Agricultural land use in AGE model production functions

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1 Introduction

Applied general equilibrium (AGE) models are nowadays extensively used to guide public policies. This is especially true with complex economic issues that involve multiple industries and interest groups. The environmental and natural resource policies has been one of the foremost areas to extensively apply AGE modeling in its analysis. Currently the issues of the climate change and the ensuing developments in the energy policies are widely studied with AGE models.

Common deficiency well known to the AGE model developers is the lack of the real estimates for the model parameters. The parameters that determine the substitutability between the inputs in the production function are the most relevant for the climate and energy policy analysis as they determine how the model handles the substitution away from harmful inputs when an environmental policy is applied. Additionally to the parameter values themselves, the way the production function is modelled is important in itself. The most common way to depict production in AGE models is with a constant elasticity of substitution (CES) function that could be nested on various levels of input use. Another less common way to model the input use is by a CRESH (Constant Ratios of Elasticity of Substitution, Homothetic) function.

Kemfert [4] first empirically studied the energy substitution of the primary factors in CES production function. She applied a non-linear estimation framework to the CES function to derive substitution elasticities for capital, labour and energy in three production nests. Van der Werf [8] found empirical evidence of the suitability of different nesting structures in CES production function and of the factor-specific technological change. The study was based on industry level panel data of a number of industrial

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countries. He found that the nest structure where energy is directly substitutable with capital-labour composite [1] fits the empirical data best and that the technological change is factor specific at least for the capital. Okagawa and Ban [6] used the same method and data as van der Werf, but got somewhat different parameter values by taking into account the individual effects for each country. They also tested what difference the estimated parameters did in an AGE model when compared to some previously used parameter values. They concluded that the estimated parameter values lead to smaller costs for the emission reductions. While both van der Werf and Okagawa and Ban used cost minimization rule to derive their system of linear regression functions, Koesler and Schymura [5] in a more recent study directly estimated CES production function by non-linear estimation procedure similar to Kemfert [4]. This procedures has advantage of not requiring separate price data but it comes with a cost of not being able to distinguish between different nest structures.

My study contributes to the modelling of agricultural production function with land use. Recently this issue has become important as AGE models have been extensively used to study the effects of biofuel policies. Increased production of biofuels has proved to be rather problematic means to mitigate climate change. Unwanted consequences for overall climate change emissions and food security are largely due to the indirect land use changes (ILUCs) that were initially left without much consideration. Nowadays the ILUCs caused by biofuel production are chiefly studied with AGE models. Yet the modelling of the agricultural production function has not been based on empirically validated functional forms or parameter values. My study therefore addresses this deficiency.

In my study I will use a panel data set of Finnish farms in order to:

1. find the preferred way to model land use in AGE model production function,
2. estimate the values of the substitution elasticities in the land use nest in Finnish AGE model.

The data is for Finnish farms and Finland as agricultural producer is rather exceptional due to climatic conditions and the agricultural policies both mandated by the EU and Finnish state itself. Therefore the generalizability of my results is certainly limited. However, I believe that for the industrial nations where agriculture is based on family farms and where government subsidizes agriculture heavily, the considerations of modelling the agricultural production function with land use are reasonably relevant.

In chapter 2 I describe the use of production functions in AGE models. The chapter 3 describes the data used for the estimation and the estimation

[1] Later on I use rather common notation to write out the nest structure. In that notation that particular nest is written out as (KL)E.

procedure itself. In chapter 4 I present the core estimation results and in chapter 5 give more detailed interpretation of the results. Chapter 6 discusses the results and concludes the work.

2 Production Function Considerations

An industry's production function describes how the output is conceived by using certain combination of primary factors and other inputs in that particular industry:

$$Y_i = Y_i(A_i, K_i, L_i, LAND_i, IM_i), \tag{1}$$

where Y_i is the output by firm i produced by using primary factors capital K_i, labor L_i and land $LAND_i$, and the intermediate inputs IM_i. A_i is the productivity parameter. I will next discuss the two most relevant production functions used in AGE models in more detail.

