunity for a technological maneuver in the short and medium run periods. However expansion of production assumes investments in new capacities.
- Technological shift and share parameters also depend on experiment horizon and should be considered for adjustment in CGE experiments.

Estimates for transport, cement industry, and residential sectors are completed as well and will be presented on the conference.

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Appendix: Details of estimation of nested CES function

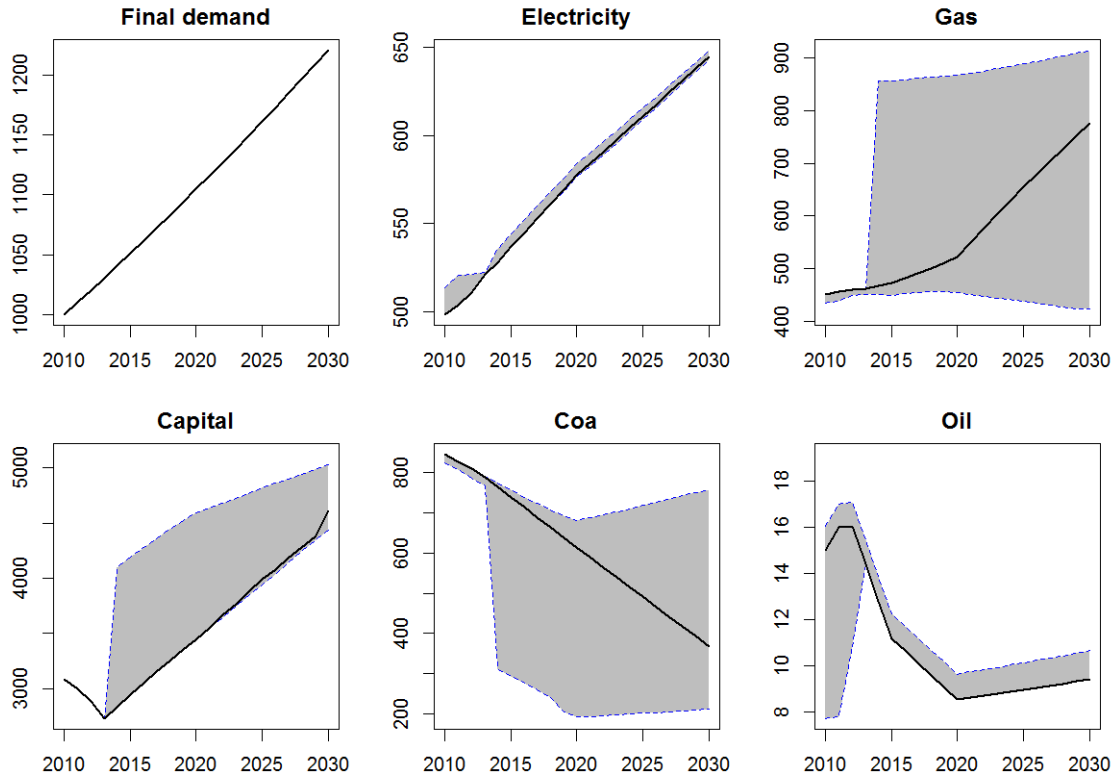


Figure 4. Final demand and fuel consumption range after numerical experiment for IIS.

