

Varying markups and income inequality in an open economy

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Abstract

Empirically and theoretically trade openness has been found to increase especially within-country wage income inequality. Recently, empirical studies have emphasized also the role of capital income, capital gains and top incomes earners in the widening of total income distributions in various countries. We contribute to the literature by analysing how a change in markups in an open economy affects different income inequality measures and unemployment rate. While the assumption of fixed markups has still been common in theoretical models that analyse the topic, in empirical studies markups have been often found to decrease when an open economy faces tougher competition. We use the general equilibrium framework of [Egger and Kreickemeier \(2012\)](#) for the analysis, which includes firm heterogeneity in productivity, fair wage setting and an analysis on the effects for different income inequality measures. We find, contrary to earlier studies, that tougher competition increases the unemployment rate, income inequality between capital income and wage income and the Gini indexes of both wage and capital income. This is due to the increase in the productivity level required to operate, which subsequently decreases the number of firms. Labour supply increases, since the more productive firms that stay in operation need less employees than the less productive firms that drop out of business. In the end, a smaller share of firm owners and employees will be able to enjoy the export premium in profits and wages and income distributions widen.

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1 Introduction

During the past decades within-country income inequality has increased in most countries (e.g. [Anand and Segal, 2008](#), [Galbraith and Kum, 2005](#), and [Harrison, McLaren, and McMillan, 2011](#)), while the trends in between-country and global income inequality have been more diverse. Widening income distributions and the concentration of money and power are claimed to increase instability in societies. For example, recently widening income inequality levels have been blamed for the political and social turmoil in the Arab countries, the Democratic Republic of Congo, Brazil, Syria and Ukraine.¹ In addition to wage income inequality, recent empirical studies have emphasized the role of top income earners ([Atkinson and Piketty, 2007, 2010](#)), capital income and capital gains (see [Atkinson and Piketty, 2010](#), and e.g. [Roine and Waldenström, 2012](#), [Biewen and Juhasz, 2010](#), and [Chi, 2012](#)) in the development of total income inequality within-countries.

The literature on firm heterogeneity has found empirical evidence that companies involved in the international markets are significantly different than non-traders. Firms provide usually the basis for people's incomes. As exporting firms pay for example higher wages than non-exporting firms, the effects of trade and firm heterogeneity on income inequality within-countries have gained attention, next to the studies on the various other factors influencing income distributions. Especially general equilibrium (GE) models with heterogeneous firms have been used for the theoretical analyses on the effects of trade on income inequality. These theoretical assessments have focused mostly on the effects of trade on wage inequality (e.g. [Helpman, Itskhoki, and Redding, 2010](#) and [Basco and Mestieri, 2013](#)) or only on capital income ([Foellmi and Oechslin, 2010](#)). A more general framework for the analysis of the effects of trade on both income types at the same time was developed by [Egger and Kreickemeier \(2012\)](#). In addition to accounting for the heterogeneity in firms, it is assumed in their framework that workers obtain a 'fair wage' that depends both on the external conditions in the labour markets and on the profits of the firm.

While most of the assumptions in the [Egger and Kreickemeier \(2012\)](#) framework match empirical findings, they have assumed that mark-ups remain unchanged from autarky to open economy. This assumption is in contrast with empirical findings on the effect of trade on mark-ups (e.g. [Epifani and Gancia, 2011](#) and [Chen, Imbs, and Scott, 2009](#)) and on the endogenous mark-up assumption in various other models.

In this paper we contribute to the analyses of the mechanisms behind the increase

¹ Jason Stearns (2012): *Dancing in the Glory of Monsters: The Collapse of the Congo and the Great War of Africa*, PublicAffairs publishing, Reprint edition (March 27, 2012). Oxfam (2014): *Working for the few: Political capture and economics inequality*, Oxfam Briefing paper 187. Global Post: 26.12.2012, "Why rising income inequality matters".

of within-country income inequality. We provide a small expansion on the theoretical analysis of [Egger and Kreickemeier \(2012\)](#) by allowing mark-ups to change after a country moves from autarky to free trade in their framework. This way we can analyse both the effects of competition increase and the effect of trade opening on income inequality in an open economy. We obtain noteworthy results that strengthen the original findings of [Egger and Kreickemeier \(2012\)](#). In the analysis we assume that both domestic firms and exporters face the same elasticity of demand (and have same level of mark-ups), but that this elasticity is higher in an open economy than it was in autarky due to higher competition in the market. We concentrate our analysis on the effects of this competition change on the differences before and after open trade on different measures of income inequality and on the unemployment level.²

Section 2 provides a review on the empirical and theoretical literature on income inequality, trade and mark-ups. In section 3 we present the theoretical framework of [Egger and Kreickemeier \(2012\)](#) together with their main results. Section 4 demonstrates the effects of a general mark-up change in a country after the opening of the economy to foreign competition. Subsection 4.1 provides analytical explanations on the effects, while subsection 4.2 shows some examples on the magnitudes of the effects. The comparative statics and numerical analyses required to illustrate these results are presented in appendix A. Section 5 concludes.

2 Literature

2.1 Income inequality and trade

According to the literature review of [Anand and Segal \(2008\)](#) the direction in global inequality is unclear. This is mainly due to the varying methods, data and definitions used in the studies. Studies looking more in detail at the decomposition of global income inequality to between- and within-country income inequality typically find that within-country income inequality has increased since the 1970s in average. Various directions have been found on the income inequality between countries, though mostly between-country inequality has been found to decline ([Ferreira and Ravallion, 2008](#); [Anand and Segal, 2008](#)). As [Ferreira and Ravallion \(2008\)](#) and [Galbraith and Kum \(2005\)](#) point out, income inequality within-countries rose in average throughout most of the world at the same time as globalisation, but not everywhere. [Harrison, McLaren, and McMillan \(2011\)](#) conclude in their literature review that more countries have witnessed an increase rather

² If mark-ups change in the EK model, also the main functional form of output changes and therefore we refrain from welfare analysis.

than a decline in within-country income inequality. Therefore, we will concentrate the rest of the literature review on the trends and reasons for the increases in within-country income inequality.

Opening of trade and globalisation have been commonly accused for the increases in within-country income inequality,³ while contrary results have been obtained as well.⁴ For example, [Rodriguez-Pose \(2012\)](#) finds empirical evidence on increase in trade leading to higher regional income inequality within-countries based on static and dynamic panel analyses from 28 countries for the time period 1975 to 2005. [Bergh and Nilsson \(2010\)](#) find similarly that trade increases within-country income inequality specifically in rich countries, and social globalization (increased contact with other cultures) in middle- and low-income countries. They used panel data from 80 countries for the years 1970 to 2005. In general, most of the relatively recent empirical studies have found a positive association between trade liberalization and an increase in within-country income inequality. However, as [Atkinson and Piketty \(2007, 2010\)](#) point out in their books covering various articles, many other aspects can affect real income distributions as well, including technological progress, social norms and in particular institutional settings.

The above mentioned findings have resulted in a growing number of empirical and theoretical analyses trying to explain the mechanisms behind the increases in within-country income inequality. People's total incomes consist of different parts, including for example wage income, capital income and social security transfers, out of which wage and capital income form typically the largest share. Various empirical studies have analysed the effects of trade (and trade in tasks) on wages and wage differences within-countries (e.g. [Klein, Moser, and Urban, 2013](#), [Van Reenen, 2011](#) and [Harrison, McLaren, and McMillan, 2011](#)). Most studies conclude that globalisation has increased wage income inequality. Similarly, theoretical analyses until now have concentrated especially on studying the effect of trade on wage income inequality.⁵ For example, [Egger and Kreickemeier \(2009\)](#), [Helpman, Itskhoki, and Redding \(2010\)](#) and [Basco and Mestieri \(2013\)](#) have all concluded with different models that trade leads to an increase in wage inequality within-countries.⁶

³ See e.g. [Rodriguez-Pose, 2012](#), [Bergh and Nilsson, 2010](#), [Meschi and Vivarelli, 2009](#), [Milanovic and Squire, 2005](#) and [Lundberg and Squire \(2003\)](#).

⁴ [Dollar and Kraay \(2004\)](#) and [Calderon and Chong \(2001\)](#) document a negative association between income inequality and trade. However, the former use rather fragmented, low-quality data from few countries, while the latter find the negative association only for developed countries and a positive association for developing countries.

⁵ [Harrison, McLaren, and McMillan \(2011\)](#) provide an excellent review on the theoretical frameworks used to study the effect of trade on within-country income inequality that were published by 2011.

⁶ Microeconomic research of [Hopkins and Kornienko \(2010\)](#) and [Hopkins \(2011\)](#) conclude also that in a tournament type of competition situation, the inequality of rewards from the tournament increases risk-taking and causes greater inequality of wealth.

While wages are the most important income source for the majority, empirical studies have emphasized also the role of top income earners (Atkinson and Piketty, 2007, 2010) and very recently the (once again) rising role of capital income and capital gains on total income inequality (see Atkinson and Piketty, 2010, and e.g. Roine and Waldenström, 2012, Biewen and Juhasz, 2010, and Chi, 2012).⁷ Nevertheless, there are only few theoretical analyses on the effects of trade on capital income inequality and on total income inequality. Foellmi and Oechslin (2010) analyse theoretically the effect of open trade on capital income. They analyse the income distributions of heterogeneous firm’s owners in less-developed countries and conclude that trade increases the incomes of initially relatively wealthy firm owners. Poorer firm owners, on the contrary, lose as profit margins shrink and access to capital is constrained. Therefore, capital income distribution widens in their model with open trade. Egger and Kreickemeier (2012) continued their previous work on wage income inequality effects of trade by analysing also the effects on firm owners. This way, they seem to have been the first to analyse the effects of trade on both wage and (one form of) capital income inequality at the same time. As will be explained later, they provide insightful views in particular on the possible effects of trade and capital income on the increasing total income inequality within-countries. Consequently, we will use their framework for our analysis.

2.2 Trade and mark-ups

Egger and Kreickemeier (2012) assume in their GE framework that when a country goes from autarky to open trade nothing happens to the mark-ups of firms and to the preferences of consumers. However, in empirical literature mark-ups have been found to decrease when trade opens. Mark-ups vary also significantly between (trading and non-trading) sectors and within sectors in a given country.⁸ For example, Epifani and Gancia (2011) find that trade openness decreases average price-costs margins (mark-ups) and increases their dispersion across industries based on US data from the year 1960 to 2000. Chen, Imbs, and Scott (2009) conclude that increased import penetration decreased prices

⁷ Roine and Waldenström (2012) conclude that the large increases found in top incomes have been mainly driven by capital income gains in Sweden over the past 20 years. The rise of top incomes has, again, increased total income inequality. Similarly, Biewen and Juhasz (2010) conclude that the rise in German total income inequality resulted from the increase in unemployment and the rising dispersion of both labour market returns and capital gains. Chi (2012) find that in urban China the contribution of capital income to the Gini index of total income has similarly increased over the recent years. Capital income forms the largest part of the top income earners’ total income in urban China and the concentration of capital income has been increasing steadily.

⁸ Tamminen and Chang (2013) and De Loecker and Warzynski (2012) find significant differences in the mark-ups of domestic versus exporting firms within sectors.

(growth rates), lowered mark-ups and lead to higher productivity due to the increased competition in European manufacturing sectors in time period 1989 to 1999 based on difference-in-difference estimations. According to the traditional pricing equations (see subsection 3.1), an increase in the number of varieties has to increase also the elasticity of substitution (and elasticity of demand) in order to obtain a decrease in prices and mark-ups. Based on the empirical studies until now, it seems realistic to assume that mark-ups change at least in average when trade opens, while evidence on mark-up heterogeneity within-sectors is not yet conclusive.

Various theoretical models have taken already an assumption of endogenous mark-ups. For example, Melitz and Ottaviano (2008) build a general equilibrium framework, where mark-ups depend on the 'toughness' of competition in each market, with larger markets inhibiting more competition and lower mark-ups. In their framework trade opening lowers average mark-ups, but increases average productivity of operating firms and welfare. In addition, for example Epifani and Gancia (2011), Bernard, Eaton, Jenson, and Kortum (2003), Ottaviano, Takatoshi, and Thisse (2002) and Asplund and Nocke (2006) have constructed trade models where mark-ups vary instead of being fixed.

3 Egger-Kreickemeier (EK) model

3.1 Closed economy

We use the Egger and Kreickemeier (2012) model without changing the main dynamics. We present them and the most important calculation steps here only for a better tractability of the model and the results. See also the original model for more detailed instructions on the calculation steps.

A population mass N is assumed, which is divided into production workers (L) and managers (M). The economy produces two types of goods: differentiated intermediate goods, q , and homogeneous final output, Y . Each firm produces one type of intermediate good. The final output Y is a CES aggregation of the differentiated intermediate goods production. In equation 1, V represents the mass of available intermediate goods M , and $0 < \rho_A < 1$ is the CES love-of-variety parameter in autarky. The subscript A is used for all parameters that are autarky specific and will change in the open economy in our analysis. The parameter ρ_A is linked to the elasticity of substitution between varieties (equals demand elasticity in this case), σ_A , by $\sigma_A \equiv 1/(1 - \rho_A)$.

$$Y = \left[M^{-(1-\rho_A)} \int_{v \in V} q(v)^{\rho_A} dv \right]^{1/\rho_A} \quad (1)$$

The profit maximisation of final output multiplied by price index,⁹ minus production costs, $\int_{v \in V} p(v)q(v)dv$, and subject to the output function leads to demand functions for intermediate goods. The demand of each variety, $q(v)$, takes the form presented in equation 2 when constant mark-up over marginal cost pricing is assumed. The price of a specific variety equals $p(v) = c(v)/\rho_A$, where $c(v) = w(v)/\varphi(v)$ is marginal costs, $w(v)$ refers to wages and φ is the productivity level.¹⁰ The mark-up over marginal costs, $\mu_A \equiv 1/(\rho_A)$, depends of the CES parameter. Total revenue $r(v)$ is derived from the demand function multiplied by the price function.

$$q(v) = \frac{Y}{M} p(v)^{-\sigma_A} = \frac{Y}{M} \left[\frac{c(v)}{\rho_A} \right]^{-\sigma_A}, \quad r(v) = \frac{Y}{M} \left[\frac{c(v)}{\rho_A} \right]^{1-\sigma_A} \quad (2)$$

The production technology in each firm requires one manager/owner and many workers. Therefore, the number of firms is the same as the number of managers and the number of varieties, M . Productivity of a person determines whether he/she will become a manager or a worker. Only the most productive individual will have high enough productivity to become a manager.