CES production function

The most typical form of a production function in an AGE model is probably the constant elasticity of substitution (CES) function, which allows for constant substitutability between the inputs. The CES production function was first introduced by Solow [7] for two primary factors. For example two input (capital and labor) CES function can be defined as:

$$Y = A_i(aK^r + (1-a)L^r)^{\frac{1}{r}}, \tag{2}$$

where r is related to the elasticity of substitution s by $s = \frac{1}{1-r}$. The elasticity of substitution measures the substitutability between inputs and can be expressed as:

$$s_{21} = \frac{\mathrm{d}\ln x_2/x_1}{\mathrm{d}\ln \frac{\mathrm{d}f}{\mathrm{d}x_1}/\frac{\mathrm{d}f}{\mathrm{d}x_2}}, \tag{3}$$

where s_{21} is the elasticity of substitution between inputs x_1 and x_2.

The CES production function is more general form of the Cobb-Douglas unit elastic production function and the fixed proportions Leontief production function [2]. Although various more general types of production functions that give better measures of substitutability [3] have been developed since the introduction of the CES function, I will concentrate on CES function in my analysis because of its applicability in the AGE models. My null hypothesis is that the there is a Cobb-Douglas type of production function for the primary factors and a Leontief for the intermediate inputs.

[2] For the Cobb-Douglas $s = 1$ and for the Leontief $s = 0$

[3] E.g. translog and generalized Leontief functions are more general representations of factor substitutability than the CES function.

Many times the inputs are nested inside the CES function in order to give more realism to the AGE production structure. It is not straightforward to tell which one of the all possible nests would be most suitable to present reality and empirical evidence is quite sparse. Indeed, most commonly researchers are lead by their intuition when they construct nested CES production functions. van der Werf [8] empirically tested how energy is nested with primary factors land and labor, which required estimation of three separate nest structures[4] He could conclude that the (KL)E suited the data best, but could not reject the non-nested form for most of the industries.

The first step in my study is to empirically test the nesting structure of the primary factors. Figure 2 summarizes the primary factor nests and their elasticities of substitution. The basic proposition is that all the primary factors appear in the same nest - (K,L,LAND). The alternative propositions are that 1) land is more naturally nested together with capital-labour composite - (K,L)LAND, 2) that land is nested with capital - (K,LAND)L or 3) that land is nested with labour composite - (L,LAND)K. The intuition to support the (K,LAND)L-nest would be that in production the capital and land are closely connected to each other - e.g. large farms would require certain level of machinery to produce the output. The second nest, (K,L)LAND is the case where machinery dictates the level of workforce needed for the output. Finally, the interpretation for the (L,LAND)K would be that the hectares the farm has, largely determines the amount of labour used. [5] The alternative propositions reduce to the basic if the upper and lower level elasticity of substitution values are equal, which can be statistically tested. I estimate the elasticities of substitution for the three alternative nesting structures after which I can test whether the basic (K,L,LAND)-structure can be safely rejected, i.e. $\sigma_{(K,L),LAND} = \sigma_{K,L}$ for the nest 4) in figure 2.