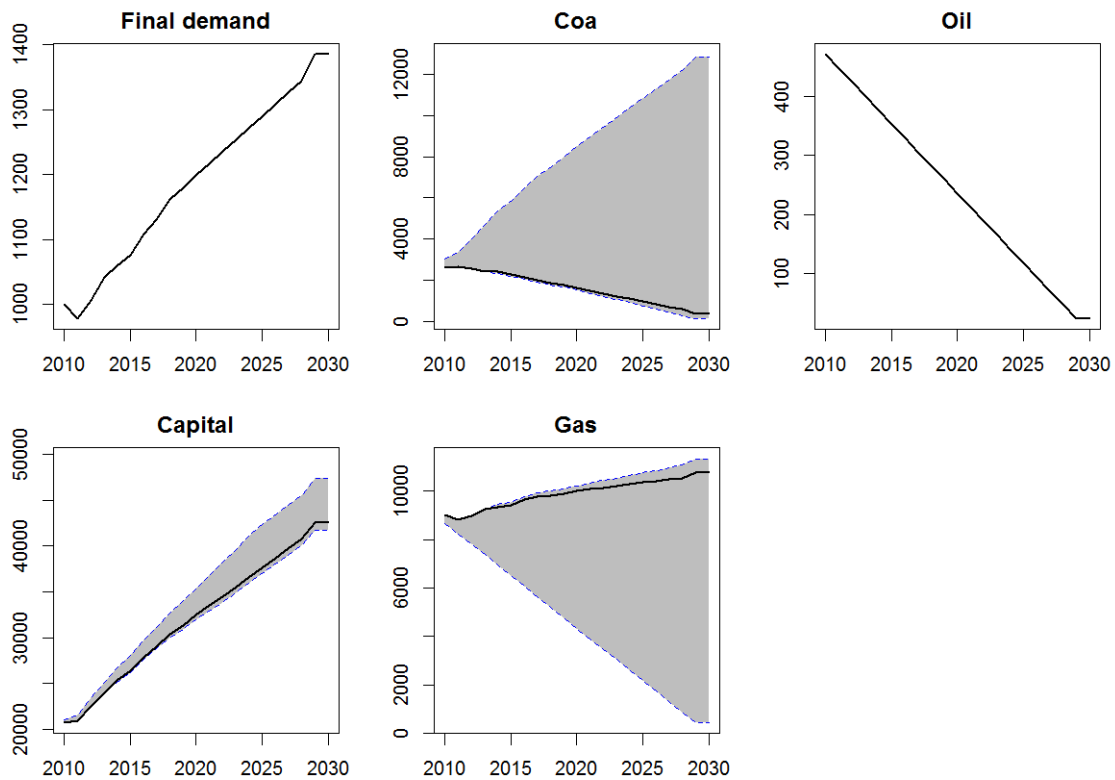


Figure 5. Final demand and fuel consumption range after numerical experiment for Electricity.