Workers are paid a fair wage $\hat{w}$ following [Akerlof and Yellen \(1990\)](#), where the wage depends on the profits of the firm and on the external reference to other firms. The external reference is defined to equal employment share of labour $(1 - U_A)$, where U_A is the unemployment level, multiplied by average wage $\bar{w}_A$. See equation 3. The fair wage increases if revenue or profits increase, unemployment decreases or average wage increases (*ceteris paribus*). This way, in this model firm profits are shared between managers and workers depending on the rent sharing parameter $\theta \in (0, 1)$.

$$\hat{w} = \left(\frac{r(v)}{\sigma_A} \right)^\theta [(1 - U) \bar{w}]^{1-\theta} \quad (3)$$

The wage formula together with the revenue function, equation 2, form a base for the relative wage and revenue rates between two firms. When the marginal cost function is taken into consideration, the relative wages and revenues depend only on the productivity levels of the firms when the firms have the same export status (which in this case is a

⁹ Price index $P = [M^{-1} \int_{v \in V} p(v)^{1-\sigma_A} dv]^{-\frac{1}{1-\sigma_A}}$ is normalised to one due to perfect competition in the final goods market.

¹⁰ The employees efficiency parameter ε included in the original marginal costs function is later in the [Egger and Kreickemeier \(2012\)](#) model set to equal always one. See p. 186-187 of [Egger and Kreickemeier \(2012\)](#) for the discussion on this.

non-exporter).¹¹

$$\frac{w(\varphi(v_1))}{w(\varphi(v_2))} = \left[\frac{r(v_1)}{r(v_2)} \right]^\theta = \left(\frac{\varphi_1}{\varphi_2} \right)^{\theta \eta_A} \quad (4)$$

$$\frac{r(\varphi(v_1))}{r(\varphi(v_2))} = \left[\frac{w(v_1)}{w(v_2)} \frac{\varphi(v_1)}{\varphi(v_2)} \right]^{1-\sigma_A} = \left(\frac{\varphi_1}{\varphi_2} \right)^{\eta_A} \quad (5)$$

The formulas include a simplification parameter, η_A , which is defined as $\eta_A \equiv (\sigma_A - 1)/[1 + \theta(\sigma_A - 1)]$. The labour productivity of firms, which equals the manager's productivity in this model, is following Pareto distribution $G(\varphi) = 1 - \varphi^{-k}$, where $k > \eta_A$. The lower the k is, the higher is the dispersion of the firms' productivity levels. Based on the Pareto distribution, the average productivity $\tilde{\varphi}$ is proportional to the cut-off productivity φ^* , which is the productivity of the lowest-producing firm.

$$\tilde{\varphi} = \left(\frac{k}{k - \eta_A} \right)^{1/\eta_A} \varphi^* \quad (6)$$

The equilibrium factor allocation is determined from the resource constraint (RC), $L_A = N - M_A$ where L_A is labour supply to production, and from the labour indifference condition (LI). The LI, equation 8, states that the average expected wage of a worker has to be the same as the profit income of the manager with the cut-off ability level φ^* (that is the productivity level of the marginal firm). All people with a higher ability will choose to be managers due to higher expected income from that position compared to being a worker. With equations 5 and 6, and the fact that with monopolistic competition aggregate labour income equals $\rho_A Y$ and aggregate profits $(1 - \rho_A) Y$ and the same multipliers for wage share and profit share apply at firm level, the ratio of average profits to marginal profits turns to the following form:

$$\frac{\pi(\tilde{\varphi})}{\pi(\varphi^*)} = \frac{(1 - \rho_A)r(\tilde{\varphi})}{(1 - \rho_A)r(\varphi^*)} = \left(\left(\frac{k}{k - \eta_A} \right)^{1/\eta_A} \right)^{\eta_A} = \frac{k}{k - \eta_A} \quad (7)$$

Given the above function on the relationship of profits and the earlier mentioned fact on the aggregate labour income, the labour indifference condition transforms from equation 8 to equation 9:

$$LI : (1 - U_A)\bar{w}_A = \pi(\varphi^*) \quad (8)$$

¹¹ As both the wage ratio and the revenue ratio presented in equations 4 and 5 depend on the relative productivity levels, the firm level variables can be linked to a respective productivity. Therefore, in the following a simplified notation is used for productivity: $\varphi_i \equiv \varphi(v_i)$.

$$LI : \frac{\rho_A Y}{L} = \frac{(1 - \rho_A) Y}{M} \left(\frac{k - \eta_A}{k} \right) \quad (9)$$

Equation 9 and the resource constraint are used to solve total labour supply L_A and the number of companies M_A in autarky. The solutions are presented in table 1, rows 3 and 4, part A. The ability level required to become a manager (operate a firm profitably) is calculated by solving φ^* from $M_A = [1 - G(\varphi^*)] N$. It results in the definition that $\varphi^* = \left(\frac{N}{M}\right)^{\frac{1}{k}}$, which holds both in autarky and in open economy. The final solution for the marginal productivity level in autarky is calculated from the previous definition and the equation of M. It is presented in table 1, row 5, part A.

Welfare is defined in a utilitarian way as income per capita, which in the model equals consumption per capita. The calculation of income per capita is based on the fact that aggregate profit income is a constant share $(1 - \rho_A) = (1/\sigma_A)$ of total income Y . Based on this, the total income Y is first defined as: $Y = \sigma_A M_A \pi(\tilde{\varphi})$.

The profit of the firm with an average productivity level, $\pi(\tilde{\varphi})$, can be determined to equal:¹²

$$\begin{aligned} \pi(\tilde{\varphi}) &= \frac{k}{k - \eta_A} \pi(\varphi^*) = \frac{k}{k - \eta_A} w(\varphi^*) = \frac{k}{k - \eta_A} \left(\frac{w(\varphi^*)}{w(\tilde{\varphi})} \right) w(\tilde{\varphi}) \\ &= \frac{k}{k - \eta_A} \left(\frac{w(\varphi^*)}{w(\tilde{\varphi})} \right) \rho_A \tilde{\varphi} = \left(\frac{k}{k - \eta_A} \right)^{\frac{\sigma_A}{\sigma_A - 1}} \rho_A \varphi^* \end{aligned} \quad (10)$$

Using the two above solutions, output per capita is solved:

$$\begin{aligned} \frac{Y}{N} &= \sigma_A M_A \pi(\tilde{\varphi}) * N^{-1} = (\sigma_A - 1) \left(\frac{k}{k - \eta_A} \right)^{\frac{\sigma_A}{\sigma_A - 1}} M_A \varphi^* N^{-1} \\ &= (\sigma_A - 1) \left(\frac{k}{k - \eta_A} \right)^{\frac{\sigma_A}{\sigma_A - 1}} \left(\frac{k - \eta_A}{k \sigma_A - \eta_A} \right)^{\frac{k-1}{k}} \end{aligned} \quad (11)$$

Due to the rent sharing mechanism, unemployment is strictly positive in the model. It is determined from the fact that aggregate total employment has to equal the sum of firm's employees. Given that $Ml(\tilde{\varphi}) = Mq(\tilde{\varphi})/\tilde{\varphi} = Y/\tilde{\varphi}$, $\frac{l(\varphi)}{l(\tilde{\varphi})} = \left(\frac{\varphi}{\tilde{\varphi}}\right)^{(1-\theta)\eta_A}$ and using the second definition of equation 11, the unemployment level can be solved from: $(1 - U_A) L_A = \frac{M_A}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} l(\varphi) dG(\varphi)$. The final solution is presented in table 1, row 6, part A.

Income inequality is measured in three ways in the EK model: 1) difference in the average expected income of workers versus managers, labelled as inter-group inequality,

¹² The steps are derived from: equation 5 with equation 6 (step 1), equation 4 with equation 6 (step 2), mark-up pricing condition for the average firm with notion that average price is one (step 4), and equation 4 with equation 6 (step 5).

2) income inequality within managers and 3) income inequality within workers. A Gini index for the total income inequality (capital and wage income together) is not calculated. Inter-group inequality is defined as the ratio of average managerial income (equals average profits) over average expected wage. Based on equation 7 and 8, inter-group income inequality is determined by the autarky equation presented in table 1, row 9. The ratio is higher than one, which means that average managerial income is higher than the expected average production worker wage.

In order to determine a Gini index for managerial income, we need to calculate first the cumulative profits of all firms with a productivity level lower than or equal to $\bar{\varphi} \in [\varphi^*, \infty]$ relative to the aggregate profits Π . Based on the productivity distribution, the proportion of firms with a productivity level smaller or equal to $\bar{\varphi}$ equals $\gamma \equiv 1 - \left(\frac{\bar{\varphi}}{\varphi^*}\right)^{-k}$. Using the latter solution, the cumulative profits relative to aggregate profits can be solved.¹³ The actual Gini index for managerial income, A_M , which by definition is between zero and one, is calculated in a standard way from $A_M = 1 - 2 \int_0^1 Q_M(\gamma) d\gamma$. The solution is presented in table 1, row 10.

The calculation of a Gini index for labour income, A_L , follows closely the steps taken in the determination of a Gini index for managerial income. Since total wages paid by a firm are proportional to the profits, the ratio of all salaries paid by firms with a productivity level lower than or equal to $\bar{\varphi} \in [\varphi^*, \infty]$ relative to the aggregate wages W is the same as the ratio of profit incomes calculated earlier. However, the share of workers, μ , employed in firms with the same productivity level out of total employment needs to be calculated first.¹⁴ With the solutions for proportional wages and employment, the cumulative wages relative to aggregate wages can be derived.¹⁵ The definition of Lorenz curve, $A_L = 1 - 2 \int_0^1 Q_L(\mu) d\mu$, is used again to obtain a solution for the Gini index for labour income presented in table 1, row 11, part A. As mentioned in Egger and Kreickemeier (2012), the Gini index for managerial income is always bigger than the Gini for labour income in autarky. This means that income inequality within managers is larger than income inequality within workers.

¹³ The cumulative profits equal:

$$Q_M(\gamma) = \frac{\Pi(\bar{\varphi})}{\Pi} = \frac{M_A}{[1-G(\varphi^*)]\Pi} \int_{\varphi^*}^{\bar{\varphi}} \pi(\varphi) dG(\varphi) = 1 - (\bar{\varphi}/\varphi^*)^{\eta_A - k} = 1 - (1 - \gamma)^{1 - \frac{\eta_A}{k}}.$$

¹⁴ The calculation steps are similar to those taken for the calculation of the unemployment level and the share is derived to equal: $\mu \equiv \frac{L(\bar{\varphi})}{(1-U)L} = \frac{M_A}{[1-G(\varphi^*)](1-U)L} \int_{\varphi^*}^{\bar{\varphi}} l(\varphi) dG(\varphi) = 1 - (\bar{\varphi}/\varphi^*)^{(1-\theta)\eta_A - k}$.

¹⁵ The cumulative wages equal: $Q_L(\mu) = \frac{W(\bar{\varphi})}{W} = 1 - \left(\frac{\bar{\varphi}}{\varphi^*}\right)^{\eta_A - k} = 1 - (1 - \mu)^{\frac{k - \eta_A}{k - \eta_A + \theta\eta_A}}$.

3.2 Open economy

The main changes in this article compared to the [Egger and Kreickemeier \(2012\)](#) article relate to the open economy solutions. In line with recent empirical findings (e.g. [Chen, Imbs, and Scott, 2009](#), [Epifani and Gancia, 2011](#)), it can be assumed that mark-ups decrease after the opening of the economy due to higher competition in the home market and in the foreign market. In this scenario we assume that both exporters and domestic companies face a higher, but identical demand elasticity in the open economy in comparison to autarky. In other words, we assume that in the open economy $\rho_T > \rho_A$, where ρ_T stands for CES parameter after opening up to trade and ρ_A stands for the CES parameter in autarky. In general, all parameters with the subscript T refer to the values after open trade with a new elasticity in comparison to the autarky solutions. All parameters and variables without a subscript are the same as in autarky. Based on the definitions used, we obtain also a new, higher elasticity of demand $\sigma_T \equiv 1/(1 - \rho_T) > \sigma_A$. The higher elasticity of demand lowers average mark-ups after open trade following the definition: $\mu_T \equiv 1/(\rho_T) < \mu_A$.

The main functional forms do not change much from the original [Egger and Kreickemeier \(2012\)](#) model. In this subsection, we derive the main solutions in the open economy in order to ease following of the later sections. We explain at the same time how open trade affects the various indicators in comparison to autarky, following closely [Egger and Kreickemeier \(2012\)](#). In section 4 we continue from the main open economy solutions and show what happens to the various indicators when mark-ups decrease.

Following the [Egger and Kreickemeier \(2012\)](#) article, the country starts trade with a similar country. The labour indifference curve changes after the opening of the economy due to a possibility for people to work as a local expert for foreign firms with salary s . Therefore, the marginal manager can choose between three options: 1) to run a firm, 2) to be a production worker or 3) to act as a local expert for a foreign firm. The labour indifference condition changes to the following form:

$$\pi(\varphi^*) = (1 - U_T)\bar{w}_T = s \quad (12)$$

Exporters are assumed to sell both domestically and abroad. Due to additional iceberg transport costs in exporting, $\tau > 1$, the total revenue of an exporter is $\Omega_T r^e(\varphi)$, where $r^e(\varphi)$ equals the domestic revenues of the exporting firm and $1 < \Omega_T \equiv 1 + \tau^{1-\sigma_T} \leq 2$.

The indifference condition for the marginal exporter with productivity φ_x^* on whether to start exporting or not is defined as:

$$\frac{\Omega_T r^e(\varphi_x^*)}{\sigma_T} - s = \frac{r^n(\varphi_x^*)}{\sigma_T} \quad (13)$$

Revenue and wage ratios' of two firms with the same productivity level, but differing export status, are determined jointly by the fair wage equation 3, the demand function and the definition for revenue in equation 2. The solutions in equation 14 point out that exporters pay higher wages than non-exporters in line with empirical findings. On the other hand, they have lower operating profits in the home market since their domestic revenues are lower than those of non-exporting firms.

$$\frac{w^e}{w^n} = \Omega_T^{\frac{\theta\eta_T}{\sigma_T-1}} > 1, \text{ and } \frac{r^e}{r^n} = \Omega_T^{-\theta\eta_T} < 1 \quad (14)$$

The total revenue of exporters, $\Omega_T^{1-\theta\eta_T} r^n(\varphi)$, is still higher than the total revenue of non-exporting firms, $r^n(\varphi)$, since the multiplier is always positive and bigger than one.

