

A generic approach to investment modelling in recursive dynamic CGE models

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Abstract

It is a common practice in recursively dynamic CGE models to maintain static expectations. Consequently, investors take current rates of return as expected future rates of return. The vexing problem with this approach is that no matter how we allocate investments across sectors and regions in the current period, it is not possible to bring the expected rates of return to equality once the equilibrium is displaced. To deal with this problem all recursively dynamic CGE models have resorted to some complex mechanisms to allocate investments across sectors and regions. By drawing on the inverse relationship between the future capital stock and its marginal productivity, this paper establishes an inverse relationship between the expected future rates of return and current investment levels and this approach has been applied to the static GTAP model (version 6.2). By doing so this paper provides an alternative to the GTAP-Dyn model.

Keywords: Recursive Dynamic CGE model, Static Expectation, Expected Rates of Return, Investment Allocation.

JEL Codes: C58, D68

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1 Introduction

A vexing issue in recursively dynamic CGE models is concerned with allocating investments across sectors and/or regions which is essentially a forward-looking problem. The difficulty arises because of the time needed to turn investment into productive capital stock is nonzero, meaning that the capital stock does not change instantaneously. The time lag necessitates that the investors make investment decisions based on future rates of return and the recursive nature of the model requires that the investors can only form expectations about the future rates of return; they cannot observe the actual solutions. A common practice is to maintain that the expectations are static and use the current rate of return as a proxy for the future rate of return. The current rate of return, however, is independent of current investments; the expected rate of return, in turn, becomes independent of the level of investment currently planned. Therefore, the expected sectoral rates of return do not necessarily decline with increasing investments. This feature has made recursively dynamic CGE models incapable of allocating investment by equalising expected sectoral rates of return.

For example, if the current rate of return in a particular sector goes up because of some exogenous change, then the system is pushed into disequilibrium. It is natural to expect a surge in investment in the sector which will reduce the expected sectoral rate of return to the level commensurate to the market equilibrium. It is, however, not possible to reduce the rate of return in the sector just by increasing investment in that sector because the current capital stock and the current rates of return are independent of current investment levels. Consequently, investment allocation based on equalisation of the (risk-neutralised) rate of return may not yield any solution.

Hence, equalisation of the rate of return has not been the preferred condition for the equilibrium allocation of investment in recursively dynamic CGE models. To guarantee a solution these models have employed a set of additional (be it ad hoc, pragmatic or a reduced form) behavioural specifications in place of the risk-neutralised rate of return equalisation rule. However, this approach remains open to the question that whether the investors in these models can still be considered rational – return maximisers. Despite the fact that the recursively dynamic models have been used in policy analysis and policy development processes for some

¹ The key idea presented in this paper was developed while I was working at the Australian Bureau of Agricultural and Resource Economics and Sciences. An earlier version of the paper, which focussed on sectoral allocation of investment in a single country CGE model, was presented at the Third National CGE Modelling Workshop 2012, Canberra. Without implicating for any errors I wish to thank Peter Dixon, Terry Walmsley, Owen Gabbitas and Jenine Dixon for their helpful comments while reviewing an earlier version of the paper. I also wish to thank Peter Warr, Premachandra Athukorala, James Giesecke and the participants of the Third National CGE Modelling Workshop and Departmental Seminar, Arndt-Corden Department of Economics, the Australian National University for their insightful comments and suggestions.

time, the apparent lack of an optimising mechanism in these models to allocate investment across sectors and regions has been a sore point.

This paper aims to take up and address this issue.

It presents a natural approach to deriving a recursive-model- consistent expected rates of return (RoR) under static expectations. These expected rates of return are inversely related with the level of current investment and thus can serve as a basis for investment allocation across sectors and regions. More importantly, the proposed approach does not require any new information; it simply uses the information currently used by these models. It is hoped that use of the proposed approach will enhance the theoretical integrity of the recursively dynamic CGE models and credibility of their solutions.

This paper first reviews the MONASH (Dixon and Rimmer 1998, 2002) and MMRF (CoPS 2007) approach to investment modelling. This is followed by a short summary of the approach taken in global models like GTAP (Ianchovichina and McDougall 2001) and GTEM (Pant 2007). These reviews show that the current approach to modelling investor behaviour in recursively dynamic models are characterised by the need to resort to some sort of reduced form mechanisms. The proposed approach is described thereafter.

2. The MONASH investment theory

Broadly speaking there are three classes of CGE models currently in use around the world - models with forward-looking agents operating mainly (not exclusively) on GAMS platform, comparative static models operating on all platforms, recursively dynamic models operating on GEMPACK platform. Our current interest is in the last class – recursively dynamic models operating on GEMPACK platform in particular, on the way they have specified the investment behaviour. These models in one way or the other owe intellectual debt to the ORANI model of the Australian economy (Dixon et al 1982), which has now evolved into MONASH family of models (MONASH, MMRF, MMRF-GREEN, TERM, etc.). As far as modelling of investment and capital accumulation dynamics is concerned, the ORANI theory has also evolved into the MONASH theory as described in Dixon and Rimmer (2002, 1998) and forms the core of investment theory used in MMRF and even in the GTAP model as well. Hence a good review of the MONASH model approach to investment modelling is expected to establish the frontier of the problem.

The key approach followed in MONASH investment and capital accumulation module can be summarised as follows.

Capital stock in MONASH is sector specific and thus there are sector-specific investments. The accumulation process is fairly standard – capital stock grows by net investment, and it takes one year to ‘install’ it. That means that the capital stock growth is fully governed by changes in investments. MONASH first determines the ‘equilibrium’ capital growth and from this derives sectoral investments. In short MONASH innovations in modelling sectoral investments can be summed up in the following three steps:

- For each sector MONASH specifies a sector-specific supply function for investible funds, stylising the behaviour of cautious financial investors. These investors, who monitor the growth in the sectoral capital stock, require higher ‘expected equilibrium rate of return’ (EEQROR) for higher growth rates in the capital stock (K_GR).

- The expected rates of return may decline with an increase in the sectoral investments. As future rates of return cannot be observed in a recursive model, the expected rates of return (EROR) are calculated for each sector either by assuming expectations are static or by iterative methods.² If iterative methods are used in the recursive models to derive expected values then the model becomes equivalent to a forward-looking model with perfect foresight. As the paper focuses on recursively dynamic models, we will ignore the iterative approach sometimes taken by MONASH models and consider the cases in which the EROR are estimated by current rates of return.