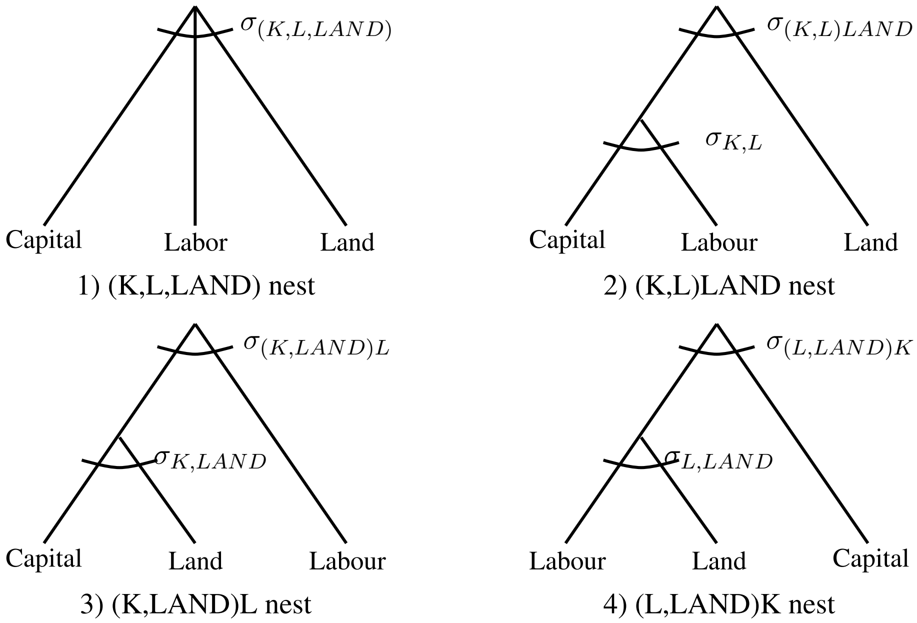
CRESH production function

CES production function is favoured in AGE models because of its reasonable generality and the practical stability. As I noted earlier, there exist more general functional forms for modelling production but they have proven difficult to apply in AGE models. One type of more general production function that actually has been used in some of the AGE models is the CRESH - Constant Ratios of Elasticity of Substitution, Homothetic by Hanoch [1]. In CRESH production functions the ratios of the elasticities of substitutions are kept constant while maintaining the homotheticity of the function. Hanoch defined the general form of CRESH function as:

[4] The nest structures were energy nested with capital and labor composite - (KL)E, energy nested with capital - (KE)L and energy nested with labor - K(LE).

[5] I intend these descriptions to be merely intuitive and devoid of any causal interpretations. Testing the causality is an interesting idea for further research of the topic.

Figure 1: Nest structures for three primary factors.



$$F(Y,\underline{x}) = \sum_{i=1}^{n} D_i[x_i/h(Y)]^{d_i} - 1 \equiv 0, \tag{4}$$

where $h(Y)$ is function of output Y that can take various forms. In addition to the general constant returns to scale (CRS) function, where $h(Y) = Y$, Hanoch proposes two alternative specifications for the estimation of CRESH function:

1. $h_1(Y)$: the general homogeneous case $h(Y) = Y^{\frac{1}{\mu}}$ [6]
2. $h_2(Y)$: a S-shaped total cost curve, e.g. $h(Y) = \exp\frac{Y}{Y_0}$, where Y_0 is the minimum average cost.

$\underline{x}$ is the vector of the factors of production and Y is the output. D_i and d_i are the parameters of the function.

The estimation of CRESH production function yields Allen-Uzawa pairwise partial elasticities of substitution. Let us denote $a_i = \frac{1}{1-d_i}$. Now, the elasticities of substitution are:

$$\sigma_{ij} = \sigma_{ji} = \frac{a_i a_j}{\sum s_k a_k}, i \neq k, \tag{5}$$

$$\sigma_{ii} = \frac{a_i^2}{\sum s_k a_k} - \frac{a_i}{s_i}, \tag{6}$$

[6] The function applies to parameter values $0 < \mu \leq 1$. The case where $\mu = 1$ is the CRS function which Hanoch treats as third separate specification.

where s_i is share parameter defined as $s_i = p_i x_i / \sum p_k x_k$.

I can compare the three factor CRESH production function with the nested and non-nested CES functions and see how they perform. In my analysis I present results for the two alternative specifications proposed in Hanoch's original article. Additional benefit of the CRESH function when compared to the CES function is that it can be much more flexibly extended to cases with more than three inputs. I will exploit this feature in my analysis when I take a look at how some important intermediate inputs are best handled in agricultural production function.

More than three inputs

It is possible to build more complex nested CES functions that have additional inputs besides the primary factors. Indeed, this is commonly done in AGE models. However, empirical testing of such functions is problematic. Firstly, the formulation used by van der Werf [8] would require to have price information on various input composites, e.g. the price for capital and labor composite. As there is no reliable way of constructing such price indecies, I find this approach unappealing. Secondly, the approach of estimating the non-linear nested function directly has the remaining problem of not being able to distinguish separate nests from each other. However, making the estimations of multiple inputs in CRESH production function framework might give at least partial solution for this problem. My intuition is that the inputs having the lowest pairwise substitution elasticities in a CRESH function, should bound together at the bottom level of the nested CES function. This intuition will be confirmed with estimation of production functions for the primary factors.

3 Material and Methods

In my analysis I use a firm level unbalanced panel data set of Finnish farms provided by the Statistics Finland. I had to supplement this data with price information and and quantity yields. I use the data to carry out econometric estimations of CES function in a linearized system of equations similar to the one used by van der Werf [8] and equivalent estimations for CRESH production function.

Data

The initial data set contains 30947 observations from six years (2004-2009) of Finnish farms. This data includes the main economic variables of these farms e.g. the income from various commodities, expenses on various production inputs, production capital and land, use of own and hired labour, and public subsidy payments received. Relevant indicator variables included those based on geographical areas (NUTS3-level and EU-subsidy classification) and eight production lines. The production lines are dairy, beef, pork, poultry, other animal production, grain and oilseed, other plant production and other production. Finnish farms are typically family farms that tend to own large share of their agricultural land regardless of the production line. Thus in animal production as well, the farms mostly own land that they use for their own feed production. I carried out the estimations for the production lines separately and omitted the production lines the other animal production and the other production from final analysis as they were rather vague and comprised only less than 2 percents of the initial observations.