The indifference condition of the marginal exporter can be rewritten with the solution for r^e from equation 14 and using $s = r^n(\varphi^*)/\sigma_T$ ¹⁶ as: $\Omega_T^{\frac{\eta_T}{\sigma_T-1}} = 1 + \left(\frac{\varphi^*}{\varphi_x^*}\right)^{\eta_T}$. Using this solution, the share of exporting firms χ_T can be calculated from the pareto distribution based on the productivity limits for exporting, φ_x^* , and for operating a firm in general, φ^* . The share of exporters can take values from nearly zero to one.

$$\chi_T = \frac{[1 - G(\varphi_x^*)]}{[1 - G(\varphi^*)]} = \left(\frac{\varphi^*}{\varphi_x^*}\right)^k = \left(\Omega_T^{\frac{\eta_T}{\sigma_T-1}} - 1\right)^{\frac{k}{\eta_T}} \quad (15)$$

Labour supply and number of firms (equals number of managers) are derived in a similar way as in the closed economy. The labour supply L_T follows from the labour indifference condition (equation 12) and the definitions that $\pi(\tilde{\varphi}) = r(\tilde{\varphi})/\sigma_T$, $\rho_T Y = (1 - U_T)L_T \bar{w}_T$ and $Y = M_T(1 + \chi_T)r^n(\tilde{\varphi})$. With these notions, the labour indifference condition turns into the following form with the average expected labour income on the right hand side and the profit level of the marginal firm in the left hand side:

$$\frac{\rho_T Y}{L_T} = \frac{(1 - \rho_T)Y}{(1 + \chi_T)M_T} \left(\frac{k - \eta_T}{k}\right)$$

Solving the above function for L results in:

$$L_T = \frac{k(\sigma_T - 1)(1 + \chi_T)}{k - \eta_T} M_T \quad (16)$$

Taking into account that part $\chi_T M$ of the labour force will work as experts for foreign firms, the resource constraint (RC) converts to $L_T = N - (1 + \chi_T)M_T$. From equation 16 and the RC, we calculate the functions for labour supply and for the number of managers/firms M. Merely the change from autarky to open trade does not affect the quantity of labour supply, but the number of firms goes down, as shown in Egger and

¹⁶ Based on equation 12.

Kreickemeier (2012). Similar to the autarky solution, the cut-off ability required to run a firm is still solved from the ratio of N to M. The number of firms in an open economy is smaller than in autarky, since the marginal productivity required to run a firm is higher. The solutions for L, M and marginal productivity in the open economy are presented in table 1, rows 3, 4 and 5, part T.

Aggregate output and welfare per capita are calculated in the same way as in the case of closed economy from the definition $Y_T = M(1 + \chi_T)r^n(\tilde{\varphi})$. The new utilian welfare in the open economy equals:

$$\frac{Y_T}{N} = (1 + \chi_T)^{\frac{1}{k}} * \frac{Y_A}{N} \quad (17)$$

As the share of exporting firms is larger than zero, output and welfare per capita are always larger with open trade in comparison to autarky. However, if mark-ups change, also the main functional form of output changes. Therefore, we concentrate on the income inequality consequences of opening up to international trade.

The level of unemployment in the open economy is calculated also in a similar way as in the closed economy case. The use of the solution for ratio $\tilde{\varphi}/\varphi^*$, as derived in the appendix A of Egger and Kreickemeier (2012), helps to solve the equations. At the end we obtain the solution presented in table 1, row 6. As presented in Egger and Kreickemeier (2012), the level of unemployment is always higher in open trade than in autarky.

The ratio of average manager income in comparison to the average expected production worker income is determined with the help of the indifference condition: $(1 - U_T)\bar{w}_T = s = \pi(\varphi^*) = r(\varphi^*)/\sigma_T$ and definition of average profits as defined earlier. The ratio is used as an indicator for the income inequality between workers and managers. It turns in to the form presented in table 1, row 9, part T. As the multiplier of the autarky level intergroup inequality is above one, intergroup inequality is higher in open economy than in autarky in the EK model.

Further, the Gini indexes for profit income (managers' income) and labour income are derived in a similar way as in the closed economy case. However, the final Gini indexes are calculated from the integral of the two segments of the Lorenz curve. First segment of the profit income Gini, $Q_M^1(\gamma)$ ¹⁷, calculates the share of profits that go to non-exporting firms. The second segment, $Q_M^2(\gamma)$ ¹⁸, derives the share of profits allocated to exporting

¹⁷ $Q_M^1(\gamma) = \frac{\Pi(\tilde{\varphi})}{\Pi} = \frac{M_T}{[1 - G(\varphi^*)]\Pi} \int_{\varphi^*}^{\tilde{\varphi}} \pi(\varphi) dG(\varphi)$. This leads to solution:

$$Q_M^1(\gamma) = \frac{k}{k + \eta_T \chi_T} \left[1 - \left(\frac{\tilde{\varphi}}{\varphi^*} \right)^{\eta_T - k} \right] = \frac{k}{k + \eta_T \chi_T} \left[1 - (1 - \gamma)^{1 - \frac{\eta_T}{k}} \right], \text{ when } \tilde{\varphi} \in [\varphi^*, \varphi_x].$$

¹⁸ $Q_M^2(\gamma) = \frac{\Pi(\tilde{\varphi})}{\Pi} = Q_M^1(\gamma) + \frac{M}{[1 - G(\varphi^*)]\Pi} \int_{\varphi_x}^{\tilde{\varphi}} \pi(\varphi) dG(\varphi)$. It leads to the further solution that:

$$Q_M^2(\gamma) = Q_M^1(\gamma) + \frac{k\Omega_T^{1 - \theta\eta_T}}{k + \eta_T \chi_T} \left[\chi_T^{1 - \frac{\eta_T}{k}} - (1 - \gamma)^{1 - \frac{\eta_T}{k}} \right] - \frac{(k - \eta_T)(\gamma - b_M)}{k + \eta_T \chi_T}, \text{ when } \tilde{\varphi} \in [\varphi_x, \infty]$$

and where $b_M \equiv 1 - \chi_T$.

firms. Detailed steps for the calculations can be found in the appendix A of [Egger and Kreickemeier \(2012\)](#). As [Egger and Kreickemeier \(2012\)](#) show, inequality of managerial income is derived again from $A_{M,T} = 1 - 2 \int_0^1 Q_M^t(\gamma) d\gamma$. The final solution is presented in table 1, row 10, part T, which shows that the inequality in profit income increases when the economy moves from autarky to an open economy.

In the determination of the labour income Gini index, the first segment, $Q_L^1(\mu)$ ¹⁹, measures the proportion of labour income going to workers employed in non-exporting firms. The second segment, $Q_L^2(\mu)$ ²⁰, measures labour income of exporting firms' employees. The detailed derivations can be found from the appendixes of the original paper. Similar to the managerial income Gini, open trade in itself increases labour income inequality as is shown in the original article. The final solution for labour income Gini is derived from $A_{L,T} = 1 - 2 \int_0^1 Q_L^t(\mu) d\mu$ and presented in table 1, row 11, part T. It is transformed to a slightly different form compared to [Egger and Kreickemeier \(2012\)](#) in order to simplify the derivatives presented in appendix A.1. See calculations before equation 36.

4 What happens when markups decrease after the opening of trade?

4.1 Summary table and main effects

In this subsection, we explain and summarise what happens in the EK model when markups change after the opening of trade in comparison to autarky, in other words, when parameter $\rho_T > \rho_A$. Table 1 summarises all the main functions from the closed economy and the open economy.²¹ In addition, it summaries briefly the sign of the derivatives of the various open economy functions with respect to ρ_T (or σ_T).

¹⁹ $Q_L^1(\mu) = \frac{M_T}{[1-G(\varphi^*)](1-U_T)L_T} \int_{\varphi^*}^{\bar{\varphi}} w^n(\varphi) l^n(\varphi) dG(\varphi)$. This leads to:

$$Q_L^1(\mu) = \frac{1}{1+\chi_T} \left[1 - \left(\frac{\bar{\varphi}}{\varphi^*} \right)^{\eta_T - k} \right] = \frac{1}{1+\chi_T} \left[1 - (1 - \Gamma\mu)^{\frac{k - \eta_T}{k - (1-\theta)\eta_T}} \right], \text{ when } \bar{\varphi} \in [\varphi^*, \varphi_x^*].$$

²⁰ $Q_L^2(\mu) = Q_L^1(b_L) + \frac{M_T \Omega_T}{[1-G(\varphi^*)]W} \int_{\varphi_x^*}^{\bar{\varphi}} w^e(\varphi) l^e(\varphi) dG(\varphi)$, leading to:

$$Q_L^2(\mu) = Q_L^1(b_L) + \frac{\Omega_T^{1-\theta\eta_T}}{1+\chi_T} \left[\chi_T^{1-\frac{\eta_T}{k}} - \left(\frac{\bar{\varphi}}{\varphi^*} \right)^{\eta_T - k} \right] = 1 - \frac{\Omega_T^{1-\theta\eta_T}}{1+\chi} \left[\frac{1-\mu\Gamma}{\Omega^{\frac{(1-\theta)\eta_T}{\sigma_T-1}}} \right]^{\frac{k-\eta_T}{k-(1-\theta)\eta_T}}, \text{ when } \bar{\varphi} \in [\varphi_x^*, \infty]$$

and where $b_L \equiv \left[1 - \chi_T^{1-(1-\theta)\eta_T/k} \right] / \Gamma_T$. See table 1 for the definition of Γ_T .

²¹ Please notice that the functional forms for average labour income and average profits are derived in equations 18 to 21 after the table.

Table 1: Summary of functions and the signs of derivatives

	Indicator	Solution in autarky (A)	Solution in open economy (T)	Sign ¹⁾ of the derivative over ρ_T/σ_T
1)	Revenue	$r^n(\varphi)$ ²⁾	$\Omega_T^{1-\theta\eta_T} * r^n(\varphi)$ ³⁾	-
2)	Share of exporters	-	$\chi_T = \left(\Omega_T^{\frac{\eta_T}{\sigma_T-1}} - 1 \right)^{\frac{k}{\eta_T}}$	(-)/(+)*
3)	Labour supply	$L_A = \frac{k(\sigma_T-1)}{k\sigma_T-\eta_T} N$	$L_T = \frac{k(\sigma_T-1)}{k\sigma_T-\eta_T} N$	(+)
4)	Number of firms	$M_A = \frac{k-\eta_A}{k\sigma_A-\eta_A} N$	$M_T = \frac{k-\eta_T}{(k\sigma_T-\eta_T)(1+\chi_T)} N$	(-)
5)	φ^*	$\varphi_A^* = \left(\frac{k\sigma_A-\eta_A}{k-\eta_A} \right)^{\frac{1}{k}}$	$\varphi_T^* = \left(\frac{(k\sigma_T-\eta_T)(1+\chi_T)}{k-\eta_T} \right)^{\frac{1}{k}}$	(+)
6)	Employment share	$(1-U_A) = \frac{k-\eta_A}{k-(1-\theta)\eta_A}$	$(1-U_T) = \frac{\Gamma_T}{(1+\chi_T)} \left[\frac{k-\eta_T}{k-(1-\theta)\eta_T} \right]$ ⁴⁾	(-)
7)	Average wage	$\bar{w}_A = \left(\frac{k-(1-\theta)\eta_A}{k-\eta_A} \right) * \Xi_A$	$\bar{w}_T = (1+\chi_T)^{\frac{1}{k}} (1-U_T)^{-1} * \Xi_T$ ⁵⁾	(+)
8)	Average profit	$\bar{\pi}_A = \rho_A \left(\frac{k}{k-\eta_A} \right)^{\frac{\sigma_A}{\sigma_A-1}} \left(\frac{k\sigma_A-\eta_A}{k-\eta_A} \right)^{\frac{1}{k}}$	$\bar{\pi}_T = \left[(1+\chi_T) \left(\frac{k}{k-\eta_T} \right) - \chi_T \right] (1+\chi_T)^{\frac{1}{k}} * \Xi_T$	(+)
9)	Between inequality	$\frac{\bar{\pi}_A}{(1-U_A)\bar{w}_A} = \frac{k}{k-\eta_A} \equiv \omega_A$	$\frac{\bar{\pi}_T}{(1-U_T)\bar{w}_T} = \frac{k}{k-\eta_T} * (1 + \frac{\eta_T\chi_T}{k})$	(+)
10)	Manager income gini	$A_M = \frac{\eta_A}{2k-\eta_A}$	$A_{M,T} = \left[\frac{\eta_T}{2k-\eta_T} \right] * \left[1 + \frac{\chi_T(2-\chi_T)(k-\eta_T)}{k+\eta_T\chi_T} \right]$	(+)
11)	Labour income Gini	$A_L = \frac{\theta\eta_A}{2[k-(1-\theta)\eta_A]-\theta\eta_A}$	$A_{L,T} = \frac{\theta\eta_T}{2[k-(1-\theta)\eta_T]-\theta\eta_T} * [1+\Lambda]$ ⁶⁾	(+)

Notes: ¹⁾ Results without brackets are based on analytical analysis and results with brackets on numerical analyses.

²⁾ Domestic revenue. ³⁾ Exporter's revenue.

$$^4) : \Gamma_T \equiv 1 + \chi_T^{\frac{k-(1-\theta)\eta_T}{k}} \left(\Omega_T^{\frac{(1-\theta)\eta_T}{\sigma_T-1}} - 1 \right).$$

$$^5) : \Xi_{S=A,T} = \left(\frac{(\sigma_S-1)^2}{\sigma_S} \right) \left(\frac{k}{k-\eta_S} \right)^{\frac{\sigma_S}{\sigma_S-1}} \left(\frac{k-\eta_S}{k\sigma_S-\eta_S} \right)^{\frac{k-1}{k}} \left(\frac{k\sigma_S-\eta_S}{k(\sigma_S-1)} \right) > 0$$

$$^6) : \Lambda \equiv \left(\frac{2k}{(\sigma_T-1)\theta} + 2k - \frac{2}{\theta} \right) \left(\frac{\chi_T - \chi_T^{2-\frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}}}{(1+\chi_T)\Gamma_T} \right) - \left(\frac{2k}{(\sigma_T-1)\theta} + 2k + 2 - \frac{2}{\theta} \right) \left(1 - \frac{1}{\Gamma_T} \right) \left(\frac{1-\chi_T^{1-\frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}}}{1+\chi_T} \right) > 0$$

* See appendix A.2, table 3, for the analysis on the sign of the derivative with different parameter values.

The derivatives' calculations are in appendix A.1. In case no analytical solution is

found on the sign of the derivative, we use numerical analyses to determine the sign with the given parameter restrictions. These numerical analyses are explained and derived in appendix A.2. As mentioned earlier, since the functional form of output changes when ρ_T changes, we concentrate on the income inequality consequences of opening up to international trade.

First of all, in case the CES parameter ρ_T increases (elasticity of demand increases and mark-ups decrease) with open trade, the total revenue of exporters decreases compared to the situation where there is no change in ρ_T after open trade. However, the multiplier for the exporter's revenue stays above one²² and exporters have larger total revenue than domestic firms even if mark-ups change. Equation 24 in the comparative statics part demonstrates this. Since the revenue differential between exporters and non-exporters is lower with the new ρ_T , the share of firms that can export decreases²³ at the same time.

Second, in addition to the decrease in the share of exporting firms, the higher ρ_T results in a lower number of firms in general. With smaller mark-ups, the productivity of the marginal firm needs to be higher than without a change in ρ_T . The (growth rate of) product prices decrease since the elasticity of demand increases (see the pricing equation in subsection 3.1). Subsequently, a smaller share of firms manage to reach the required level of productivity to produce profitably with the lower prices and mark-ups. Out of the population of N , fewer people will be managers and fewer can work as local experts for foreign firms. Therefore, labour supply, $L_T = N - (1 + \chi_T)M_T$, increases.