At the capital growth rate for which $EEQROR = EROR$ holds, the market for sectoral investible funds clears in each period. In actual application, however, the model maintains that the expected equilibrium rate of return (EEQROR), the RoR required by the financial investors, and the expected rate of return (EROR), as seen by the real investors, is assumed to be equalised only in the long run. In the short run disequilibrium may persist and the disequilibrium gap is assumed to close gradually, and the speed of correction is set by a parameter of modeller's choice.

2.1 The basics

2.1.1 Capital accumulation

Capital accumulation is given by

$$K_{j,t+1} = (1 - D_j)K_{j,t} + I_{j,t} \quad (1)$$

with standard definitions of the terms appearing in all equations, K for capital stock, I for investment, D for the depreciation rate, j is a sector identifier and t refers to the year (time).

Define

$$K_GR_{j,t} = \left(\frac{K_{j,t+1}}{K_{j,t}} - 1 \right) \quad (2)$$

then, it follows from (1) and (2) that

$$K_GR_{j,t} = \frac{I_{j,t}}{K_{j,t}} - D_j. \quad (3)$$

Equation (3) describes the trajectory of annual growth rate of the sector specific capital stock, provided the initial values and the time path of sectoral investments are known. The growth rate of the sectoral capital stock will be higher the higher is the level of current investment. The capital stock growth rate will be zero if the ratio of investment to current capital stock is exactly equal to the depreciation rate.

The MONASH strategy is to determine the equilibrium $K_GR_{j,t}$ for each sector first and then use equation (3) to determine $I_{j,t}$.

MONASH has smartly developed a supply side and a demand side of investible funds which in 'equilibrium' determine $K_GR_{j,t}$ via a market clearing process. While doing so it also imposes

² MONASH also allows modelling of rational expectation by solving the model iteratively until the expected values of the variables, whose initial values are given by static expectation, equal to the actual model solution. Perhaps because of the implied computational burden, MMRF allows static expectation only.

guide rails for changes in sectoral capital stocks, so that there is always a feasible and unique solution.

2.1.2 The supply side of investible fund

To stylise the supply side behaviour MONASH introduces few new concepts:

Bounds for the growth rate of sectoral capital stock

$K_GR_MIN_j$ – minimum possible rate of growth of capital, and is set at the negative of the depreciation rate in industry j ($-D_j$).

$TREND_j$ – the industry's historical normal capital growth rate. This is an observed growth rate (not the average growth rate over a period) in the industry capital stock over the historical period.

$K_GR_MAX_j$ – the maximum feasible growth rate of industry capital stock. This number is normally set by adding a positive number, such as 0.06 or 0.1, to the $TREND$ rate.

By forcing $K_GR_MIN_j \leq K_GR_j \leq K_GR_MAX_j$ the MONASH model imposes a first discipline (guiderails) on how investments will be allocated across industries. It requires that gross investment cannot fall below zero, (which means that the net investment can be equal to the negative of the capital stock that is depreciated so that capital stock can decline by the amount of depreciation only). Similarly, no matter what, the annual growth in the industry capital stock cannot exceed K_GR_MAX

The key information required to set up a supply function is the $TREND$ rate of capital growth (and K_GR_MAX) by sector. Estimation of these growth rates for each sector could be challenging.

MONASH also introduces a variety of rates of return:

$RORN_j$ – the industry's historical normal rate of return, an estimate of the average rate of return that applied over the historical period in which the industry's average rate of capital growth was $TREND_j$.

$EEQROR_j$ – the expected equilibrium rate of return and

$EROR_j$ – the expected rate of return.

In addition, there are other rates of return types used in the model, but for the present purpose the above list will suffice.

Note that $RORN_j$ is another key piece of information required to be estimated.

Given these parameters of the supply side behaviour, the sectoral fund-supply function is described by an inverse logistic function:

$$EEQROR_j = RORN_j + (1/C_j) * [\ln(K_GR_j - K_GR_MIN_j) - \ln(K_GR_MAX_j - K_GR_j) - \ln(TREND_j - K_GR_MIN_j) + \ln(K_GR_MAX_j - TREND_j)] \quad (4)$$

where, C_j is a positive parameter whose calibration procedure is described and discussed in section 2.5.

Equation (4) can also be written as

$$e^{EEQROR_j} = e^{RORN_j} * \left(\frac{\frac{K_{GR_j} - K_{GR_MIN_j}}{K_{GR_MAX_j} - K_{GR_j}}}{\frac{TREND_j - K_{GR_MIN_j}}{K_{GR_MAX_j} - TREND_j}} \right)^{1/C_j} \quad (5)$$

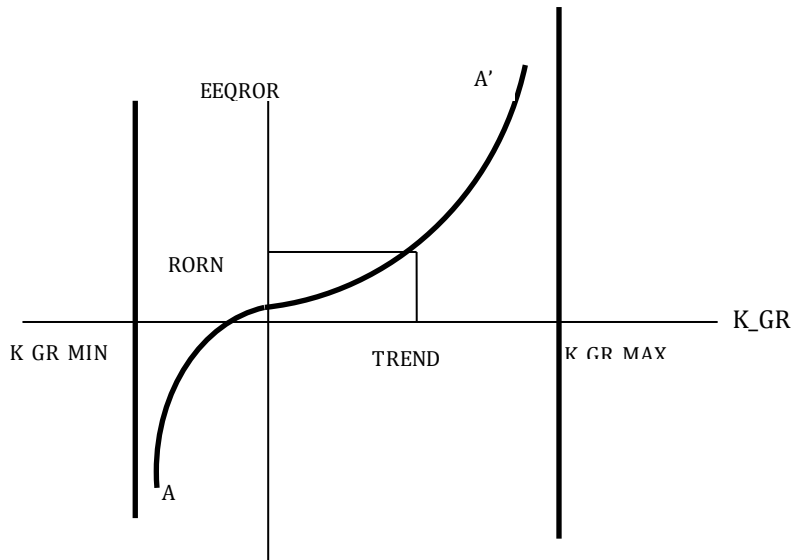
A simple algebraic manipulation of (5) shows that

$$K_{GR_j} \begin{matrix} \leq \\ > \end{matrix} TREND_j \text{ as } EEQROR_j \begin{matrix} \leq \\ > \end{matrix} RORN_j \quad (6)$$

The condition (6) summarises the property of the supply function which states that the suppliers of fund in the MONASH model would be tempted to finance investments in a sector above its TREND level if it is expected to deliver higher than the normal rate of return in equilibrium, and so on.