The master data described above needed to be supplemented in several ways. The most important supplement is the data of the working hours[7] of which 6801 observations coincide with the master data. In order to have sufficient data of yearly changes for the working hours, I imputed the missing working hours by using total output, farm cultivated land area, capital stock, hired labour expenses, production line and year as regressors [8]. The estimations of the CES production function requires the use of percentage changes of the main variables and therefore all the farms that had only one observation had to be omitted. After the omission 25679 observations remained. For the CRESH estimation larger data could be used as it does not require yearly changes.

The master data does not include any price information, which I needed to retrieve from other sources. The output in data is divided to main products that are: grains (includes wheat, rye, barley and oats), potatoes, sugarbeet and other plant products (consisting mostly of rapeseed) and animal products (milk, eggs and meat). I formed output price index based on yearly average prices. For the grains I used average area production as the weight by NUTS3 regional classification. Land prices are the median agricultural land prices in the NUTS3-area. Wages I combined from the average wage claims for the own work and average hired labor wages in agriculture. I calculated the user cost of capital similarly to van der Werf [8]: foregone interest (nominal interest rate) plus depreciation minus capital gain (increase or decrease in capital stock during the year). The interest rate is the one by Bank of Finland and depreciation rate and capital gain I was able to derive from the data for individual farms. [9]

Table 1 summarizes the main aspects of the data by production line.

[7]The working hours data was provided by Centre of the Ministry of Agriculture and Forestry.

[8]I implemented the imputation process with Amelia II package in R. Amelia uses bootstrap-based EMB algorithm for the imputations. See Honaker et al. (2012) [2] for more details.

[9]The computation of user cost of capital showed few implausible extreme values. Therefore I imposed restriction for the data that omitted bottom and top 5 % of the observations based on the computed user cost of capital values, which served also to omit the major outliers.

Production line	N	Capital stocks, euros	Yearly working hours	Land area, hectares
Beef	2123	144289	3780	53
Dairy	8823	161271	4602	53
Grains and oilseeds	4176	90262	2871	61
Other plant production	2093	88062	3468	46
Pork	2357	227388	4356	68
Poultry	606	295909	4663	61

Table 1: Data summary

We see that the differences in land use are small between production lines. We can also see that the grain and oilseed production is the least labour intensive production line and that pork and poultry are the most capital intensive. Figure 2 presents the size distribution of farms in their land use by production line.

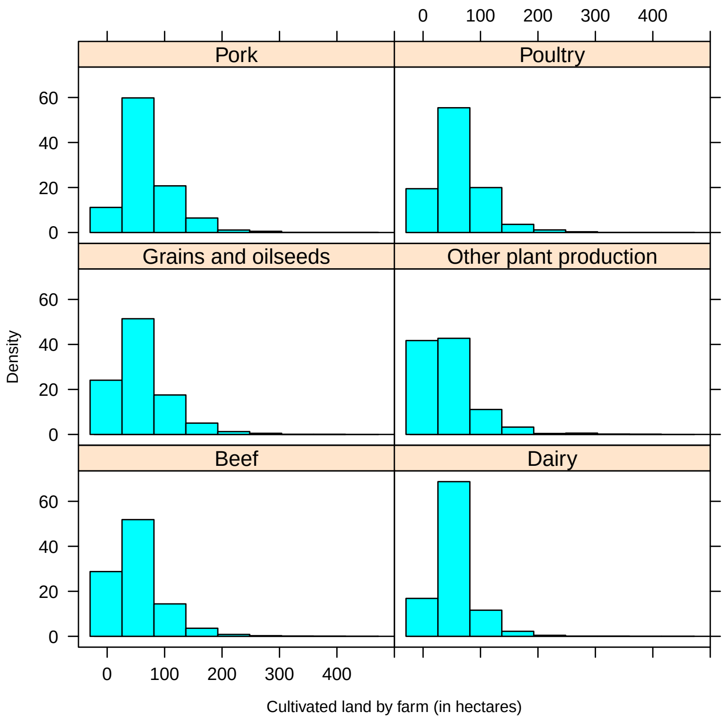


Figure 2: Land use by production line.