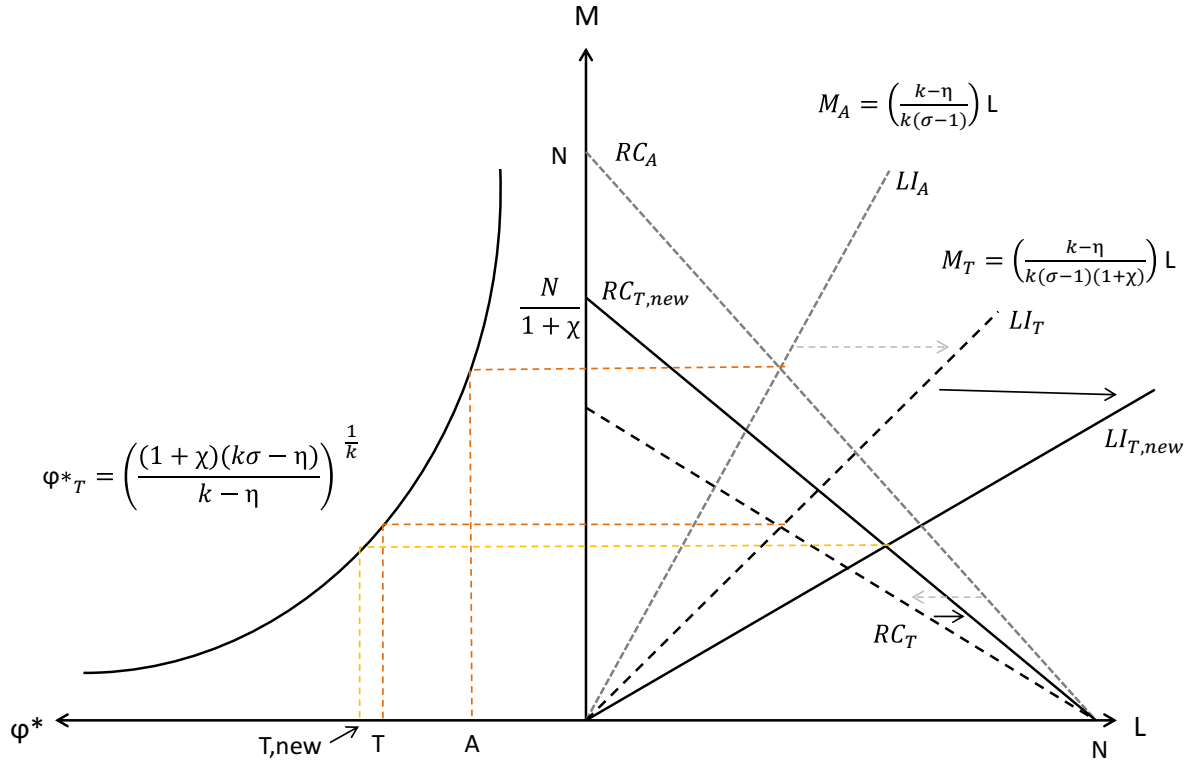
Graph 1 demonstrates these effects in a similar way as Egger and Kreickemeier (2012) did in the original article. See also equations 27, 28 and 29 in appendix A.1 and numerical analyses in appendix A.2 for proof.

The equilibrium values of L and M are determined together by the labour indifference (LI) condition (equations 9 and 16 in autarky and open economy respectively) and the recourse constraints (RC), which illustrate the possible divisions of population to workers and managers provided that share χ_T will work as foreign experts in open economy. The LI line is already lower with open trade than in autarky (LI_A vs. LI_T) and shifts further to $LI_{T,new}$ as ρ_T increases. The resource constraint turns slightly to the right after the change in mark-ups. This results from the lower share of exporting firms after the new mark-up. However, the RC line will still stay below the autarky level. The new equilibrium for L and M is found from the intersection of the $LI_{T,new}$ and $RC_{T,new}$ lines, the solid lines. Based on the derivatives, the shift in LI_T has always a larger effect than

²² Omega is defined to be larger than one.

²³ There are few parameter values with which the derivative of χ_T with respect to sigma is positive, but as table 3 shows, these are rather unusual value combinations for the different parameters. Therefore, it can be concluded that in most cases, the share of exporters decreases when σ_T increases.

the change in the RC line. At the equilibrium labour supply is higher than previously and the number of firms is lower. The equivalent productivity level required to produce, on the left side of the graph, is slightly higher.



²⁴ Retailers could become also larger with more negotiation power, which increases similarly the elasticity of demand.

and their employees went down. By now, there are relatively few firms producing flour compared to history and the existing firms operate with few employees, many machines and very low per unit costs. For example, in the year 1993 Finland had already less than 100 firms producing flour products. Between 1993 and 2007 the number of these firms went further down by 27 percent.²⁵ At the same time, the number of employees in the industry went down, while the demand for flour per person has probably not changed considerably over the last hundred years and the Finnish population has increased.

With the previously explained changes in labour supply, the number of firms and marginal productivity, unemployment rate is found to increase in the EK model when mark-ups decrease. This result is contrary to what has been considered until now. For example, [Blanchard and Giavazzi \(2003\)](#) come to the exact opposite conclusion. They find that when the number of firms is endogenous (as it is in the EK model), the unemployment level will decrease when product markets are deregulated and the market power of firms diminishes. The main difference between their GE model and the EK model is the inclusion of firm heterogeneity and fair wages in the latter. The reason for this result in the EK model is straightforward. Considering again the flour production industry, we see that there are three main reasons for the increase in the share of unemployed people:

1. The firms that continue operations are more productive, but due to the higher productivity level they need less employees to obtain the same level of revenue than less productive firms that drop out of competition;
2. The number of firms that employ people decreases; and
3. Labour supply increases after the elasticity of demand increases.

Due to these mechanisms, more workers from the labour supply will stay unemployed compared to the situation where there is no change in mark-ups after trade opening. See appendixes [A.1](#) and [A.2](#) for proof.²⁶

With regards to income inequality, detailed analyses on the changes in the average labour wage and average profits clarify the mechanisms of the EK model. The function for average labour wage is not derived in [Egger and Kreickemeier \(2012\)](#), but can be solved from the definition of average expected income: $\bar{w}_i(1 - U_i) = \frac{\rho_i Y_i}{L_i}$, $i \in [A, T]$. The same definition applies both in autarky and in open economy, but the level of Y is different in the open economy even if ρ_T would not change. Using the autarky and open

²⁵ Source: Statistics Finland, firm statistics by industry, 1993-2007.

²⁶ In the analyses of the functional forms, it should be noticed that while the share of exporting firms is smaller after a change in the mark-ups, the inequality of $\Gamma_T < 1 + \chi_T$ in the employment rate equation in row 6 of table [1](#) holds also after an equal change in mark-ups.

economy solutions for Y and L, as presented in subsections 3.1 and 3.2 respectively, the average labour wage in autarky is:

$$\begin{aligned}\bar{w}_A &= \frac{\rho_A Y_A}{L_A(1 - U_A)} = \frac{\rho_A(\sigma_A - 1) \left(\frac{k}{k - \eta_A}\right)^{\frac{\sigma_A}{\sigma_A - 1}} \left(\frac{k - \eta_A}{k\sigma_A - \eta_A}\right)^{\frac{k-1}{k}} N}{\frac{k(\sigma_A - 1)}{k\sigma_A - \eta_A} N * \frac{k - \eta_A}{k - (1 - \theta)\eta_A}} \\ &= \left(\frac{(\sigma_A - 1)^2}{\sigma_A}\right) \left(\frac{k}{k - \eta_A}\right)^{\frac{\sigma_A}{\sigma_A - 1}} \left(\frac{k - \eta_A}{k\sigma_A - \eta_A}\right)^{\frac{k-1}{k}} * \\ &\quad \left(\frac{k\sigma_A - \eta_A}{k(\sigma_A - 1)}\right) \left(\frac{k - (1 - \theta)\eta_A}{k - \eta_A}\right) \quad (18)\end{aligned}$$

In comparison, with open trade, the average salary paid to employed workers is:

$$\begin{aligned}\bar{w}_T &= \frac{\rho_T Y_T}{L_T(1 - U_T)} = \frac{\rho_T(1 + \chi_T)^{\frac{1}{k}} * (\sigma_T - 1) \left(\frac{k}{k - \eta_T}\right)^{\frac{\sigma_T}{\sigma_T - 1}} \left(\frac{k - \eta_T}{k\sigma_T - \eta_T}\right)^{\frac{k-1}{k}}}{\frac{k(\sigma_T - 1)}{k\sigma_T - \eta_T} * (1 - U_T)} \\ \bar{w}_T &= (1 + \chi_T)^{\frac{1}{k}} \left[\frac{1 + \chi_T}{\Gamma_T}\right] \left(\frac{k - (1 - \theta)\eta_T}{k - \eta_T}\right) \left(\frac{(\sigma_T - 1)^2}{\sigma_T}\right) * \\ &\quad \left(\frac{k}{k - \eta_T}\right)^{\frac{\sigma_T}{\sigma_T - 1}} \left(\frac{k - \eta_T}{k\sigma_T - \eta_T}\right)^{\frac{k-1}{k}} \left(\frac{k\sigma_T - \eta_T}{k(\sigma_T - 1)}\right) \quad (19)\end{aligned}$$

Even without a change in the markups between autarky and open trade, the average wage increases with open trade since the first two terms of function 19 are both above 1 (as shown in Egger and Kreickemeier, 2012). In case markups decrease and a larger fraction of revenue is allocated to employees, the average wage in the economy increases further. See equation 32 and subsection A.2 in the appendix for proof.

Similar to the average labour wage, final solutions for average profits in autarky and open economy are not included in Egger and Kreickemeier (2012), but can be derived from the other definitions. In autarky average profits equal:

$$\begin{aligned}\bar{\pi}_A &= \pi(\tilde{\varphi}) = \rho_A \left(\frac{k}{k - \eta_A}\right)^{\frac{\sigma_A}{\sigma_A - 1}} \varphi^* \\ &= \rho_A \left(\frac{k}{k - \eta_A}\right)^{\frac{\sigma_A}{\sigma_A - 1}} \left(\frac{k\sigma_A - \eta_A}{k - \eta_A}\right)^{\frac{1}{k}} \quad (20)\end{aligned}$$

When the average managerial income is compared to the average wage of a production worker²⁷ in autarky, the managers are found to obtain a higher average income. With

²⁷ In comparison, in the EK model average manager income is compared to expected average worker income $\bar{w}(1 - U)$, see row 9, table 1.

open trade the average profits equal:

$$\begin{aligned}
\pi_T &= (1 + \chi_T) r^n(\tilde{\varphi}) / \sigma_T - \chi_T s = (1 + \chi_T) \frac{\left(\frac{k}{k-\eta_T}\right) \sigma_T \pi(\varphi^*)}{\sigma_T} - \chi_T \pi(\varphi^*) \\
\bar{\pi}_T &= \left[(1 + \chi_T) \left(\frac{k}{k-\eta_T}\right) - \chi_T \right] \bar{w}_T (1 - U_T) \\
&= \left[(1 + \chi_T) \left(\frac{k}{k-\eta_T}\right) - \chi_T \right] (1 + \chi_T)^{\frac{1}{k}} \left(\frac{(\sigma_T - 1)^2}{\sigma_T} \right)^* \\
&\quad \left(\frac{k}{k-\eta_T} \right)^{\frac{\sigma_T}{\sigma_T-1}} \left(\frac{k-\eta_T}{k\sigma_T-\eta_T} \right)^{\frac{k-1}{k}} \left(\frac{k\sigma_T-\eta_T}{k(\sigma_T-1)} \right) \quad (21)
\end{aligned}$$

The average managerial income is higher in open trade than in autarky.²⁸ If mark-ups fall, the average managerial income increases further. This results from the fact that the fewer firms that continue to operate are more productive. Therefore, even the manager's lower share of revenue results in a higher average income for them. See equation 33 and subsection A.2 in the appendixes for proof.

Finally, we analyse the income inequality effects of a change in mark-ups and in elasticity of demand. The opening of trade by itself increases the income inequality between managers and production workers. In case mark-ups decrease with open trade, the inter-group inequality increases further (see equation 34 and numerical analysis). This results from the wage setting equation, which stresses the external labour market conditions in addition to the firms' profits. In other words, the average profits of the operating firms go up more than the average expected labour wage, $\bar{w}_T(1 - U_T)$. While the average labour wage, $\bar{w}_T$, increases, the share of employed people decreases at the same time. The comparison of average profits to average expected labour wage shows that the increase in $\bar{w}_T$ is not sufficient to compensate for the decrease in $(1 - U_T)$.

A decrease in the mark-ups also results in a higher Gini index for profit income in comparison to the open economy situation without a change in mark-ups. See equation 35 in the comparative statics section and the numerical analyses. Since there are fewer firms that can export and materialize higher total profits (based on the higher total revenue of exporters), the share of managers that can enjoy the export premium on profits decreases. At the same time, a higher share of firms will operate only in the domestic market with lower total profits. This widens the income distribution and increases the Gini index of profit income.

²⁸ This is based on the following inequality:

$$\left[(1 + \chi_T) \left(\frac{k}{k-\eta_T}\right) - \chi_T \right] (1 + \chi_T)^{\frac{1}{k}} \left(\frac{(\sigma_T-1)^2}{\sigma_T} \right) \left(\frac{k-\eta_T}{k\sigma_T-\eta_T} \right)^{\frac{k-1}{k}} \left(\frac{k\sigma_T-\eta_T}{k(\sigma_T-1)} \right) > \rho_A \left(\frac{k\sigma_A-\eta_A}{k-\eta_A} \right)^{\frac{1}{k}}$$

Similarly, the decrease in mark-ups increases the Gini index of labour income inequality. The reason is very similar to the case of profit income changes. A higher share of workers are employed in non-exporting firms, which pay lower salaries than exporting firms. The distribution of labour income becomes broader and the Gini index increases. See comparative statics, equation 37 for the derivations and subsection A.2 in the appendixes for the numerical analyses.