As can be seen from (5) $EEQROR_j \rightarrow \infty$ as $K_{GR_j} \rightarrow K_{GR_MAX_j}$. That is, the suppliers of fund need sufficiently high expected equilibrium rate of return in order to induce them to finance the investment that lets the capital stock grow at the maximum allowable rate. Similarly, we can see that if $EEQROR_j \rightarrow -\infty$, then $K_{GR_j} \rightarrow K_{GR_MIN_j}$. That is, if the expected equilibrium rate of return falls to a very low level, then the capital stock will diminish only by the depreciation rate.³

Figure 1 Supply curve for investible funds



The curve AA' in figure 1 depicts the behaviour of the funds-supplier as represented by equation (5). The supplier will not supply funds to finance investments that put the capital growth rate above its maximum rate irrespective of the expected equilibrium rates of return it may offer, and similarly the suppliers cannot make the capital stock decline by more than the rate of its depreciation by reducing the funds for investment. Their supply of investible funds is bounded.

³ What will the sector do with the capital stock if the price of the output of a sector fall significantly, or slapped with some new tax so that the scale of operation, rental rate and the rate of return, in turn, falls significantly? One possible answer is that the sector will reduce employment of other factors sufficiently and increase the capital intensity of production to keep the undepreciated capital stock employed provided the elasticity of factor substitution is sufficiently large. Otherwise, sector specificity of capital stock could pose a challenge in keeping the rental rate nonnegative in all possible situations, particularly when some sectors face a significant negative (damaging) shock.

2.1.3 The demand side of investible fund:

The 'real investors', who buy equipment, know that the present value of their expected rates of return is given by:

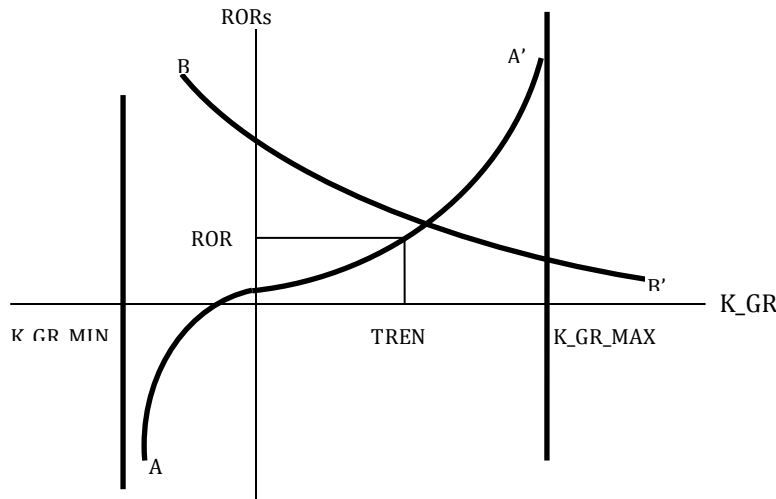
$$EROR_{jt} = -1 + \frac{E(Q_{j,t+1})}{\Pi_{jt}} * \frac{1}{1+i} + (1 - D_j) * \frac{E(\Pi_{j,t+1})}{\Pi_{j,t}} * \frac{1}{1+i} \quad (7)$$

Where Q is the rental rate and Π is the replacement cost of capital stock (price of capital goods) and i is the nominal interest rate. E is the expectation operator that determines the currently held expectation of future (next period) value of the variable. The presence of tax has been ignored for simplicity.

Calculation of the rate of return relative to the interest rate is the preferred option of MONASH model.⁴

It is noteworthy that if an ordinary (current value) rate of return were equal to the interest rate, then the corresponding EROR given by (7) would be zero. Any positive rate of return given by (7) therefore provides appropriate motivation for additional investments in the sector.⁵

Figure 2 demand for funds under perfect foresight and equilibrium in the sectoral financial market



As capital accumulates in the next period with increased investment in the current period, it is natural to anticipate that the rental rates in the next period will fall. Thus in a perfect foresight model the EROR will fall with investment and hence with an increase in K_{GR} as well. This implies a downward sloping demand curve reflecting a negative relationship between K_{GR}

⁴ Alternatively, the expected rates of return can be calculated using current price rental and value of depreciated capital. In this case, the expected rates of return need to exceed the interest rate or the opportunity cost of capital for the investment project to be viable. This is the approach taken by this paper.

⁵ Given this definition of the expected rates of return, whether the curve AA' in figure 1 should pass through origin depends on whether there is some kind of risk premium attached to the sector or not. If there is no risk premium attached to sector, then the curve would pass through the origin.

and EROR. Because of the falling ROR, the investors demand more funds only if the cost of borrowings are lower. The curve BB' in figure 2 is a representation of such behaviour of the real investors with perfect foresight.

2.2 The Equilibrium

The investment equilibrium is attained at the intersection of the AA' and BB' curve for the respective sectors, where the capital growth the financiers are happy to fund at the expected equilibrium rate of return is just equal to the capital growth the investors desire to have offering the same expected rate of return. That is, equilibrium is attained at the capital stock growth rate when

$$EEQROR_j = EROR_j. \quad (8)$$

2.3 Expectation formation

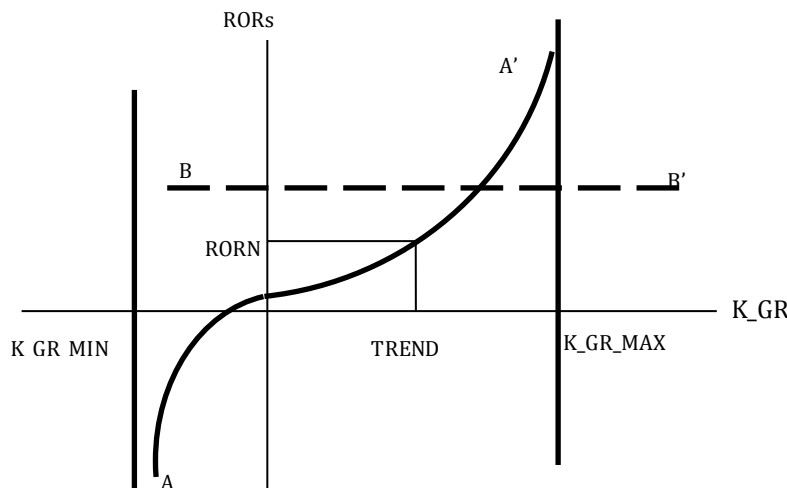
One of the key features of recursively dynamic models is that the agents cannot have perfect foresight. Although MONASH has developed an iterative procedure to calculate the perfect foresight equivalent expected rate of return, the procedure is very computation intensive. A common practice is to use static expectations, which means that the future will look the same as the current period. In other words the values of all market determined variables for the next period, including prices, are expected to remain the same as in the current period.