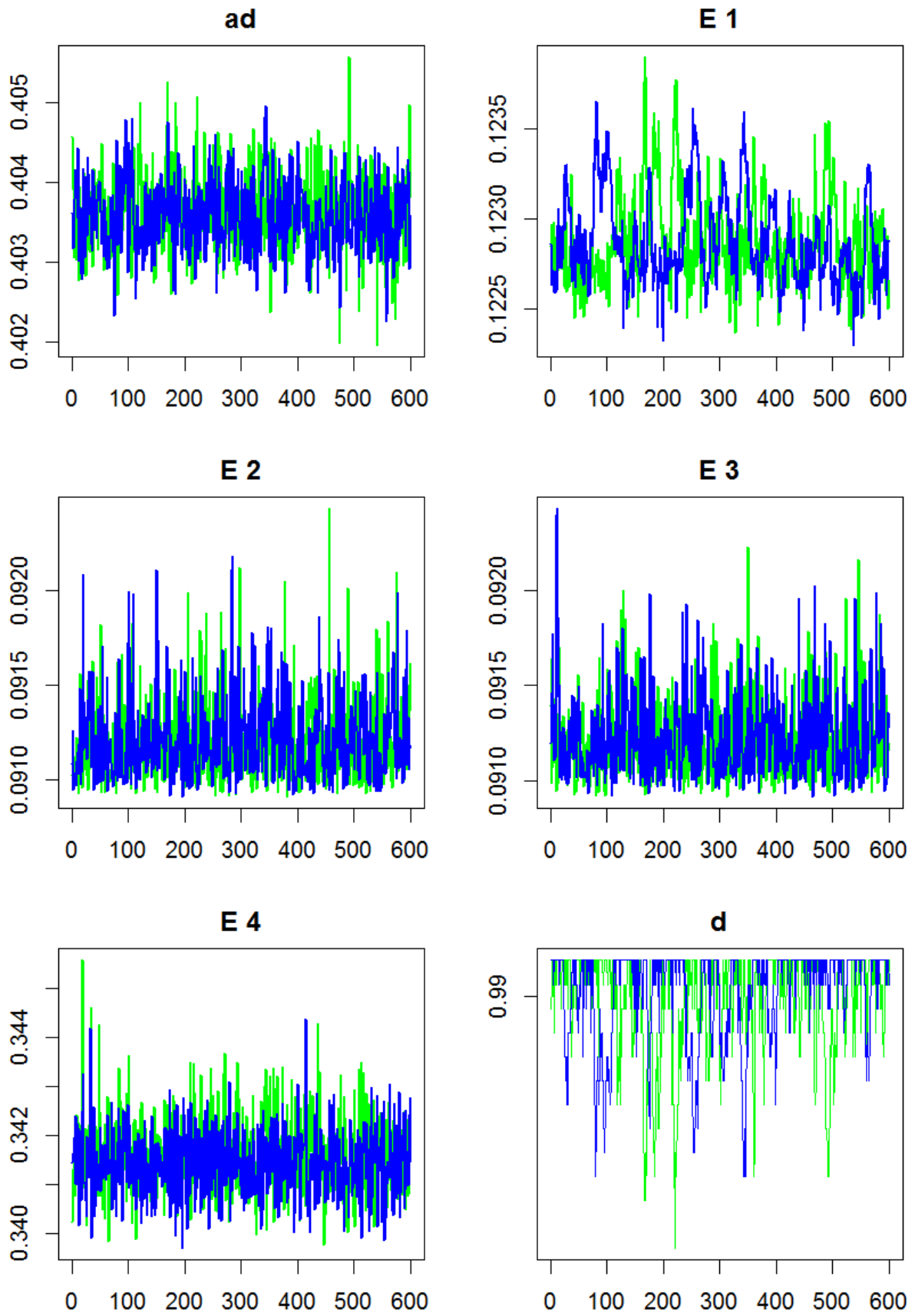


Figure 6. MCMC estimation nested CES parameter for IIS 2010.