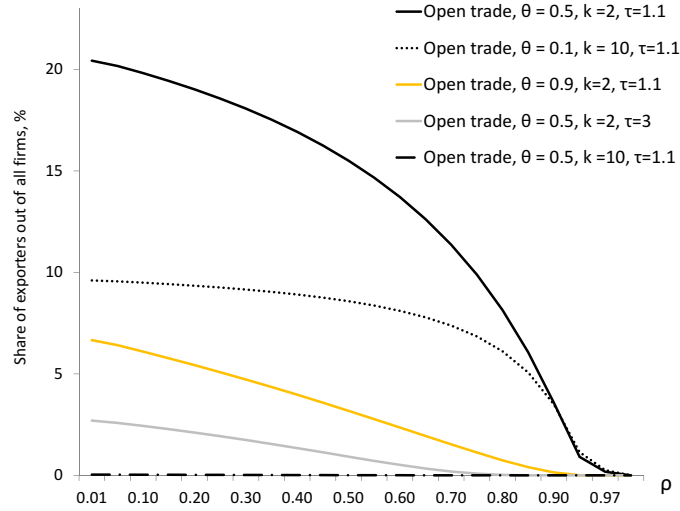
4.2 Magnitude of the effects

Once we understand the dynamics behind the main effects, there is still the question of how extensive the effects are. In order to magnify and compare them, figures from 2 up to 6 provide few examples. They show the share of exporting firms, the level of unemployment rate, the rate of the two Gini indexes and intergroup inequality with different values of ρ_T and other parameters. With respect to the evaluation of the other three parameters, it should be remembered that the higher the parameter θ is, the higher emphasis employees give to the profits of the firm in their wage demands in comparison to the external conditions in the labour markets. The lower the value of k , the more spread out is the distribution of firms' productivity levels. Last, the higher is τ , the higher are transport costs for the exporters.

First of all, figure 2 displays the share of exporting firms with different parameter values. The lines show the decrease in the share of exporting firms as ρ_T increases and that the absolute levels vary with the different parameter values. With a relatively low transport cost level of 10 percent ($\tau = 1.1$), the higher is the dispersion of the productivity distribution (the lower k is), the larger is the share of exporting firms with the different values of ρ_T . This is visible by comparing line $\theta = 0.5, k = 2, \tau = 1.1$ to line $\theta = 0.5, k = 10, \tau = 1.1$ in the figure. On the other hand, a higher emphasis on profits (a higher θ) with a given level of k , results in a lower share of exporting firms since exporting firms need to pay higher wages than with less emphasis on their profits. This increases their costs and pulls the productivity level requirement higher (compare line $\theta = 0.5, k = 2, \tau = 1.1$ to line $\theta = 0.9, k = 2, \tau = 1.1$, and line $\theta = 0.1, k = 10, \tau = 1.1$ to line $\theta = 0.5, k = 10, \tau = 1.1$ in figure 2). The level of transport costs has also a considerable effect on the share of exporting firms in the EK model (compare the solid black and grey lines) similar to other theoretical and empirical studies' findings. In general, as mark-ups approach one, the share of exporting firms approaches zero.

Second, with regards to the unemployment rates, we can compare the effect of competition increase on the absolute unemployment rate versus the effects of trade opening and trade costs decrease. Figure 3, part (a) shows first the effect of the movement from

Figure 2: Share of exporters, %, with different parameter values



autarky to open trade versus the effect from the competition increase on the rate of unemployment. In addition, part (b) of the same figure provides a view on the level of unemployment when trade costs decrease vs. when competition increases with different parameter values.

The movement from autarky to open trade is not a percentage change in any variable. Therefore its comparison to the effect of competition increase depends on the change in ρ_T , but also on the magnitude of the various parameters. Part (a) of figure 3 shows that e.g. in case A, see orange lines, the unemployment rate increases slightly more when the economy goes from autarky to open trade than when competition increases (when ρ_T increases from 0.3 to 0.35). With other parameter values in case B, the effect of competition increase on the unemployment rate seems to be bigger than the effect of trade opening. See the blue lines. Overall, it cannot be concluded whether the move from autarky to trade or the change in mark-ups has a larger impact on the level of the unemployment rate.

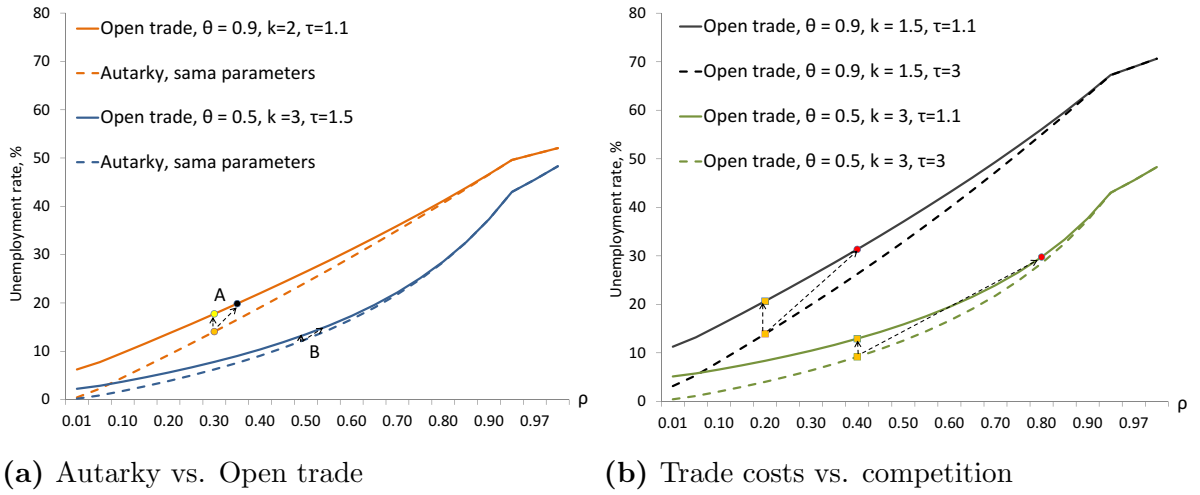
Part (b) of figure 3 demonstrates the increase in the unemployment rate in two cases when trade costs decrease (as concluded by Egger and Kreickemeier, 2012) versus when competition increases. Compare the solid lines with lower transport costs to the dotted lines with higher transport costs. The other parameters are kept constant in the lines with the same colour. If ρ_T increases as much as trade costs decrease (95 percent²⁹) and ρ_T goes e.g. from 0.2/0.4 to around 0.4/0.8, the unemployment rate increases more than it does from the change in trade costs. See the difference between the two orange dots

²⁹ Trade costs are 200 percent when $\tau = 3$ and 10 percent when $\tau = 1.1$, which equals a 95 percent drop in the rate of trade costs.

versus the difference between the orange and the red dot in the solid lines.

The flour industry example can help to understand the unemployment rates with the highest values for ρ_T , which seem somewhat extreme. At the moment that the elasticity of demand is very high, only few mills would be able to survive in the competition and they will produce all the flour demanded. These few firms will have a high productivity level and need only few employees (in addition to the machines). Most of the people that used to work in the sector will be unemployed or employed in another sector. The sectoral unemployment rate could reach even 70 percent if these dynamics would happen in a short time. So high values for ρ_T are not likely to occur on average at the level of the whole economy, but for some specific industries they can hold.

Figure 3: Unemployment rate, %, with different parameter values



(a) Autarky vs. Open trade

(b) Trade costs vs. competition

Third, figures 4 and 5 provide examples on the levels of the Gini indexes in a similar way. The difference between the autarky vs. open trade values depend again of the various parameters. In general, the profit income Gini increases slightly more from any change in competition than from the movement from autarky to open trade with the shown parameter values (see cases A and B in part (a) of figure 4). Labour income Gini, presented in figure 5, increases in both presented cases somewhat more from the movement from autarky to open trade than the profit income Gini. The effect of competition increase depends on the absolute change in ρ_T and on the value of the other parameters.

With regards to the comparison of the competition increase to the decrease of trade costs in parts (b), a change in τ affects especially the profit income Gini bit less than an equal percentage change in the value of ρ_T . On the other hand, a change in ρ_T from 0.1 to nearly 0.2 (or 0.2 to 0.4) has a smaller or equal effect on the labour income Gini than a 95 percent decrease in trade costs (see the cases presented in part (b) of figure 5).

To conclude, these comparisons' results depend on the level of the other parameters (k and θ) and on the initial level of ρ_T . Overall, the higher ρ_T is, the higher the inequality within managers and workers since a smaller share of the firms export.

Figure 4: Profit income Gini with different parameters

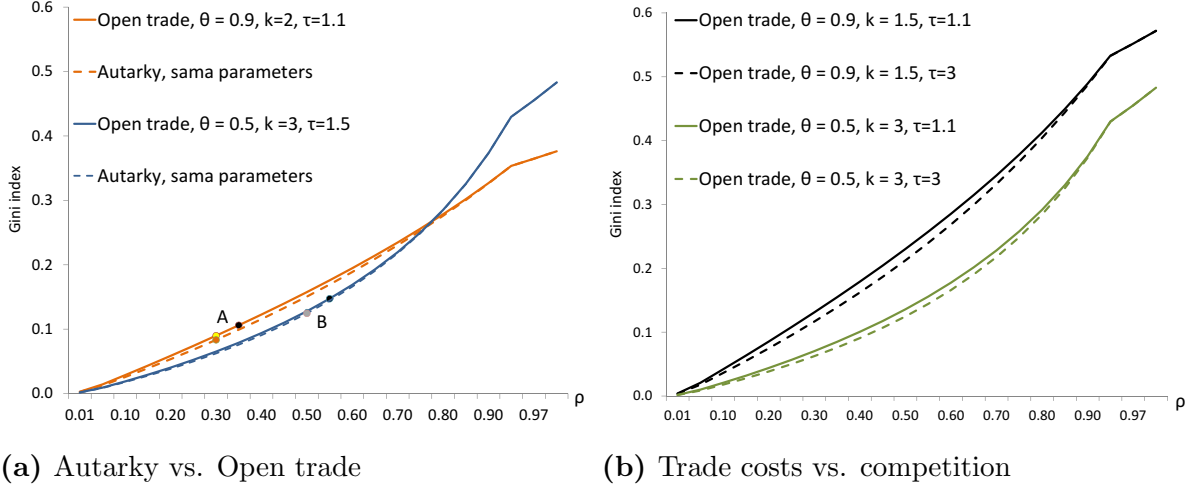
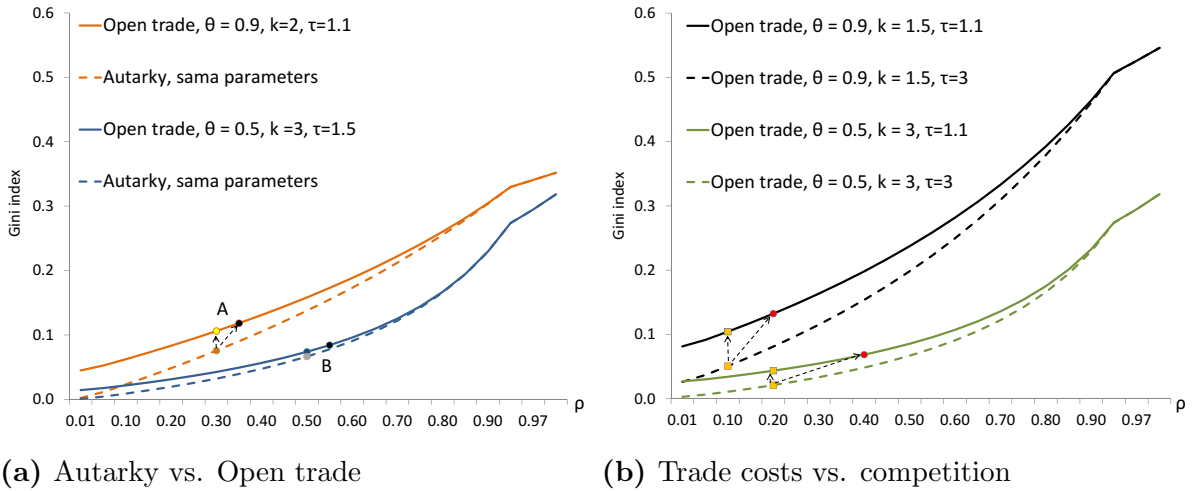


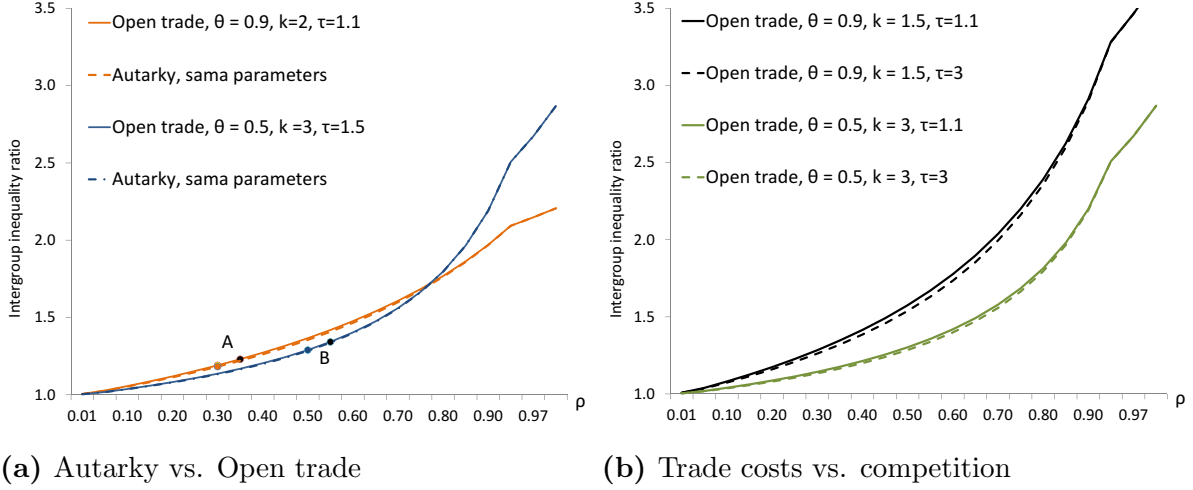
Figure 5: Labour income Gini with different parameters



Last, figure 6 presents comparisons for the ratio of average profits to average expected wages. The effect of competition increase on the ratio seems to be again slightly bigger in both presented cases (A and B, part (a) of the figure) than the effect from the movement from autarky to open trade. In effect, the differences in the intergroup inequality ratios between autarky and open trade are so small that they are hardly visible in the figure. Part (b) of the figure presents that an equal change in ρ_T , as compared to τ , results in a bigger change in the ratio in both presented cases. This is detectable by comparing the

solid and the dotted lines with same value of ρ_T and e.g. a change in ρ_T from 0.4 to 0.8 in the solid lines. In general, the level of the other parameters affects the absolute changes and levels significantly and no definite conclusions can be drawn from the comparisons.

Figure 6: Intergroup inequality ratio with different parameters



5 Conclusions

During the past decades within-country income inequality has increased in most countries, while the trends in between-country and global income inequality have been more diverse. Widening income distributions and the concentration of money and power have been claimed to increase instability in societies. This has resulted in a wide interest especially on the link between trade and the increasing wage income inequality within-countries. In addition, the role of top incomes and capital income as contributors to the rising total income inequality have gained attention recently. Likewise, they have been found to increase total income inequality significantly.