With this specific assumption, equation (7) becomes

$$EROR_{jt} = -1 + \left(\frac{Q_{jt}}{\Pi_{jt}} + (1 - D_j) \right) * \frac{1}{1+i} \quad (9)$$

Note that in deriving (9) all expected values on the right side of (7) are replaced by their current values. But this specification also means that the expected rate of return will not be affected by the level of investment undertaken in the current period because current rental rates will be independent of the size of future capital stocks. This means that the 'demand' curve, BB', in figure 2 will be a horizontal line, as shown in figure 3.

Figure 3 Investment allocations in MONASH with static expectation



MMRF-GREEN (Adams et al., 2003 p.44) and MMRF (CoPS, 2007 p. 88) further assume that real after tax interest rate will be zero (this paper ignores the presence of tax for simplicity), which means that the nominal interest rate is equal to the rate of inflation.⁶ In a modelling exercise of the real economy, where the inflation rate is exogenously fixed and set to zero, interest rate discounting that appears in (9) becomes irrelevant and thus the expected rate of return can be written as

$$EROR_{jt} = \frac{Q_{jt}}{\Pi_{jt}} - D_j \quad (9')$$

Clearly, the expected rate of return given by (9') is independent of the growth in current investment and consequent growth in the sectoral capital stock (here the small GE impact that may have on the sectoral rental rate, Q_{jt} , and on the price of sectoral capital goods, Π_{jt} , of changes in investment expenditure made by the sector are ignored). It is therefore useful to note the change in the shape of the BB' curve between the figures 2 and 3.

Given that the supply curve, AA', is continuous and defined over the entire range of the expected rate of return, there will be a feasible solution irrespective of the position of the demand curve BB'. This is the beauty of the MONASH scheme. The solution will always fall on the supply curve and thus it will be bounded and continuous.

2.4 Modelling of slow adjustments

Given that sectoral rates of return on capital may change for many reasons, the BB' curve will shift as a result. This will bring a change in the equilibrium growth rate of the capital stock. Whether the adjustment will be instantaneous or will take few periods to complete could be a matter for debate. MONASH model has made a provision to allow for partial adjustment of the sectoral equilibrium rates of return by modifying equation (8), which can be described as follows.

Assume that either the investors are slow to make necessary adjustments in the level of investments they plan to undertake or the fund managers are slow in responding when the market conditions change or both. Consequently, the condition (8) does not hold at all times. It holds with the following adjustments:

$$EEQROR_{jt} = EROR_{jt} + DISEQ_{jt} \quad (10)$$

and

$$DISEQ_{jt} = (1 - \phi_j)DISEQ_{j,t-1} \quad (11)$$

where, $0 \leq \phi_j \leq 1$ is a parameter. MONASH default value for this parameter is 0.5.

Combining (10) and (11) gives the partial adjustment mechanism

$$EEQROR_{jt} - EROR_{jt} = (1 - \phi_j)(EEQROR_{j,t-1} - EROR_{j,t-1}) \quad (12)$$

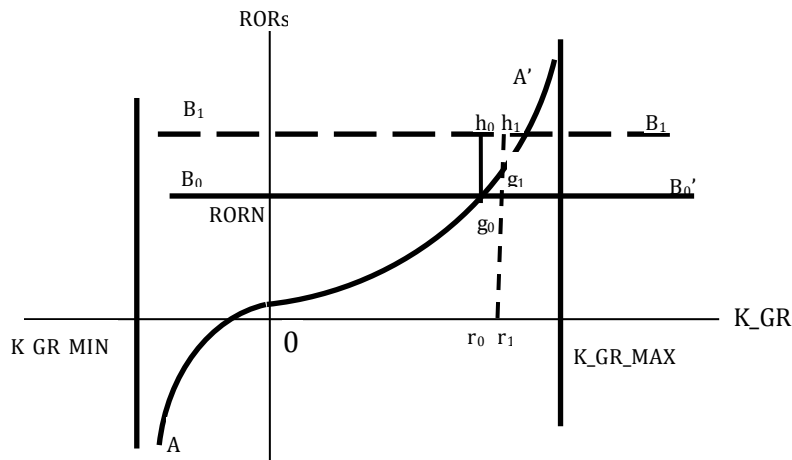
⁶ This assumption is not necessary provided the expected rates of return are matched with the cost of capital in one form or the other while allocating investments to sectors. Equation (9') can simply be viewed as an expression for the ordinary rate of return based on static expectations and current prices. See also footnote 3.

That the gap that arises between the EEQROR and EROR as a result of changed market conditions will be gradually eliminated. In specific terms, with the default value for $\phi_j = 0.5$, one-half of the gap will be eliminated each period by adjusting the level of investment.

The adjustment mechanism defined by equation (12) is illustrated by figure 4.

Assume that initial demand for fund curve is given by B_0B_0' and the supply of fund is given by the AA' curve. The equilibrium is attained at point g_0 with equilibrium capital stock growth given by r_0 , which is also the TREND rate, and the capital stock earning the normal rate of return.