After omissions of implausible values like negative amounts etc., I was able to utilize 7804 observations for the CES estimations and 20178 for

CRESH estimations in the six production lines. The higher amount of observation available for the CRESH estimations is mainly due to it not requiring data of yearly changes. The panel was unbalanced with many farms having only observation.

Econometric model

CES function

My analysis of the CES function follows that of van der Werf [8] cost minimization approach that leads to estimation of linearized system of equations and allows comparison between the nests and factor specific technical change. The data needed for the analysis consists of the quantities (Q) and prices (P)of the three primary factors and total output, which I indicate with upper case letters, and their yearly percentage changes, which I indicate with lower case letters (q and p) [10]. For illustratory purposes, I define the model for estimating the (K,L)LAND nest (panel 2 in figure 2). The model to be is estimated can be described as the following system of linear functions:

$$y_1 = \alpha_1 + \beta_1 x_1 + \epsilon_1, \tag{7}$$

$$y_2 = \alpha_2 + \beta_{21} x_{21} + \beta_{22} x_{22} + \epsilon_2, \tag{8}$$

$$y_3 = \alpha_3 + \beta_{31} x_{31} + \beta_{32} x_{32} + \epsilon_3, \tag{9}$$

where the dependent variables are $y_1 = q_{LAND} - q_{OUTPUT}$, $y_2 = p_K + q_K - d\ln(P_K K + P_L L)$ and $y_3 = p_L + q_K - d\ln(P_K K + P_L L)$, where $d\ln X$ denotes the first difference of the natural logarithm of X. The independent variables are $x_1 = p_{OUTPUT} - p_{LAND}$, $x_{21} = x_{31} = d\ln(P_K K + P_L L) - p_{OUTPUT} - q_{OUTPUT}$, $x_{22} = p_K - p_{OUTPUT}$ and $x_{32} = p_L - p_{OUTPUT}$. ϵs are the error terms. The following cross-equation restritions apply: $\beta_{22} = \beta_{32}$ and $\beta_{21} = \beta_{31} = -\beta_{22}/(1 - \beta_1)$. [11]

The elasticities of substitution are thus defined as: $\sigma_{(K,L)LAND} = \beta_1$ and $\sigma_{K,L} = 1 - \beta_{22}$. The paramaters for factor specific technical change are defined as: $a_{LAND} = \alpha_1/(\beta_1 - 1)$, $a_L = -\alpha_2/\beta_{22}$ and $a_K = -\alpha_3/\beta_{22}$.

[10] The yearly percentage changes measured as discrete time approximation of natural logarithmic differences, e.g. $y = \ln Y(t) - \ln Y(t-1)$

[11] The last restriction is non-linear and complicates the analysis. This complication can however be circumvented by solving the first equation separately and using the estimates in the following two equations that are solved as a system. This procedure relies on the weak separability of the nested CES function.

Production line	(K,L)LAND	(K,LAND)L	(L,LAND)K
Beef	0.08	0.17	0.48
Dairy	0.03	0.06	0.53
Grains and oilseeds	0.04	0.16	0.34
Other plant production	0.14	0.27	0.37
Pork	0.01	0.03	0.48
Poultry	0.14	0.19	0.43

Table 2: Goodness of fit values for nested primary factors CES functions.

CRESH function

After assuming cost minimization we can estimate the CRESH production function by system of linear equations:

$$\log x_i = b_{i1} \log x_1 + b_{iy} \log h(Y) + b_i \log \frac{p_i}{p_1} + b_{i0} (i = 2, ..., n), \tag{10}$$

where the subindex i denotes a factor input different from factor input chosen as numeraire (subindex 1). In the general homogeneous case $b_{iy} \log h(Y)$ is replaced by $B^1_{iy} \log Y$, where $B^1_{iy} = \frac{1}{\mu}(1 - (\frac{a_i}{a_1}))$. In the S-shaped total cost curve case $b_{iy} \log h(Y)$ is replaced by $B^2_{iy} Y$, where $B^2_{iy} = \frac{1}{Y_0}(1 - (\frac{a_i}{a_1}))$.

4 Results

CES function

Table 2 shows the goodness of fit values [12] for the three alternative CES nest structures of primary factors for the six production lines. For all of the CES cases, the nest that has land substitutable with labor in the lower level and land-labor composite substitutable with capital in the upper level (the (L,LAND)K nest), obtains the highest goodness of fit value. The second best appears to be the (K,LAND)L nest while (K,L)LAND nest had the worst goodness of fit.