Theoretical analyses have focused mostly on the effects of trade on wage inequality (e.g. [Egger and Kreickemeier, 2009](#), [Helpman, Itskhoki, and Redding, 2010](#) and [Basco and Mestieri, 2013](#)) or only on capital income ([Foellmi and Oechslin, 2010](#)). A more general framework for the assessment of the effects of trade on both income inequality types at the same time was developed by [Egger and Kreickemeier \(2012\)](#). In addition to accounting for the heterogeneity in firms, it is assumed in their framework that workers obtain a 'fair wage' that depends both on the external conditions in the labour markets and on the profits of the firm. While most of the assumptions in the [Egger and Kreickemeier \(2012\)](#) framework match empirical findings, they have assumed that mark-ups remain unchanged

after a country opens for trade after autarky. This assumption is in contrast with recent empirical findings on the effect of trade on mark-ups (e.g. [Epifani and Gancia, 2011](#), [Chen, Imbs, and Scott, 2009](#)) and on the endogenous mark-up assumption in various other models.

In this article we contribute to the analyses on the mechanisms behind the increase of within-country income inequality. We provide a small expansion on the theoretical analysis of [Egger and Kreickemeier \(2012\)](#) by allowing mark-ups to change after a country leaves autarky in their framework. In the analysis we assume that both domestic firms and exporters face the same elasticity of demand (and have same level of mark-ups), but that this elasticity is higher in an open economy than it was in autarky due to increased competition in the market.

Our results strengthen the original findings of [Egger and Kreickemeier \(2012\)](#). We find that increased competition in an open economy will increase unemployment rate, the Gini index of profit income, the Gini index of wage income and the inequality between profit and wage income. Especially the result on unemployment rate is in contrast to an earlier study on the effect of competition increase and deregulation of markets, based on a model that does not take into account firm heterogeneity or have 'fair wage' setting. However, the reasons for these results in the EK model are straightforward. With smaller mark-ups (higher ρ_T), the productivity of the marginal firm needs to be higher than without a change in ρ_T . Only a smaller number of managers/firms reach the required level of productivity to produce profitably. Also, a smaller share of firms reach the higher marginal productivity level required to export. Out of the population, fewer people can be managers and fewer can work as local experts for foreign firms. Therefore, labour supply increases, but the operating firms with higher (labour) productivity need less employees than the firms with a lower productivity levels that drop out of the competition. Unemployment level increases.

With the lower mark-ups and a larger fraction of firm's revenue being allocated to employees, the average wage in the economy increases further than with the mere opening of trade. In addition, as the fewer firms that still operate in the markets are more productive, even the managers' lower share of revenue results in a higher average income for them. The average profits of the operating firms increase more than the average expected labour wage due to the fair wage setting that emphasises the external conditions in addition to the firm's profits. Subsequently, intergroup inequality increases. The share of high profits earning managers decreases since there are fewer firms that can export and materialize higher total profits. The distribution of profit income widens. The Gini index of wage income increases as well, as a higher share of employees work for non-exporting firms, which pay lower salaries than exporting firms.

As an example on these types of dynamics from increased competition in reality, one can think of the trends in the milling industry. While the demand for flour per person has most likely not changed much, the number of mills and employees in the industry has decreased significantly over the years since trade and transport costs have decreased and competition increased. Similarly, if mark-ups decrease on average in an open economy, these dynamics can be present in a smaller scale.

The absolute changes in the different indicators depend on the values of the various parameters in the model/economy. In some cases trade opening after autarky seems to have a bigger effect on the indicators' level than the competition increase, while in other cases the opposite holds. In most studied cases a decrease in mark-ups, which is as big as the decrease in trade costs in percentage terms, leads to a larger change in the indicator than the decrease in trade costs. However, this effect depends on the initial level of ρ_T and on the values of other parameters. Opposite results are found as well.

To conclude, our results provide additional insights on the possible reasons for the found increases in total income inequality within-countries and in the rise of top incomes and capital income inequality. In future, an actual endogenous mark-up function could be introduced in the model e.g. in [Melitz and Ottaviano \(2008\)](#) style. In addition, since consumers typically care about the quality of products besides the price, quality-adjusted demand models could be considered. In recent literature mark-ups have been found to differ also between exporting firms and domestic firms in the same sector (e.g. [Tamminen and Chang, 2013](#), and [De Loecker and Warzynski, 2012](#)), which could affect these dynamics as well.

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A Appendix: Comparative statics

A.1 Derivatives

The way how the change in mark-ups affects the main solutions do not depend only on the way ρ_T affects the solutions, but also on how it affects the various parameters that depend on ρ_T . Therefore, we start the comparative statics analyses from the effect of ρ_T on the other parameters used in the solutions.

First of all, in case ρ_T increases, the price demand elasticity (and substitution) increases based on:

$$\frac{\partial \sigma_T}{\partial \rho_T} = \frac{1}{(1 - \rho_T)^2} > 1 \quad (22)$$

Mark-ups μ_T , on the other hand, decrease in case ρ_T increases:

$$\frac{\partial \mu_T}{\partial \rho_T} = -\frac{1}{(\rho_T)^2} < 0 \quad (23)$$

In the following, the other functions derivatives are done with respect to σ_T . As presented earlier, if ρ_T increases, σ_T increases as well. The derivations of specifically the more complicated functions are clearer when done based on the changes of σ_T .

The exporter multiplier $\Omega_T \equiv 1 + \tau^{1-\sigma_T}$ is smaller in case ρ_T increases in comparison to the case where there is no change in ρ_T after open trade as presented by function 24.

$$\frac{\partial \Omega_T}{\partial \sigma_T} = -\tau^{1-\sigma_T} \ln(\tau) < 0 \quad (24)$$

Similarly, the difference between the total revenue of exporters, $\Omega_T r^e(\varphi) = \Omega_T^{1-\theta\eta_T} r^n(\varphi) = (1 + \tau^{1-\sigma_T})^{\frac{1}{1+\theta(\sigma_T-1)}} r^n(\varphi)$, and the total revenue of non-exporting firms, $r^n(\varphi)$, decreases if σ_T (and ρ_T) increases. This is demonstrated by the derivative of the exporters' revenue multiplier with respect to sigma in equation 25. The function is always negative, as both parts inside the brackets are negative and the multiplier is positive.

$$\begin{aligned} \frac{\partial \Omega_T}{\partial \sigma_T} = (1 + \tau^{1-\sigma_T})^{\frac{1}{1+\theta(\sigma_T-1)}} * \left[\frac{-\theta}{[1 + \theta(\sigma_T - 1)]^2} \ln(1 + \tau^{1-\sigma_T}) \right. \\ \left. - \frac{\tau^{1-\sigma_T} \ln(\tau)}{[1 + \theta(\sigma_T - 1)] * (1 + \tau^{1-\sigma_T})} \right] < 0 \quad (25) \end{aligned}$$

The derivative of export share with respect to σ_T is slightly more complicated as σ_T appears in the powers of the function in various parts. First, the function needs to be derived as a function of only σ_T , instead of a function of σ_T and η_T . Secondly, logarithm

of the function is taken in order to simplify the derivation. The full derivative is named $D\chi_T$, since it is needed later on in the derivations of other functions. The signs of the first few derivatives are clear from the functional forms. However, both of the first two terms inside the brackets of derivative 26 are negative (as $\ln(\Omega_T^{\frac{\eta_T}{\sigma_T-1}} - 1)$ is defined to be positive but less than 1). This way the first multiplication is positive, while the last terms are all negative. No analytical solution can be found on the comparison of the positive vs. negative parts values. In other words, no analytical solution can be obtained on the sign of the derivative with respect to sigma. Therefore, we test the sign of the derivative numerically with in total nearly 98 million combinations of different values for ρ_T , θ , k and τ . See section A.2 for the results. So, mostly, when mark-ups decrease, the share of firms that can export decreases.

$$\chi_T = \left(\Omega_T^{\frac{\eta_T}{\sigma_T-1}} - 1 \right)^{\frac{k}{\eta_T}} = \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma_T-1)}} - 1 \right)^{\frac{k(1+\theta(\sigma_T-1))}{\sigma_T-1}}$$

$$\ln \chi_T = \frac{k(1 + \theta(\sigma - 1))}{\sigma - 1} * \ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right)$$

$$\begin{aligned} \frac{\partial \chi}{\partial \sigma} * \frac{1}{\chi} &= \frac{(\sigma - 1)k\theta - k\theta\sigma + k\theta - k}{(\sigma - 1)^2} * \ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right) \\ &\quad + \frac{k(1 + \theta(\sigma - 1))}{\sigma - 1} * \frac{d}{d\sigma} \left(\ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right) \right) \end{aligned}$$

$$\begin{aligned} \frac{\partial \chi}{\partial \sigma} * \frac{1}{\chi_T} &= \frac{(\sigma - 1)k\theta - k\theta\sigma + k\theta - k}{(\sigma - 1)^2} * \ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right) \\ &\quad + \frac{k(1 + \theta(\sigma - 1))}{\sigma - 1} * \frac{(1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}}}{(1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1} * \frac{d}{d\sigma} \left(\frac{1}{1 + \theta(\sigma - 1)} \ln (1 + \tau^{1-\sigma}) \right) \end{aligned}$$

$$\begin{aligned} \frac{\partial \chi}{\partial \sigma} &= \chi_T * \left[\left(\frac{k\theta}{(\sigma - 1)} - \frac{k\theta\sigma - k\theta + k}{(\sigma - 1)^2} \right) * \ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right) \right. \\ &\quad \left. + \frac{k(1 + \theta(\sigma - 1)) * (1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} * \left(\frac{-\tau^{1-\sigma} * \ln(\tau)}{(1 + \tau^{1-\sigma})(1 + \theta(\sigma - 1))} - \frac{\ln(1 + \tau^{1-\sigma})\theta}{(1 + \theta(\sigma - 1))^2} \right)}{(\sigma - 1) * \left[(1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right]} \right] \end{aligned}$$

$$D\chi_T \equiv \frac{\partial \chi}{\partial \sigma} = \chi_T * \left[\left(\frac{-k}{(\sigma-1)^2} \right) * \ln \left((1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right) \right. \\ \left. + \frac{\left(-\tau^{1-\sigma} * \ln(\tau) * k * (1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 - \frac{\ln(1+\tau^{1-\sigma})k\theta(1+\tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}}}{(1+\theta(\sigma-1))} \right)}{(\sigma-1) * \left[(1 + \tau^{1-\sigma})^{\frac{1}{1+\theta(\sigma-1)}} - 1 \right]} \right] \quad (26)$$

Similarly, the sign of the derivative of labour supply with respect to sigma in equation 27 is not very clear cut and no analytical solution is found for the sign of the derivative again. It depends especially on the magnitudes of the parameters k and θ . For the derivation, the labour supply equation is first transferred again as a function of σ_T . However, based on the numerical analyses in subsection A.2 the derivative is always positive. In other words, labour supply increases in case there is an increase in ρ_T and consequently in σ_T after open trade.

$$L_T = \frac{k(\sigma_T - 1)}{k\sigma_T - \eta_T} N = \frac{k(\sigma_T - 1)}{k\sigma_T - \frac{\sigma_T - 1}{1+\theta(\sigma_T - 1)}} N = \frac{k(\sigma_T - 1)(1 + \theta(\sigma_T - 1))}{k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1} N \\ \frac{\partial L_T}{\partial \sigma} = N * \left[\frac{k + 2k\theta(\sigma - 1)}{k\sigma_T + (k\sigma_T\theta - 1)(\sigma_T - 1)} - \frac{k(\sigma_T - 1) [1 + \theta(\sigma_T - 1)] * (k + k\theta(2\sigma - 1) - 1)}{[k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1]^2} \right] \quad (27)$$

The derivative of the number of firms/managers with regards to σ_T is presented in equation 28. Within the derivation the earlier result on $\frac{\partial \chi_T}{\partial \sigma} \equiv D\chi_T$ has been used as defined in equation 26. For the ease of calculations, the function of M_T is also first transferred to logarithmic terms and as a function of σ_T . Due to the fact that $D\chi_T$ can take both positive and negative values, the last part of the equation can be either positive or negative (though in most cases positive, see subsection A.2). No easy comparison can be made on the magnitude of the different parts' values in the derivative. Therefore, we use numerical analysis to value the sign of the derivative with different parameter values. Based on the results, presented in subsection A.2, the sign of the derivative is always negative.

$$\ln M_T = \ln \left(\frac{k - \eta_T}{k\sigma_T - \eta_T} \right) - \ln(1 + \chi_T) + \ln N \\ \ln M_T = \ln \left(\frac{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} \right) - \ln((1 + \chi_T)) + \ln N$$

$$\begin{aligned} \frac{\partial M}{\partial \sigma} * \frac{1}{M} &= \frac{1}{\frac{k-\eta_T}{k\sigma_T-\eta_T}} * \left[\frac{\theta - \frac{1}{k}}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} \right. \\ &\quad \left. - \frac{[1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)] * (1 + 2\theta\sigma - \theta - \frac{1}{k})}{[\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)]^2} \right] - \frac{1}{1 + \chi_T} * \frac{\partial \chi_T}{\partial \sigma} \end{aligned}$$

$$\begin{aligned} \frac{\partial M}{\partial \sigma} &= \frac{N}{1 + \chi_T} * \\ &\quad \left[\frac{\theta - \frac{1}{k}}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} - \frac{(1 + 2\theta\sigma - \theta - \frac{1}{k}) [1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)]}{[\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)]^2} \right] \\ &\quad - \left[\frac{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} \right] \frac{N * D\chi_T}{[1 + \chi_T]^2} \quad (28) \end{aligned}$$

The definition of the marginal productivity required to run a firm is very close to the definition of M . Therefore, its derivative is also a very close one. Similarly to the previous derivative, the sign is analysed with numerical methods. Based on them, the derivative is always positive. So, if mark-ups decrease in open trade and therefore σ_T increases, the marginal productivity required to run a firm increases. Again, the use of the above defined $D\chi_T$ shortens the derivation significantly.

$$\varphi^* = \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right)^{\frac{1}{k}}$$

$$\frac{\partial \varphi^*}{\partial \sigma} = \frac{1}{k} \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right)^{\frac{1}{k}-1} * \frac{d}{d\sigma} \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right)$$

$$\begin{aligned} \frac{\partial \varphi^*}{\partial \sigma} &= \frac{1}{k} \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right)^{\frac{1}{k}-1} * \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right) * \\ &\quad \left\{ \frac{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} * \right. \\ &\quad \left[\frac{(1 + 2\theta\sigma - \theta - \frac{1}{k})}{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)} - \frac{[\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)] * (\theta - \frac{1}{k})}{[1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)]^2} \right] \\ &\quad \left. + \frac{D\chi_T}{1 + \chi_T} \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial \varphi^*}{\partial \sigma} = & \frac{1}{k} \left(\frac{(k\sigma_T - \eta_T)(1 + \chi_T)}{k - \eta_T} \right)^{\frac{1}{k}} * \\ & \left\{ \left[\frac{(1 + 2\theta\sigma - \theta - \frac{1}{k})}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} - \frac{(\theta - \frac{1}{k})}{[1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)]} \right] \right. \\ & \left. + \frac{D\chi_T}{1 + \chi_T} \right\} \quad (29) \end{aligned}$$