Figure 4 Partial adjustments in the capital stock growth and the equilibrium rates of return in MMRF and MONASH models



Suppose that an exogenous change in the market condition raised the expected rate of return causing an upward shift on the demand curve to B_1B_1' . The disequilibrium gap between the expected equilibrium rates of return (consistent with the supply curve) and the expected rate of return (consistent with the new demand curve) is given by the distance g_0h_0 . Instead of increasing the growth rate of the sectoral capital stock to the point where the two curves intersect, the new equilibrium will yield a growth rate of r_1 , so that $g_1h_1 = (\frac{1}{2})g_0h_0$. The remaining gap will be closed in subsequent periods by following similar steps as travelled in the current period until EROR and EEQROR are equalised. Note that the BB' curve will have to shift again, as the base capital stock has changed and expected rates of return has to fall with the increase in the capital stock. Eventually it will have to come back to its original level so that the capital stock in equilibrium is growing at the TREND rate, earning the normal rate of return.

The mechanism that shifts the curve BB' appropriately to re-establish the long run 'steady state' equilibrium is quite crucial.

2.5 Calibration of the supply function parameter

To complete the derivation of the sector specific investment allocation mechanism of the MONASH models requires the calibration of the parameter C_j of equation (4). The size of this parameter determines how sensitive the suppliers of funds would be to changes in the sectoral capital stock and hence determines the slope (and the shape) of the curve AA' , the smaller the value of C_j , the larger the gap between the required rate of return from the normal rate of return.

Noting that RORN is a constant, differentiation of both sides of equation (4) with respect to K_GR gives

$$C_j = \left[\frac{\partial EEQROR_j}{\partial K_GR_j} \right]^{-1} * \frac{K_GR_MAX_j - K_GR_MIN_j}{(K_GR_MAX_j - TREND_j)(TREND_j - K_GR_MIN_j)} \quad (13)$$

Equation (13) clearly shows that the slope of the AA' curve must be known to parameterise C_j . Since the equation holds at all points along the curve, the calibration can be done by evaluating the derivatives at the point whose values are known, such as the combination of the TREND growth rate and the normal rate of return, RORN, for the sector. As the value of the term $\left[\frac{\partial EEQROR_j}{\partial K_GR_j} \right]^{-1}$ for each industry is not available, MONASH draws on other models such as Murphy model (Powell and Murphy, 1997) and TRYM (Jilek et al., 1993) to get an idea of the average sensitivity of capital growth to variations in expected rates of return across all industries. MONASH obtained an estimate (denoted by SMURF) of the average value over all industries of the sensitivity of the growth variations in expected rates of return, defined for all j ,

$$\left(\frac{\partial EEQROR_j}{\partial K_GR_j} \Big|_{K_GR_j = TREND_j} \right)^{-1} = SMURF \quad (14)$$

Making use of equation (14), the equation (13) can be written as

$$C_j = SMURF * \frac{K_GR_MAX_j - K_GR_MIN_j}{(K_GR_MAX_j - TREND_j)(TREND_j - K_GR_MIN_j)} \quad (15)$$

which completes the calibration of the supply side parameter and shows the complexity involved in numerically specifying the funds supply function in the MONASH models.

2.6 A summary of the MONASH/MMRF approach

In a technical sense the MNOASH/MMRF approach to investment modelling can be summarised in the following key points:

- The supply curve AA' or the equation (4) that describes the behaviour of cautious financial investors is the 'key player' in the MONASH/MMRF scheme of investment modelling.
- By virtue of this curve, the growth in sectoral capital stock has a unique solution; it is bounded and continuous. This is a desirable feature as it always guarantees a unique solution.
- With sector specific supply functions, it is possible to determine a sector specific investment without noticing any direct competition from other sectors. The inter-sectoral competition for funds can be assumed to have been subsumed in the nature of the sectoral supply functions. This allows sector specific rates of return to remain different, which could be explained by referring to risk differences across sectors.
- Implementation of the sector specific supply function, however, requires estimates of the TREND growth rate of sectoral capital stock, the normal rates of return (RORN), the upper bound for capital growth (K_GR_MAX) and the value of the parameter measuring the sensitivity of the capital growth to variations in the expected rates of return or the parameter C_j . They are not less challenging.

3. Investment allocation in global models: GTAP and GTEM

3 Investment modelling in Global CGE models

3.1 Investment modelling in the dynamic GTAP model

At the core of the dynamic GTAP (GTAP-dyn) investment theory lie the following concepts and relationships:

For each region the following variables are defined

RORGTARG – the suppliers of the fund have a target rate of return

RORGROSS – defined as the ratio of current rental rate to the price of capital,

RRG_RORG – required rate of growth in the rate of (unspecified, but RORGROSS is a logical candidate) return.

Using these three variables the dynamic GTAP model postulates that

$$RRG_RORG = LAMRORG * \log\left(\frac{RORGTARG}{RORGROSS}\right) \quad (16)$$

where *LAMRORG* is a coefficient of adjustment.⁷

Equation (16) implies that given the target rate of return, the investors (suppliers of the fund) determine the direction of change in the region specific capital stock once they know the current gross rate of return. Higher than the target rate of return calls for a reduction on the gross rate of return, which can be achieved by increasing the regional capital stock, hence more investment. Alternatively, a lower gross rate than the target rate of return requires the gross rate to rise, which means a reduction in the capital stock is required.

The demand side of the investment allocation process is described by a slightly modified ORANI-type specification of the expected rate of return. To specify this function the dynamic GTAP introduces another set of new variables and new concepts for each region:

RORGREF – denotes a reference rate of return, comparable to the normal rate of return in MONASH

QKF - a reference capital stock that can grow at the rate of '*khat*' without affecting the reference rate of return. Clearly, '*khat*' is comparable to the growth rate TREND in MONASH.⁸

RORGEXP – expected gross rate of return

QK – planned capital stock for the next period

With these tools, the dynamic GTAP specifies the relation

⁷ If *LAMRORG* is taken as an inverse of the time taken to grow *RORGROSS* to *RORGTARG*, then clearly *RRG_RORG* gives the rate of exponential growth required. Hence the term required rate of growth of the rate of return.

⁸ It is important to note that these comparisons are made at the conceptual level. It can also be seen that equation (16) and equation (5) play the same role, representing the behaviour of the suppliers of the investible fund.

$$\frac{RORGEXP}{RORGREF} = \left(\frac{QK}{QKF}\right)^{-RORGFLEX} \quad (17)$$

where RORGFLEX is a positive parameter.

Linearising (17) and ignoring any changes in the QKF for simplification results in

$$ERG_RORG = -RORGFLEX * RG_QK \quad (18)$$

where ERG_RORG is the growth rate in the expected rate of return and RG_QK is the growth rate in the capital stock.