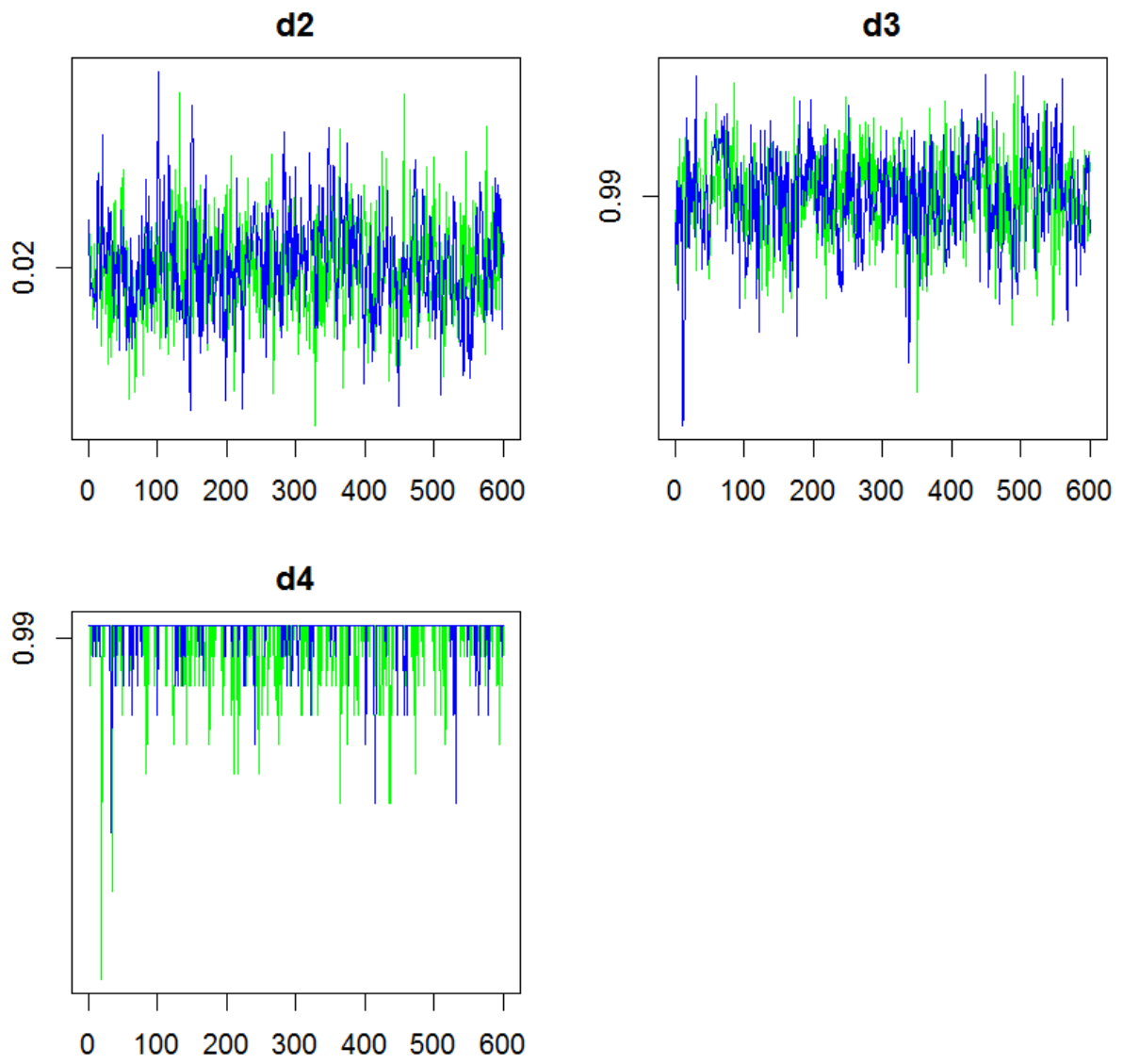


Figure 7. MCMC estimation nested CES parameter for IIS 2010.

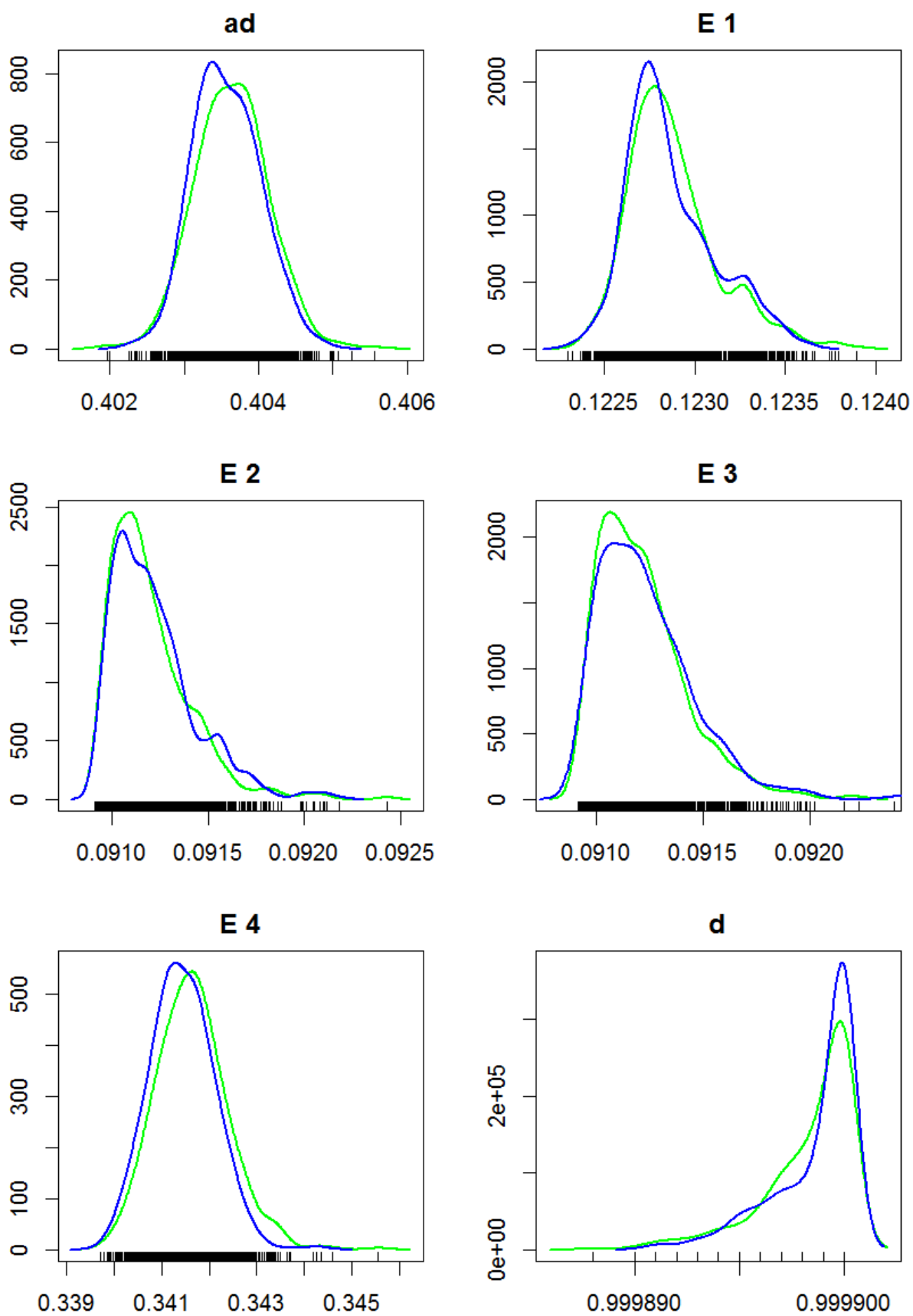


Figure 8. Kernel of estimation nested CES parameter for IIS 2010.