The comparison of alternative CES nesting structures to the general, non-nested primary factor production function shows that the general nesting structure cannot be rejected for most of the production lines in the best nesting structure (L,LAND)K. The non-nested version seems to be different from the two less successful nests. The dairy and grains and oilseeds differ the most from the non-nested structure. Table 3 reports the p-values of two-sided t-test for testing $\sigma_{upper} = \sigma_{lower}$.

[12] The goodness of fit values for the system of equations are based on the method by Jitthavech [3]

Production line	(K,L)LAND	(K,LAND)L	(L,LAND)K
Dairy	0.00	0.00	0.00
Beef	0.09	0.06	0.66
Pork	0.00	0.00	0.80
Poultry	0.00	0.00	0.13
Grains and oilseeds	0.00	0.00	0.24
Other plant production	0.02	0.00	0.26

Table 3: p-values for common elasticity tests.

Production line	(K,L)LAND $\sigma_{(K,L)LAND}$	(K,L)LAND $\sigma_{K,L}$	(K,LAND)L $\sigma_{(K,LAND)L}$	(K,LAND)L $\sigma_{K,LAND}$	(L,LAND)K $\sigma_{(L,LAND)K}$	(L,LAND)K $\sigma_{L,LAND}$
Dairy	0.56	0.97	0.48	0.94	0.95	0.98
Beef	1.04	1.00	1.04	0.97	0.98	0.97
Pork	0.15	0.97	0.40	0.96	0.93	0.94
Poultry	1.74	0.99	1.27	0.96	1.06	0.99
Grains and oilseeds	0.72	0.99	1.07	0.98	0.96	0.99
Other plant production	1.08	1.00	1.36	0.98	0.96	1.00

Table 4: Estimated elasticities of substitution - CES function.

Table 4 summarizes the estimates for the elasticity of substitution values. We can see that the (L,LAND)K-nest that has the best goodness of fit value on the other hand has elasticities of substitution very close to unity. The values of the higher and lower level elasticities of substitution are not far from each other either. Therefore, the most suitable nest seems to be very close to Cobb-Douglas unit elasticity function although there is statistical significance. Table 5 shows the values of two-sided t-test for the Cobb-Douglas function.

CRESH function

Table 6 summarizes the goodness of fit values for the CRESH function. Homog. refers to the general homogenous specification of CRESH function and S-shaped to the S-shaped total cost curve specification. In all of the cases the CRESH function yields better goodness of fit values than the best performing CES nest. Including fertilizer in the production function improved

Production line	(K,L)LAND $\sigma_{(K,L)LAND}$	(K,L)LAND $\sigma_{K,L}$	(K,LAND)L $\sigma_{(K,LAND)L}$	(K,LAND)L $\sigma_{K,LAND}$	(L,LAND)K $\sigma_{(L,LAND)K}$	(L,LAND)K $\sigma_{L,LAND}$
Dairy	0.00	0.03	0.00	0.00	0.00	0.05
Beef	0.11	0.93	0.33	0.00	0.43	0.28
Pork	0.00	0.22	0.00	0.00	0.00	0.01
Poultry	0.00	0.91	0.01	0.02	0.19	0.84
Grains and oilseeds	0.00	0.58	0.00	0.02	0.09	0.68
Other plant production	0.02	0.96	0.00	0.23	0.23	0.94

Table 5: Tests of Cobb-Douglas function.

Production line	Homog. (prim)	S-shaped (prim)	Homog. (prim+fert)	S-shaped (prim+fert)
Beef	0.55	0.53	0.50	0.49
Dairy	0.49	0.49	0.45	0.45
Grains and oilseeds	0.45	0.45	0.48	0.48
Other plant production	0.38	0.40	0.44	0.43
Pork	0.54	0.53	0.49	0.49
Poultry	0.51	0.51	0.46	0.47

Table 6: Goodness of fit values for nested CRESH functions.

goodness of fit for grain and oilseed and other plant production but not for animal husbandry. There is no big difference between the two alternative CRESH specifications.