Clearly equation (18) established a negative relationship between the growth rate in the capital stock and expected rate of return. RG_QK is the choice variable of the 'real' investors.⁹

The equilibrium is established if

$$ERG_RORG = RRG_ROR. \quad (19)$$

That is, if the required growth in the regional rate of return is equal to the growth rate in the expected regional rate of return, then the regional allocation of investment is in equilibrium.

Leaving the calibration process, further extensions and generalisations of the approach aside, it is easy to see that the core investment theory of the dynamic GTAP model needs to rely on specification of additional functions (16) and (17) relating few new and unique variables.

It may be useful to note that the allocation is not based on equalisation of some sorts of rates of return across regions but equalising different rates in each region separately. Regional equilibrium conditions involve region specific target rate driven growth in the gross rate of return and the growth rate on the expected rates of return that is based on an external parameter and a 'reduced form' function.

There is an apparent subtle difference on the time to which the left and right hand side of equation (19) refer to. The left hand side refers to the rate of growth in the expected rate of return (that is supposed to prevail in the future) and the right hand side refers to the required growth rate on the current gross rate of return. It is not quite clear why these two rates are required to be equal to deliver an equilibrium outcome. It may be argued, however, that the right hand side of equation (19) refers to the change in the rate of return 'desired' by the fund suppliers that can only be met next year and the left hand side refers to the change in the rate of return that the 'real' investors expect to deliver next year. The need to specify equations (16) and (17) arose simply because being a recursively dynamic model the investors in the dynamic GTAP model also did not have the ability to see how the future rates of return will fall as investment in a region is increased.

3.2 Investment modelling in GTEM

Relative to the dynamic GTAP investment modelling in GTEM is rather simple and straightforward. Being a recursively dynamic model, GTEM also faces the same problem as faced by MONASH, MMRF, or the dynamic GTAP models. It also relies on a reduced form specification of the investment function, and does so in a simplistic way.

⁹ It may be useful to note that RG_KB in GTAP notation is the same as RG_K in standard MONASH notation.

For each country or region, the level of investment is given by

$$I_REAL(r) = \Theta(r)\delta K(r)e^{\rho(r)*(ROR(r)-GLOBAL_i)} \quad (20)$$

Where the parameter

$$\Theta(r) = \exp \left\{ -\varpi \cdot \frac{\Phi(r)*DEBT(r)}{GDP(r)} \right\} \quad (21)$$

$\rho(r)$	is a positive parameter, normally set to 10
ϖ	is a positive parameter with default value 0.5
$\Phi(r)$	Exchange rate - units of local currency unit per unit of global currency
$I_REAL(r)$	Real investment
$K(r)$	Capital stock, sometimes replaced by real GDP, as the scale variable.
$ROR(r)$	Current rate of return is serving as the expected rate of future return
$GLOBAL_i$	an interest rate that clears the global financial market,
r	a generic element of the set Region, and
δ	is the depreciation rate.

Capital stock in GTEM is perfectly mobile across sectors and therefore there is one regional capital stock. Equation (20) determines the level of real investment allocated to each region. The parameter Θ imposes a penalty on a country that accumulates excessive debt relative to its GDP. The key principle embedded in (20) is that given the size of the economy, investment also responds to the current rate of return relative to the cost of borrowing, which is the $GLOBAL_i$. If domestic rate of return in each region is equal to the global interest rate and the debt level of each region is zero, then the regional real investment will be equal to the depreciation of the current capital stock. In other words, the capital stock will either remain constant over time or grow in proportion with investment.

A positive gap between current rates of return and the global cost of borrowing creates an extra demand for investible funds in each region, causing global demand for funds to rise. For a given supply of global funds, the global cost of borrowing rises with the demand. Consequently, the demand for investible fund falls. And new equilibrium is established with higher $GLOBAL_i$ and reallocation of investment across regions. It allows competition for funds across regions and competition for capital stock across sectors within a country.

Investment allocation in GTEM also requires parameters that are estimated. Increasing investments cannot reduce the expected rates of return because current rates are used under static expectation. On year-on-year pathway, the actual capital stock is updated and the rates of return will reflect the accumulation of capital stock. Rates of return would be falling with capital accumulation with a lag in an ex post sense. This relation cannot be used by investors while making investment decisions. In this sense, maximisation of the rates of return on overall investment has not yet been used as an objective of the investors, and is therefore suboptimal.

4. An alternative approach

In all of the approaches discussed so far the key issue has been the inability to capture responsiveness of the expected rate of return to changes in the level of current investment based on the information contained in the model. Current investment alters the capital stock of tomorrow, and thus affects the rental rates of tomorrow which cannot be observed in recursively dynamic models. As a result these models have not been able to use this fundamental relationship in allocating investment across sectors and regions. Instead, the models resorted to some reduced form specification of the behaviour of fund suppliers to help determine the sectoral or national investment levels.

This section shows that it is, however, possible to derive an expression for the expected rate of return based on production function parameters of the sectors and their properties, various equilibrium conditions with static expectation.

The expected rate of return thus derived is responsive to the level of investment planned and is naturally consistent with the internal structure of the model. These relationships provide the opportunity to allocate investments across sectors by equalising the expected rate of return or the changes in the expected rate of return.

This approach will provide a basis to claim that investors in recursively dynamic CGE models are rational, return maximisers.

4.1 Allocation of investment in GE models with perfect capital mobility

Global models such as GTAP and GTEM are characterised by perfect mobility of capital across sectors within a region and immobility of installed capital across regions. Consequently different sectors within a region face the same market rental price of capital, while the rental rate across regions may differ. Given that the profit maximizing condition requires that the expected rental rate be equal to the expected value of marginal product in each sector, it is therefore useful to aggregate the expected rental functions across all sectors.

Once the expected sectoral rental rate functions are successfully aggregated, equation (34) and (35) can be applied to derive the expected rates of return at the economy-wide level as the construction cost (or purchase price) of capital and the depreciation rate is defined for the region as a whole. This section outlines the aggregation procedure.

Since decline in the rates of return would not be the same across sectors even if the capital stock is perfectly mobile, we need to consider what happens to sectoral rates of return if the regional capital stock were increased by one unit and then use this information to obtain the extent of fall in the regional rates of return.