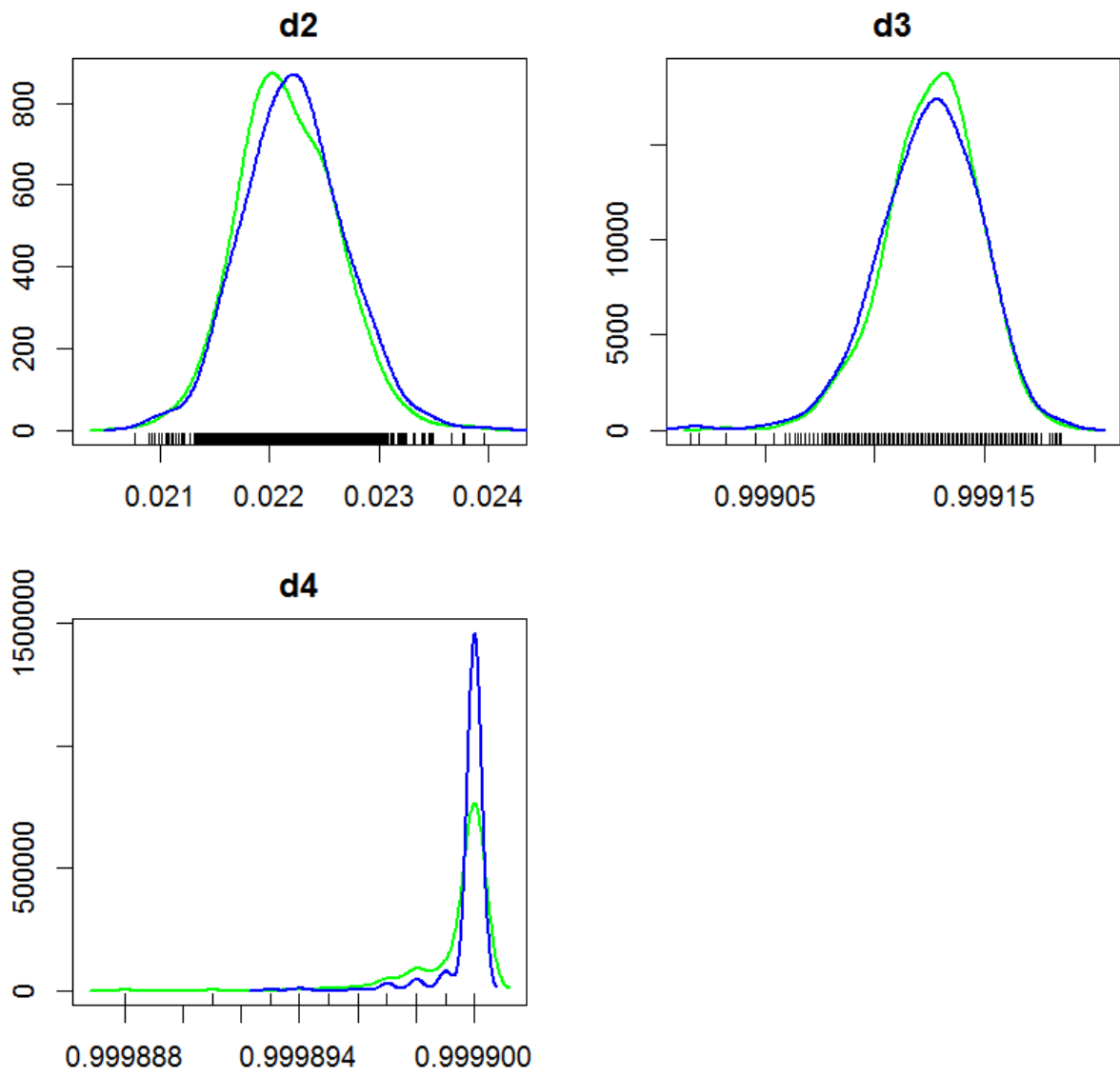


Figure 9. Kernel of estimation nested CES parameter for IIS 2010 (app.).