Tables 7 and 8 display the values for elasticities of substitution for the general homogenous CRESH function specification for the primary factor and fertilizer estimations, respectively. We can see that for the primary factors the lowest substitutability is between land and labour in all of the cases. This is consistent with the results for the CES nests: the nest that combines land and labour at the lowest level had highest goodness of fit value. The values themselves are however generally much lower than in the CES cases. Highest substitutability can be seen in grains and oilseeds and other plant production. These were the production lines that got improved goodness of fit values when the fertlizer was included in analysis. For all of the production lines we can see that fertlizer is the least substitutable input. Except for beef and dairy, fertilizer is least substitutable with land.

5 Analysis

My analysis has thusfar showed that the evaluation of AGE model production functions might be reasonable to implement with the CRESH functional form. I was able to gain better goodness of fit values for the analysis of CRESH than for the equivalent nested CES function. Furthermore, the CRESH specification is much more flexible when extended to more than three inputs. The CES analysis requires data of yearly changes in order to able to distinguish separate nesting structures. This is clearly an advantage as for any suitable data set for the CES estimation, there will be at least twice as many observations available to implement CRESH analysis.

The evidence shows that the agricultural land should be modelled to be least substitutable with labour. This is finally quite intuitive result - fixed amount of land requires relatively fixed amount of labour. In CES cases this results shows as (L,LAND)K nest ended up having the best goodness of fit. Furthermore, in CRESH case the substitutability between land and labour

V1	V2	Capital	Labour	Land
Beef	Capital	-1.53	0.594	0.798
	Labour	0.594	-14.3	0.458
	Land	0.798	0.458	-0.457
Dairy	Capital	-1.32	0.437	0.802
	Labour	0.437	-9.16	0.378
	Land	0.802	0.378	-0.535
Grains and oilseeds	Capital	-5.34	0.956	1.2
	Labour	0.956	-45	0.952
	Land	1.2	0.952	-0.3
Other plant production	Capital	-3.33	0.792	0.938
	Labour	0.792	-16.2	0.522
	Land	0.938	0.522	-0.299
Pork	Capital	-1.8	0.833	0.866
	Labour	0.833	-23.9	0.515
	Land	0.866	0.515	-0.451
Poultry	Capital	-0.945	0.78	0.598
	Labour	0.78	-16.4	0.23
	Land	0.598	0.23	-0.412

Table 7: Estimated elasticities of substitution - general homogenous CRESH function for primary factors.

seems to be even more restricted than in the CES case which actually closely resembles unitary elastic Cobb-Douglas case.

For the AGE models in general, the CRESH functional form appears to be appealing way to model the agricultural land use. I could not find any strong evidence for including the fertilizer use along the primary factors. In many cases its substitutability is low and close to Leontief case of zero substitutability that is commonly used in AGE models for intermediate inputs.

6 Discussion

I will conclude my study later by including the use of energy and feed inputs in CRESH functional form. Testing the effects of newly modelled land use in an AGE model for e.g. some biofuel policy is also natural continuation for the work.

V1	V2	Capital	Labour	Land	Fertilizer
Beef	Capital	-1.55	0.601	0.805	0.171
	Labour	0.601	-14.5	0.463	0.0986
	Land	0.805	0.463	-0.464	0.132
	Fertilizer	0.171	0.0986	0.132	-10.1
Dairy	Capital	-1.36	0.432	0.828	0.183
	Labour	0.432	-9.29	0.389	0.0862
	Land	0.828	0.389	-0.557	0.165
	Fertilizer	0.183	0.0862	0.165	-9.92
Grains and oilseeds	Capital	-5.08	1.23	1.13	0.297
	Labour	1.23	-45.3	0.891	0.235
	Land	1.13	0.891	-0.286	0.215
	Fertilizer	0.297	0.235	0.215	-17.2
Other plant production	Capital	-3.21	0.903	0.888	0.411
	Labour	0.903	-16.5	0.499	0.231
	Land	0.888	0.499	-0.288	0.227
	Fertilizer	0.411	0.231	0.227	-16.5
Pork	Capital	-1.7	0.924	0.804	0.349
	Labour	0.924	-24.2	0.479	0.208
	Land	0.804	0.479	-0.423	0.181
	Fertilizer	0.349	0.208	0.181	-14.3
Poultry	Capital	-0.792	0.856	0.488	0.233
	Labour	0.856	-16.6	0.189	0.0902
	Land	0.488	0.189	-0.337	0.0514
	Fertilizer	0.233	0.0902	0.0514	-12.6

Table 8: Estimated elasticities of substitution - general homogenous CRESH function for primary factors and fertilizer use.

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