Assume that a region has n sectors with n different production functions. This means that there will be n different rental equations like (23). Using the sectoral index (ignoring time) and writing the first order condition for cost minimization in general form, one gets

$$Q_j = P_j F_j(K_j; \dots) \quad j=1, 2, \dots, n. \quad (22)$$

where $F_j(K_j; \dots)$ is the function yielding the marginal physical product of capital in sector j , the first derivative of a CRS production function with respect to K_j .

Inverting equation (22) we get

$$K_j = G_j(Q_j/P_j) \quad j=1, 2, \dots, n. \quad (23)$$

where G_j is the inverse function of F_j .

Equation (23) expresses the demand for capital services as a function of the rental rate and the price of output. This equation allows horizontal aggregation (see figure 5 for a simplified illustration) of the sectoral demand for capital services at each rental rate. The aggregated function anticipated for period $t+1$ can be written as

$$K_{t+1}^e = \sum_{j=1}^n G_j(Q_{t+1}^e/P_{j,t+1}^e) \quad (24)$$

where Q is the economy wide rental rate of capital faced by all sectors.

To examine the effect of an increase in the economy wide stock of capital on the economy-wide rental rate we differentiate equation (24) totally and get, ignoring the $t+1$ subscript,

$$100 * dK^e = \sum_{j=1}^n \frac{\partial G_j}{\partial (Q^e/P_j)} \cdot \frac{(Q^e/P_j)}{G_j} \cdot \frac{G_j}{(Q^e/P_j)} \cdot (Q^e/P_j) \cdot (q^e - p_j) \quad (25)$$

Where lower case letters are percentage change forms of the variables denoted by corresponding uppercase letters and under static expectation $P_{j,t+1}^e = P_{j,t}$.

With simplification, equation (25) reduces to

$$\frac{dK^e}{K} = \left(\sum_{j=1}^n \eta_{KK}^j \frac{G_j}{K} \right) q^e - \left(\sum_{j=1}^n \eta_{KK}^j \frac{G_j}{K} p_j \right) \quad (26)$$

Equation (26) can also be written as

$$q_t^e = \frac{1}{\left(\sum_{j=1}^n \eta_{KK}^j \omega_{jt} \right)} \cdot [K_GR_t + \sum_{j=1}^n \eta_{KK}^j \omega_{jt} p_{jt}] \quad (27)$$

where $\omega_{jt} = \frac{K_{jt}}{K_t}$, is the share of sector j in the employment of the economy-wide capital stock, K , and η_{KK}^j is the own price elasticity of demand for capital services.

Therefore parameterisation of equation (27) reduces to finding the own price elasticity of demand for capital services. The elasticity of factor substitution and factor cost shares come handy here.

Since, it is known from Uzawa (1962) and Sato and Koizumi (1973) that¹⁰

$$\eta_{ij} = S_j \sigma_{ij} \quad \text{for all factors } i \text{ and } j. \quad (28)$$

Where S_j is the cost share of factor j and σ_{ij} is the Allen-Uzawa elasticity of substitution between factors i and j .

¹⁰ Allen-Uzawa elasticity of substitution has been used here. For further details about alternative forms of elasticity of substitution see Blackorby and Russell (1989), Stern (2008) and Blackorby et al. (2007). Some additional refinements in the light of this literature will improve the framework suggested in this paper.

Since input demand functions are homogenous of degree zero, Euler's theorem implies that

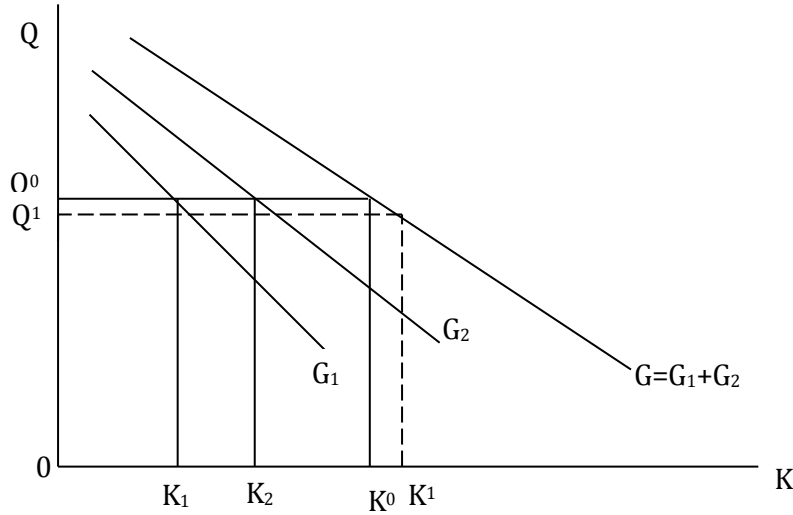
$$\sum_j \eta_{ij} = 0, \quad (29)$$

for each factor i . It therefore follows that

$$\eta_{KK} = -\sum_{j \neq K} S_j \sigma_{Kj} \quad (30)$$

which completes the numerical specification of equation (27).

Figure 5 Aggregating sectoral demand curves for capital services



A change form of the relationship represented by the equation (26) is depicted by the dashed lines in figure 5. A small increase in the national capital stock (from K^0 to K^1) in the next period via increased net investment in the current period can be expected to lower the economy-wide rental rate in the next period (from Q^0 to Q^1) under static expectation.

The calibration of the term in the parenthesis of equation (27) requires, as explained above, the elasticity of factor substitution and cost share of factors in each sectors and sectoral share in the national employment of capital stock. These information are either already available in the model database or can be derived using the database. No new information is required.

4.2 Expected regional rates of return on investment

Equation (27) can be used to estimate changes in the expected rental rate under static expectation when the capital stock changes because of investment. In MONASH notations equation (27) helps us to determine $E(Q_{r,t+1})$, where $E(Q_{r,t+1})$ is the regional average rental rates expected to prevail in the next period.

Instead of equation (9), the expected rate of return on investment without discounting can be written as

$$EROR_{rt} = -1 + \frac{E(Q_{r,t+1})}{\Pi_{rt}} + (1 - D_r) * \frac{E(\Pi_{r,t+1})}{\Pi_{r,t}} \quad (31)$$

Since, under static expectations prices do not change, so

$$E(\Pi_{r,t+1}) = \Pi_{r,t} \quad (32)$$

the expected rate of return, as defined by (31), can thus be written as

$$EROR_{rt} = \frac{E(Q_{r,t+1})}{\Pi_{rt}} - D_r \quad (33)$$

Equation (33) is the same as the equation (9') used in the MMRF model except that the MMRF model uses $E(Q_{r,t+1}) = Q_{r,t}$. Here it is proposed that $E(Q_{r,t+1})$ be given by the level form of equation (31).