Equation 30 shows the derivative of employment share (out of total labour supply) with regards to sigma. The derivative is split into two main parts and the earlier result on $D\chi_T$ is used again to shorten it. Analytical solution on the sign of the derivative is not again found. Based on numerical analyses, the derivative $D(1 - U_T)$ is negative with the given restrictions on the parameters. In other words, employment decreases and unemployment increases if sigma increases. As labour supply increases but the number of firms decreases, unemployment increases.

$$\begin{aligned} 1 - U_T = & \frac{\Gamma}{(1 + \chi_T)} * \left[\frac{k - \eta_T}{k - (1 - \theta)\eta_T} \right] \\ = & \frac{1 + \chi_T^{\frac{k - (1 - \theta)\eta_T}{k}} (\Omega_T^{\frac{(1 - \theta)\eta_T}{\sigma_T - 1}} - 1)}{(1 + \chi_T)} * \left[\frac{k - \eta_T}{k - (1 - \theta)\eta_T} \right] \end{aligned}$$

$$\ln(1 - U_T) = \ln \left[1 + \chi_T^{\frac{k - (1 - \theta)\eta_T}{k}} (\Omega_T^{\frac{(1 - \theta)\eta_T}{\sigma_T - 1}} - 1) \right] - \ln(1 + \chi_T) + \ln \left[\frac{k - \eta_T}{k - (1 - \theta)\eta_T} \right]$$

$$\begin{aligned} \ln(1 - U_T) = & \ln \left[1 + \chi_T^{1 - \frac{(1 - \theta)(\sigma_T - 1)}{k(1 + \theta(\sigma_T - 1))}} ((1 + \tau^{1 - \sigma_T})^{\frac{(1 - \theta)}{1 + \theta(\sigma_T - 1)}} - 1) \right] \\ & - \ln(1 + \chi_T) + \ln \left[\frac{k(1 + \theta(\sigma_T - 1)) - \sigma_T + 1}{k(1 + \theta(\sigma_T - 1)) - (1 - \theta)(\sigma_T - 1)} \right] \end{aligned}$$

$$\begin{aligned} D(1 - U_T) \equiv \frac{\partial(1 - U_T)}{\partial \sigma} = & (1 - U_T) \left\{ \frac{1}{\Gamma} * \frac{d}{d\sigma} \Gamma - \frac{D\chi_T}{(1 + \chi_T)} + \right. \\ & \left. \left(\frac{k\theta - 1}{k(1 + \theta(\sigma_T - 1)) - \sigma_T + 1} - \frac{k\theta + \theta - 1}{k(1 + \theta(\sigma_T - 1)) - (1 - \theta)(\sigma_T - 1)} \right) \right\} \quad (30) \end{aligned}$$

, where $\Gamma = 1 + \chi_T^{1 - \frac{(1 - \theta)(\sigma_T - 1)}{k(1 + \theta(\sigma_T - 1))}} ((1 + \tau^{1 - \sigma_T})^{\frac{(1 - \theta)}{1 + \theta(\sigma_T - 1)}} - 1)$ and:

$$\begin{aligned}
\frac{d}{d\sigma}\Gamma &= (\Gamma - 1) * \left[\left(\frac{-(1-\theta)}{k[1+\theta(\sigma_T-1)]^2} \right) \ln(\chi_T) \right. \\
&\quad \left. + \left(1 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))} \right) \frac{D\chi_T}{\chi_T} \right] + \chi_T^{1-\frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}} * (1 + \tau^{1-\sigma_T})^{\frac{(1-\theta)}{1+\theta(\sigma_T-1)}} * \\
&\quad \left[\frac{-\tau^{1-\sigma_T} \ln(\tau)(1-\theta)}{[1+\theta(\sigma_T-1)](1+\tau^{1-\sigma_T})} - \frac{\theta(1-\theta) \ln(1+\tau^{1-\sigma_T})}{[1+\theta(\sigma_T-1)]^2} \right] = D\Gamma \quad (31)
\end{aligned}$$

Before continuing with the derivatives of the different income inequality measures, we solve what happens to average wage and average managerial income when mark-ups change, i.e. when ρ_T and σ_T change in comparison to autarky. The earlier solutions $D\chi_T$ and $D(1 - U_T)$ are used to shorten the derivative and the original function presented in table 1 are first transferred to logarithmic forms. The derivative of average wage with respect to sigma in equation 32 is always positive based on numerical analyses. In other words, average wage increases if mark-ups decrease (i.e. sigma increases).

$$\begin{aligned}
\ln \bar{w}_T &= \frac{1}{k} \ln(1 + \chi_T) + \ln \left(\frac{(\sigma_T - 1)^2}{\sigma_T} \right) \\
&\quad + \frac{\sigma_T}{\sigma_T - 1} \ln \left(\frac{k[1 + \theta(\sigma_T - 1)]}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right) \\
&\quad + \frac{k-1}{k} \ln \left(\frac{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} \right) + \\
&\quad \ln \left(\frac{k(\sigma_T - 1)(1 + \theta(\sigma_T - 1))}{k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1} \right) - \ln(1 - U_T)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{w}_T}{\partial \sigma} = & \bar{w}_T \left[\frac{D\chi_T}{k(1+\chi_T)} + \left(\frac{\sigma_T^2 - 1}{\sigma_T(\sigma_T - 1)^2} \right) \right. \\
& + \frac{\sigma_T}{\sigma_T - 1} * \left[\frac{\theta}{(1 + \theta(\sigma_T - 1))} - \frac{k\theta - 1}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right] \\
& + \left(\frac{-1}{(\sigma_T - 1)^2} \right) \ln \left(\frac{k}{k - \eta_T} \right) \\
& + \frac{k - 1}{k} * \left[\frac{\theta - \frac{1}{k}}{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)} - \frac{(1 + 2\theta\sigma - \theta - \frac{1}{k})}{[\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)]} \right] \\
& + \left[\frac{k + 2k\theta\sigma - k\theta - 1}{[k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1]} - \frac{1 + 2\theta(\sigma - 1)}{(\sigma_T - 1)[1 + \theta(\sigma_T - 1)]} \right] \\
& \left. - \frac{D(1 - U_T)}{1 - U_T} \right] \quad (32)
\end{aligned}$$

Similar to the average wage income, the average managerial income increases if mark-ups decrease. This is due to the fact that the derivative of average profits with regards to sigma in equation 33 is also always positive based on numerical analyses.

$$\begin{aligned}
\ln \bar{\pi}_t = & \ln \left[(1 + \chi_T) \left(\frac{k}{k - \eta_T} \right) - \chi_T \right] + \frac{1}{k} \ln(1 + \chi_T) + \ln \left(\frac{(\sigma_T - 1)^2}{\sigma_T} \right) \\
& + \frac{\sigma_T}{\sigma_T - 1} \ln \left(\frac{k[1 + \theta(\sigma_T - 1)]}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right) \\
& + \frac{k - 1}{k} \ln \left(\frac{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)}{\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)} \right) \\
& + \ln \left(\frac{k(\sigma_T - 1)(1 + \theta(\sigma_T - 1))}{k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1} \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{\pi}_T}{\partial \sigma} = & \bar{\pi}_T \left[\frac{1}{(1 + \chi_T) \left(\frac{k}{k - \eta_T} \right) - \chi_T} * \right. \\
& ((1 + \chi_T) \left[\frac{k\theta}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} - \frac{(k\theta - 1)k[1 + \theta(\sigma_T - 1)]}{[k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)]^2} \right] \\
& + \left[\frac{k}{k - \eta_T} - 1 \right] D\chi_T) \\
& + \frac{D\chi_T}{k(1 + \chi_T)} + \left(\frac{\sigma_T^2 - 1}{\sigma_T(\sigma_T - 1)^2} \right) + \\
& \frac{\sigma_T}{\sigma_T - 1} * \left[\frac{\theta}{(1 + \theta(\sigma_T - 1))} - \frac{k\theta - 1}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right] \\
& + \left(\frac{-1}{(\sigma_T - 1)^2} \right) \ln \left(\frac{k}{k - \eta_T} \right) \\
& + \frac{k - 1}{k} * \left[\frac{\theta - \frac{1}{k}}{1 + \theta(\sigma_T - 1) - \frac{1}{k}(\sigma_T - 1)} - \frac{(1 + 2\theta\sigma - \theta - \frac{1}{k})}{[\sigma_T[1 + \theta(\sigma_T - 1)] - \frac{1}{k}(\sigma_T - 1)]} \right] \\
& \left. + \left[\frac{k + 2k\theta\sigma - k\theta - 1}{[k\sigma_T(1 + \theta(\sigma_T - 1)) - \sigma_T + 1]} - \frac{1 + 2\theta\sigma - 2\theta}{(\sigma_T - 1)[1 + \theta(\sigma_T - 1)]} \right] \right] \quad (33)
\end{aligned}$$

The intergroup inequality I_G is measured by the ratio of average managerial income to average expected labour income. In order to solve what happens if sigma changes, the function is transformed again to a logarithmic form and as a function of sigma. This tells us also separately what happens to the autarky level inequality ($k/(k - \eta_T)$) and to the 'open economy multiplier' of the inequality ratio. The first part of the derivative 34 within the main brackets is positive according to numerical analysis. This means that already in autarky intergroup inequality would increase if mark-ups decrease, since less firms manage to operate in the market. The average profits, i.e. the average managerial income, is higher due to the higher productivity of the operating firms even though the share of the revenue that goes to the managers is lower. Therefore, also the ratio of average profit to average expected wage income is higher. In addition, the second part of the derivative is positive. So, in total the derivative is positive according to the numerical tests. Therefore, intergroup inequality increases both in autarky and in open economy when mark-ups decrease.

$$\frac{\bar{\pi}}{(1 - U)\bar{w}} = \frac{k}{k - \eta_T} * \left(1 + \frac{\eta_T \chi_T}{k} \right) \equiv I_G$$

$$\begin{aligned}
\ln I_G &= \ln\left(\frac{k}{k - \eta_T}\right) + \ln\left(1 + \frac{\eta_T \chi_T}{k}\right) \\
&= \ln\left(\frac{k[1 + \theta(\sigma_T - 1)]}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)}\right) + \ln\left(1 + \frac{\sigma_T - 1}{k(1 + \theta(\sigma_T - 1))} * \chi_T\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial I_G}{\partial \sigma} * \frac{1}{I_G} &= \left[\frac{\theta}{(1 + \theta(\sigma_T - 1))} - \frac{k\theta - 1}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right] \\
&\quad + \frac{k(1 + \theta(\sigma_T - 1))}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)\chi_T} * \frac{\sigma_T - 1}{k(1 + \theta(\sigma_T - 1))} \\
&\quad * \chi_T * \left[\left(\frac{1}{\sigma_T - 1} - \frac{\theta}{(1 + \theta(\sigma_T - 1))} \right) + \frac{D\chi_T}{\chi_T} \right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial I_G}{\partial \sigma} &= I_G \left(\left[\frac{\theta}{(1 + \theta(\sigma_T - 1))} - \frac{k\theta - 1}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right] \right. \\
&\quad \left. + \frac{\sigma_T - 1}{k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)\chi_T} * \chi_T * \left[\left(\frac{1}{\sigma_T - 1} - \frac{\theta}{(1 + \theta(\sigma_T - 1))} \right) + \frac{D\chi_T}{\chi_T} \right] \right) \quad (34)
\end{aligned}$$

The income inequality between managers is defined in table 1, row 10, part T. In order to find the derivative of it with regards to sigma, the function is derived again as a function of only sigma and in logarithmic terms.

$$\begin{aligned}
\ln A_{M,T} &= \ln\left[\frac{\eta_T}{2k - \eta_T}\right] + \ln\left[1 + \frac{\chi_T(2 - \chi_T)(k - \eta_T)}{k + \eta_T \chi_T}\right] \\
&= \ln\left(\frac{\sigma_T - 1}{2k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)}\right) \\
&\quad + \ln\left[1 + \frac{\chi_T(2 - \chi_T) * [k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)]}{k(1 + \theta(\sigma_T - 1)) + (\sigma_T - 1)\chi_T}\right]
\end{aligned}$$

Function 35 provides the final form of the derivative. The sign of the derivative is always positive based on numerical analyses and shows that the income inequality Gini

of managerial income increases if sigma increases.

$$\begin{aligned}
\frac{\partial A_{M,T}}{\partial \sigma} = & A_{M,T} \left[\left(\frac{1}{\sigma_T - 1} - \frac{2k\theta - 1}{2k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)} \right) \right. \\
& + \frac{\chi_T(2 - \chi_T) * [k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)]}{(2\chi_T - \chi_T^2 + 1)k[1 + \theta(\sigma_T - 1)] - [\chi_T(1 - \chi_T)(\sigma_T - 1)]} * \\
& \left. \left[\frac{(2 - 2\chi_T) * D\chi_T}{\chi_T(2 - \chi_T)} \right. \right. \\
& \left. \left. + \left(\frac{k\theta - 1}{[k(1 + \theta(\sigma_T - 1)) - (\sigma_T - 1)]} - \frac{k\theta + \chi_T + (\sigma_T - 1)D\chi_T}{[k(1 + \theta(\sigma_T - 1)) + (\sigma_T - 1)\chi_T]} \right) \right] \right] \quad (35)
\end{aligned}$$

Similar to the previous parts, the Gini of labour income is first transferred as a function of sigma. By reshuffling the terms in the original form of the function, we find a new way to define the function. Function 36 is used as the basis for the derivative with respect to sigma.

$$\begin{aligned}
A_{L,T} = & \frac{\theta\eta_T}{2[k - (1 - \theta)\eta_T] - \theta\eta_T} \\
& * \left[1 + \frac{2(k - \eta_T)\chi_T(1 - \chi_T^{1-(1-\theta)\eta_T/k}) - 2[k - (1 - \theta)\eta_T](\Gamma_T - 1)(1 - \chi_T^{1-\eta_T/k})}{(1 + \chi_T)\theta\eta_T\Gamma_T} \right]
\end{aligned}$$

$$\begin{aligned}
A_{L,T} = & \frac{\theta\eta_T}{2[k - (1 - \theta)\eta_T] - \theta\eta_T} * \\
& \left[1 + \left(\frac{2(k - \eta_T)}{\theta\eta_T} \right) \left(\frac{\chi_T(1 - \chi_T^{1-(1-\theta)\eta_T/k})}{(1 + \chi_T)\Gamma_T} \right) \right. \\
& \left. - \left(\frac{2[k - (1 - \theta)\eta_T]}{\theta\eta_T} \right) \left(\frac{\Gamma_T - 1}{\Gamma_T} \right) \left(\frac{1 - \chi_T^{1-\eta_T/k}}{1 + \chi_T} \right) \right]
\end{aligned}$$

$$\begin{aligned}
A_{L,T} = & \frac{\theta}{\frac{2k}{(\sigma_T - 1)} + (2k + 1)\theta - 2} \\
& * \left[1 + \left(\frac{2k}{(\sigma_T - 1)\theta} + 2k - \frac{2}{\theta} \right) \left(\frac{\chi_T - \chi_T^{2 - \frac{(1-\theta)(\sigma_T - 1)}{k(1 + \theta(\sigma_T - 1))}}}{(1 + \chi_T)\Gamma_T} \right) \right. \\
& \left. - \left(\frac{2k}{(\sigma_T - 1)\theta} + 2k + 2 - \frac{2}{\theta} \right) \left(1 - \frac{1}{\Gamma_T} \right) \left(\frac{1 - \chi_T^{\frac{\sigma_T - 1}{k(1 + \theta(\sigma_T - 1))}}}{1 + \chi_T} \right) \right]
\end{aligned}$$

$$\ln A_{L,T} = \ln \left(\frac{\theta}{\frac{2k}{(\sigma_T-1)} + (2k+1)\theta - 2} \right) + \ln[1 + \Lambda] \quad (36)$$