Expressing equation (33) in the change form we get

$$100 * deror_{rt} = \frac{Q_{rt}^e}{\Pi_{rt}} (q_{rt}^e - \pi_{rt}) \quad (34)$$

Initially $Q_{rt}^e = Q_{rt}$.

4.3 Regional allocation of investment

Based on these expected rates of sectoral return, investment can therefore be allocated to sectors so that the (per cent) change in the expected rates of return are equalised.

That is, equilibrium in the investible fund market would require that

$$deror_{rt} = d\rho_t + d\lambda_{rt}, \quad \text{for all } r. \quad (35)$$

where $d\rho_t$ is the change in the equilibrium interest rate determined by the world market conditions, and $d\lambda_{rt}$ is exogenously determined change in the region specific risk premium.

Equations (27), (34) and (35) together determine expected rental rate for the next period, q_t^e ; change in the expected regional rate of return on capital stock, $deror_{rt}$; and the regional level of investments that equalise the expected rates of return across regions allowing for the differences in risk premiums, accounting for other imperfections in the capital market.

As long as the capital market is open and the regional average rate of return remains positive, no matter how small, the investment allocation problem will be solved by equations (27), (33) and (34). All regions with increases in the rate of return greater than the least one will get higher allocation, and changes in the expected rates of return the two equilibria will be equalised across regions. If this equalised rate is greater than the global change in the interest rate all sectors will receive additional investments until the equilibrium condition is established.

If, however, a region faces an adverse market conditions such that the rental price of the capital stock has to be negative to keep the current stock fully employed, then we have a situation that is not suitable for a model which requires that all prices have to remain positive (no free goods). To keep the rental rate positive either there is a need for free disposal of capital and/or a change in the production technology that increases capital intensity or substitutability or interregional mobility of capital needs to be modelled. Free disposal is, in some sense, equivalent to a shut-down of some firms within the sectors. Normally, higher factor substitution and trade in commodities with flexible prices will assure that such a situation does not arise.

4.4 Some analysis of regional investment demand

Assume that the relative risk premium remain unchanged, then this means that the equilibrium condition (35) can be rewritten as

$$deror_{rt} = d\rho_t, \quad \text{for all } r. \quad (36)$$

Equation (34) then implies that

$$q_{rt}^e = \pi_{rt} + 100 * \frac{\pi_{rt}}{Q_{rt}^e} \cdot d\rho_t \quad (37)$$

Using equation (37) into equation (27) we obtain

$$K_GR_t = \left(\sum_{j=1}^n \eta_{KK}^j \omega_{jt} \right) * \left[\pi_{rt} + 100 * \frac{\pi_{rt}}{Q_{rt}^e} \cdot d\rho_t \right] - \sum_{j=1}^n \eta_{KK}^j \omega_{jt} p_{jt} \quad (38)$$

Using a linearised version of equation (1) into equation (38) yields the regional allocation of investment as

$$i_j = \frac{\left(\sum_{j=1}^n \eta_{KK}^j \omega_{jt} * (\pi_{rt} + 100 * \frac{\pi_{rt}}{Q_{rt}^e} \cdot d\rho_t - p_{jt}) \right)}{S_I_j} - \frac{S_K0_j}{S_I_j} k0_j \quad (39)$$

Where

$S_I_j = \frac{I_j}{K1_j}$, and $S_K0_j = \frac{(1-D_j)K0_j}{K1_j}$ are shares of gross investment and depreciated current capital stock of current period in the next period's capital stock. $k0_j$ and i_j are percentage change on the current capital stock and gross investment of the current period.

Clearly equation (39) yields a neat formula to allocate investments across sectors in a small open economy where rates of return are initially in equilibrium. This formula does not involve any expected values of any variables neither does involve any unknown parameters.

Equation (39) shows that investment increases in a sector that has experienced growth in its output price relative to the growth in the price of its new capital stock. Current price induced growth in investment would be moderated by the growth in the current capital stock induced by the investment of previous year.

5. Conclusion

This paper identified a key problem faced by all recursively dynamic CGE models in allocating investible funds across sectors and regions if they also assumed that expectations are static. It was the effective independence of expected rate of return from the current level of investment which meant that the expected sectoral rates of return remained unchanged irrespective of the level of investment allocated to the sector. As a result, equalisation of expected rates of return could not be used as a condition for equilibrium allocation of investible funds. To solve this problem in a pragmatic way, these models have resorted to some additional specifications that were not necessarily based on an optimising framework. This has been a sore point for the modelling community as a whole.

To address this problem, this paper derived a consistent expression for the expected rates of return in recursively dynamic GE models with static expectation. The derivation is based on the property of the production function that the marginal product of capital declines as the capital

stock accumulates. With static expectation on all market prices, the expected cost minimising rental rates and the rate of return declines with investment. The expected rate of return thus derived displays an inverse relationship with the level of sectoral or national investment, as the case may be.

This opens up the possibility of allocating investments across sectors and regions so that the expected rates of return are equalised and provides a possible solution to the vexing problem faced by all recursively dynamic CGE models, whether they are national or global.

If the modelling framework was forward looking and the agents had the perfect foresight, then the expected rate of return could have been the actual rate of return that would have prevailed in the next period. This environment would have allowed all prices, output levels and factor employment to fully adjust and the investment allocation could have been based on the equalisation of the actual sectoral or regional rates of return. In a recursively dynamic modelling environment, however, accounting of all these adjustments is not possible. The procedure outlined above holds the employment of all other factors constant while deriving the marginal product of capital stock and, in turn, in deriving the response of expected rates of return with respect to the sectoral level of investment. The expected rates of return estimated this way can be taken as the lower bound as the marginal product of capital can be expected to be higher with increased employment of other factors with the capital stock. Therefore, the approach proposed suffers from the usual limitations of recursive dynamic models. It can, nevertheless, be hoped that this approach takes recursively dynamic CGE modelling a small step ahead and provides a new basis for further development.

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