, where

$$\begin{aligned} \Lambda \equiv & \left(\frac{2k}{(\sigma_T-1)\theta} + 2k - \frac{2}{\theta} \right) \left(\frac{\chi_T - \chi_T^{2 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}}}{(1 + \chi_T)\Gamma_T} \right) \\ & - \left(\frac{2k}{(\sigma_T-1)\theta} + 2k + 2 - \frac{2}{\theta} \right) \left(1 - \frac{1}{\Gamma_T} \right) \left(\frac{1 - \chi_T^{1 - \frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}}}{1 + \chi_T} \right) \end{aligned}$$

Γ has been defined earlier in sub-section 3.2 and its derivative $\frac{\partial \Gamma}{\partial \sigma} \equiv D\Gamma$ is defined in function 31. The derivative of labour income Gini with respect to sigma is derived in two parts as:

$$\frac{\partial A_{L,T}}{\partial \sigma_T} = A_{L,T} \left\{ \left(\frac{2k}{\left[\frac{2k}{(\sigma_T-1)} + (2k+1)\theta - 2 \right] (\sigma_T-1)^2} \right) + \frac{1}{[1 + \Lambda]} \frac{d}{d\sigma_T} \Lambda \right\} \quad (37)$$

,where

$$\begin{aligned}
\frac{d}{d\sigma_T}\Lambda = & \left(\frac{-2k}{(\sigma_T - 1)^2\theta} \right) \left(\frac{\chi_T - \chi_T^{2 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}}}{(1 + \chi_T)\Gamma_T} \right) + \left(\frac{2k}{(\sigma_T - 1)\theta} + 2k - \frac{2}{\theta} \right) * \\
& \left\{ \frac{D\chi_T - \chi_T^{2 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}} * \left\{ \left[\frac{(\theta-1)}{k[1+\theta(\sigma_T-1)]^2} \right] \ln\chi_T + \left(2 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))} \right) \frac{D\chi_T}{\chi_T} \right\}}{(1 + \chi_T)\Gamma_T} \right. \\
& \left. - \frac{\left(\chi_T - \chi_T^{2 - \frac{(1-\theta)(\sigma_T-1)}{k(1+\theta(\sigma_T-1))}} \right) (D\chi_T\Gamma_T + (1 + \chi_T)D\Gamma_T)}{[(1 + \chi_T)\Gamma_T]^2} \right\} \\
& - \left(\frac{-2k}{(\sigma_T - 1)^2\theta} \right) \left(1 - \frac{1}{\Gamma_T} \right) \left(\frac{1 - \chi_T^{\frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}}}{1 + \chi_T} \right) \\
& - \left(\frac{2k}{(\sigma_T - 1)\theta} + 2k + 2 - \frac{2}{\theta} \right) \left(\frac{D\Gamma_T}{\Gamma_T^2} \right) \left(\frac{1 - \chi_T^{\frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}}}{1 + \chi_T} \right) \\
& - \left(\frac{2k}{(\sigma_T - 1)\theta} + 2k + 2 - \frac{2}{\theta} \right) \left(1 - \frac{1}{\Gamma_T} \right) * \\
& \left\{ \frac{-\chi_T^{\frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}} \left[\left(\frac{-1}{k(1+\theta(\sigma_T-1))^2} \right) \ln\chi_T + \left(1 - \frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))} \right) \frac{D\chi_T}{\chi_T} \right]}{1 + \chi_T} \right. \\
& \left. - \frac{\left(1 - \chi_T^{\frac{\sigma_T-1}{k(1+\theta(\sigma_T-1))}} \right) D\chi_T}{[1 + \chi_T]^2} \right\}
\end{aligned}$$

The derivative 37 is always positive with the given parameter restrictions (see subsection A.2). It means that labour income Gini index increases if mark-ups decrease. This is due to the fact that a smaller share of workers are employed in exporting firms, which pay higher wages due to their higher productivity. This shows already from the fact that average wage of employed people has increased. So, the distribution of production worker salaries widens after mark-ups decrease.

A.2 Numerical analyses

Due to the fact that the signs of various derivatives are not clear from the function forms of the derivatives, numerical tests on the values are calculated with a total of nearly million different combinations of ρ_T , θ and k and over 98 million different combinations

of ρ_T , θ , k and τ . The nearly million combinations of only ρ_T , θ and k are used for the analysis of the derivatives which do not include τ . Otherwise, the full sample of 100 million combinations of different parameter values is used.

In order to analyse whether the positive parts are larger than the negative parts, we test the values of the different derivatives with all possible combinations of the parameter values of ρ_T , θ , k and τ . Both ρ_T and θ have clear restrictions on the values they can take. They both need to be strictly between 0 and 1. We divide the range 0.01-0.99 to 99 points with 0.01 between every step. This provides us already with a total of $99 \times 99 = 9801$ different parameter value combinations. The possible values of parameters k and τ are less clear. Both of the previously mentioned parameters have only a lower bound: $k > \eta$ and $\tau > 1$. In order to set some kind of upper limit for both of these parameters, we investigate the empirical estimates of these parameters from literature. Further, as mentioned earlier, it is defined in the model that $1 < \Omega_T \equiv 1 + \tau^{1-\sigma_T} \leq 2$. However, with some values of ρ_T and τ , $\Omega_T \equiv 1 + \tau^{1-\sigma_T}$ is so close to one that at the level of 52 decimals it is rounded to be exactly one. Based on the restriction, we need to rule out also all combinations of ρ_T , θ , k and τ that result in this numerical, forbidden case of Ω_T that is rounded to be one. In the following, these cases have been marked by noting that the derivative is missing.

Empirical estimates on the value of k are relatively small and close to each other. Most of the found estimates with advanced countries' data range from around 1 to around 2.³⁰ However, for China significantly higher estimates have been found with range from 0.8 to 24 and average at 7.9 (Hsieh and Ossa, 2011). In our model k needs to be larger than η_T even after there is a change in ρ_T . In other words, k has to equal at minimum η plus a tiny value. Already this restriction results in values for k that can be anywhere between 0.01 and nearly 50 depending on the values of ρ_T and θ . Especially the value of 50 for k appears high in comparison to the empirical findings. Therefore, we assume that k is mostly relatively close to η , but we test also for cases where k is even $\eta + 10$. In that case, the maximum value of k is nearly 60, which is 300 times larger than the average empirical estimates of k and still more than 2 times as large as the highest empirical estimate (from China). We include in total 100 different values for the difference of k and

³⁰ See e.g. Del Gatto, Mion, and Ottaviano (2006), Del Gatto, Ottaviano, and Pagnini (2008), Eaton, Kortum, and Kramarz (2011), Helpman, Melitz, and Yeaple (2004), Luttmer (2007) and Gabaix (2009). Out of mentioned literature Del Gatto, Mion, and Ottaviano (2006), Del Gatto, Ottaviano, and Pagnini (2008), Eaton, Kortum, and Kramarz (2011) estimate productivity distributions with European data, while Luttmer (2007) and Gabaix (2009) have studied firm size distributions that are assumed to be directly linked to the productivity of the firms. Helpman, Melitz, and Yeaple (2004) analysed sales distributions, which have a tail index interlinked to productivity distributions' tail index if both are distributed according to pareto distribution. In general, most studies conclude that pareto distribution is a good proxy for the distribution of firm productivity.

η in the estimations with most of the values between 0 and 4.³¹ With the 100 values for the difference of k and η , we have in total 980,000 different combinations of ρ_T , θ and k . The absolute values of k have a mean of 3.4, standard deviation of 2.6, minimum value of 0.02 and maximum value of 59.7.

The value of $\tau > 1$ is similarly not restricted from above. While the value of τ affects mostly only $1 < \Omega_T \equiv 1 + \tau^{1-\sigma_T} \leq 2$, there are some derivatives which include τ in itself. The iceberg transport costs, which τ measures, can vary from country to country depending on which trading partners are in question. Similarly, in addition to the actual transport costs, τ includes typically also costs from tariffs and non-tariff measures, NTMs, (Helpman, Melitz, and Yeaple 2004). Via new technologies and negotiations on the abolishing of tariffs and NTMs, these iceberg transport costs change over time and they have fallen significantly in most countries. However, in this type of theoretical consideration, we will consider various different possibilities for their level. Several attempts have been made to assess the level of trade costs in tariff equivalents with different methodologies and datasets. During history, tariff equivalents of up to 350 percent have been found (Jacks, Meissner, and Novy, 2011), but most of the found estimates on the average level of tariff equivalents in different countries at different times lie in the range of few percents up to 170 percent.³² In general, the literature has studied tariff equivalents, which means that in order to translate them to the τ in this model, 1 needs to be added to all values. Therefore, based on literature, we expect the iceberg transport costs to vary from 1.01 to around 10 in reality, but we will additionally test for the derivative's signs if τ is artificially high, at a maximum of 1000 (meaning a 90000 percent tariff equivalent). In total, we test for 100 different values of τ , with most of them in the range of 1.01 to 5³³ and a total of 98 million different combinations on the values of parameters ρ_T , θ , k and τ .

See table 2 for the results of the numerical tests on the values of the different derivatives. Based on the around 98 million tests (=100*100*99*99) with different parameter

³¹ The 100 values for the difference of k and η increase gradually in the distance between the value and most of the values are between 0.01 and 4 with relatively small steps in between every value. In addition, few larger values have been included to account for any abnormally large k 's. The included values are: 1) 10 values from 0.01 to 0.1 with steps of 0.01 in between every value, 2) 78 values from 0.15 to 4 with 0.05 between every value, and 3) 12 values from 4.5 to 10 with 0.5 between every value.

³² See e.g. Anderson and van Wincoop (2004), Novy (2013) and Jacks, Meissner, and Novy (2011).

³³ The 100 values for the value of τ increase gradually in the distance between the values and most of the values are between 0.01 and 5 with relatively small steps in between every value. In addition, few larger values have been included to account for any artificially high trade costs. The included values are: 1) 5 values from 1.01 to 1.05 with 0.1 between every value, 2) 59 values from 0.1 to 4 with 0.05 between every value, 3) 25 values from 4.1 to 6.5 with 0.1 between every value 4) 7 values from 7 to 10 with 0.5 between every value, and 5) values 25, 50, 100 and 1000.

value combinations, all other derivatives have always the same sign (the cases where $\chi_T \approx 0$ at 52 decimal level have been ruled out by the model restrictions) except for the derivative $D\chi_T$. The sign of derivative $D\chi_T$ is analysed further in table 3. All results presented in table 2 and their economic explanations are discussed already in the subsection A.1 along each derivative's functional form.

Table 2: Numerical test on the sign of derivatives with respect to sigma

Derivative of ..	No of combinations	Mean	Max	Min	Share of inadmissible results, %	Share of admissible results Negative, % Positive, %	
Share of exporters	98,010,000	-0.015	2.60E-16	-0.251	3.034	99.995	0.005*
Labour supply	980,100	0.542	2.03E-06	27.63	0.000	0.000	100.00
Number of firms	98,010,000	-0.468	-2.02E-06	-18.659	3.034	100.00	0.000
φ^*	98,010,000	1.E+13	2.5E+17	9.70E-07	3.034	0.000	100.00
Employment share	98,010,000	-0.351	-9.2E-06	-32.17	3.034	100.00	0.000
Average wage	98,010,000	7.E+42	1.3E+47	0.001	3.034	0.000	100.00
Average profit	98,010,000	1.E+43	2.8E+47	0.002	3.034	0.000	100.00
Between inequality	98,010,000	66	210764	0.00001	3.034	0.000	100.00
Inequality, manager	98,010,000	0.400	44.60	0.00001	3.034	0.000	100.00
Inequality, labour	98,010,000	0.37	42.84	0.00001	3.034	0.000	100.00

Notes:* Analyses on the cases where the derivative is positive are in another table.

Analyses on the cases where $D\chi_T$ is positive in table 3 show that the derivative is positive only in few exceptional cases. In fact, in all of the cases, the parameter value of ρ_T is the last value with which a derivative for χ_T exists as χ_T is already tiny. If ρ_T increases further (while the other parameter values keep constant), χ_T is rounded to zero numerically and we cannot calculate the value of $D\chi_T$ anymore. Table 3 shows these special cases and the number of parameter combinations that result in a positive value for $D\chi_T$. Therefore, despite these few parameter value combinations that provide a positive value for $D\chi_T$, it can be concluded that with most realistic values for the different parameters, the derivative of χ_T is negative. In other words, the share of exporters decreases if most of the tested cases.

Table 3: Cases where $D\chi_T$ is positive

No of cases	Value tau	Value rho	Value theta	$(k - \eta_T)$	Note:
200	2.1	0.98	[0.01, 0.02]	[0.01, 10]	If rho>0.98 & tau=2.1, $\chi_T \approx 0$
100	3	0.97	0.01	[0.01, 10]	If rho>0.97 & tau=3, $\chi_T \approx 0$
200	3.05	0.97	[0.01, 0.02]	[0.01, 10]	If rho>0.97 & tau=3.05, $\chi_T \approx 0$
300	3.1	0.97	[0.01, 0.03]	[0.01, 10]	If rho>0.97 & tau=3.1, $\chi_T \approx 0$
100	4.4	0.96	0.01	[0.01, 10]	If rho>0.96 & tau=4.4, $\chi_T \approx 0$
400	4.5	0.96	[0.01, 0.04]	[0.01, 10]	If rho>0.96 & tau=4.5, $\chi_T \approx 0$
400	4.6	0.96	[0.01, 0.04]	[0.01, 10]	If rho>0.96 & tau=4.6, $\chi_T \approx 0$
100	6.3	0.95	0.01	[0.01, 10]	If rho>0.95 & tau=6.3, $\chi_T \approx 0$
100	6.5	0.95	0.01	[0.01, 10]	If rho>0.95 & tau=6.5, $\chi_T \approx 0$
600	10	0.94	[0.01, 0.06]	[0.01, 10]	If rho>0.94 & tau=10, $\chi_T \approx 0$
1900	1000	0.84	[0.01, 0.19]	[0.01, 10]	If rho>0.84 & tau=1000, $\chi_T \approx 0$

Notes: The approximation $\chi_T \approx 0$ refers to the numerical value with 52 decimals.