

Economic Projection with Non-homothetic Preferences: The Performance and Application of a CDE Demand System

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Abstract

Computable general equilibrium modeling has been used for decades in studying implications of various counterfactual scenarios. Nevertheless, whether the model responses are consistent to empirically observed income and own-price demand elasticities remains a key challenge in many modeling exercises. To address this issue, the Constant Difference of Elasticities (CDE) demand system has been adopted by the GTAP model since the 1990s. However, perhaps due to complexities of the system, the applications of CDE system in other general equilibrium models are less common. Furthermore, how well the system can represent the given elasticities is rarely discussed or examined in existing literature. The study aims at bridging these gaps by revisiting calibration details of the system, exploring conditions with better calibration performance, and presenting strategies to incorporate the system into GTAP8inGAMS, a global computable general equilibrium model written in GAMS and MPSGE modeling languages. The study finds that the calibrated elasticities of the CDE system can be matched to the targeted elasticities more accurately if the sectorial resolution is higher, targeted own-price demand elasticities are lower, and targeted income demand elasticities are higher. It also verifies that for the GTAP8inGAMS with a CDE system, the model responses can successfully replicate the calibrated elasticities under various price and income shocks.

Key words: model calibration; non-homothetic preference; constant difference of elasticities

1. INTRODUCTION

In computable general equilibrium (CGE) modeling, it has been identified that price and income elasticities of demand are crucial in determining the sectorial growth pattern and economic impacts of various policies (Hertel, 2012). This suggests that while a typical Constant Elasticity of Substitution (CES) function is still widely used in modeling final consumption (Sancho, 2009; Annabi et al., 2006; Elsenburg, 2003), the property of having unitary income elasticities of demand is often considered as highly inflexible. Also, in a single-nest CES setting, after applying the Cournot's aggregation, it can be shown that the sectorial expenditure shares will fully determine the variation in own-price elasticities of demand, which is quite restrictive as well.

To capture the observed non-homothetic preferences with income elasticities of demand diverging from unity, one approach is to use the Linear Expenditure System (LES) such as the Stone-Geary preference (Geary, 1950; Stone, 1954). The LES system can be calibrated to income elasticities of demand compatible to a valid demand system. In addition, with a special multi-nest structure, the desired own-price elasticities of demand can be matched perfectly through calibration (Perroni and Rutherford, 1995).¹ The shortcoming of LES, however, is that due to constant marginal budget shares with respect to income, the limit property of LES is still constant-return-to-scale, and therefore the underlying income elasticities of demand will approach one as income grows.

An alternative option to model non-homotheticity is to utilize the Constant Difference of Elasticities (CDE) demand system proposed by Hanoch (1975). With implicit additivity, a N -commodity CDE system has N expansion parameters and N substitution parameters to achieve a more general functional form than the single nest CES case. The N expansion parameters make it possible to incorporate various income elasticities of demand across commodities/sectors, and the income elasticities will remain at their given levels as income changes ("commodity" and "sector" are used interchangeably in this study). On the other hand, compared to a single-nest CES setting, the N substitution parameters allow modelers to come up with a somewhat better representation for the targeted own-price demand elasticities.

One caveat of CDE applications, paradoxically, comes from the constancy of each income elasticity regardless of income levels. While this feature might not severely contradict empirical evidence for developed countries, existing studies have found that, for instance, income elasticities of some food items in developing countries tend to decrease as income grows (Haque, 2005; Chern et al., 2003). In some cases, economic growth may turn luxury goods into necessities (Zhou et al., 2012). To overcome this, with more income response parameters, Rimmer and Powell (1996) presents an implicit directly additive demand system (AIDADS) that allows income elasticities of demand to vary logistically. Nevertheless, AIDADS has a narrow range of substitution across goods, and due to theoretical and computational reasons, AIDADS

¹ While Perroni and Rutherford (1995) focuses on homothetic preferences, it points out that the multi-nest strategy achieving a perfect match in own-price elasticities calibration also works for non-homothetic preferences.

applications are limited to within 10 commodities/sectors (Reimer and Hertel, 2004). As a result, these applications are much limited and project-specific. In contrast, despite of some limitations, the CDE system seems to be more applicable as a generic setting for modeling non-homothetic preferences.

While CGE models such as GTAP (Hertel and Tsigas, 1997) and ENVISAGE (van der Mensbrugghe, 2008) have been using CDE systems in modeling final consumption behaviors, perhaps due to the complexities in both calibration and implementation, other CDE applications are less common so far.² In addition, when studying the responses of CGE models with non-homothetic preferences, some research tends to focus more on the implications of income elasticities of demand on future projection (e.g., Yu et al. (2004)). However, the roles of own-price elasticities of demand should be carefully examined as well, as own-price demand elasticities could also influence projections and may become even more crucial under some policy shocks. Further, while existing literature points out that to ensure the regularity of a well-behaved demand function, calibrating a CDE system to the targeted elasticities that are observed might be infeasible (Hertel, 2012; Huff et al., 1997), how well the system can match those elasticities is beyond the discussion of most existing literature. One exception is Liu et al. (1998), which presents the differences between targeted and calibrated elasticities. Nevertheless, under what conditions the calibration performance can be improved are beyond the scope of Liu et al.

To bridge these gaps, the goal of this study is twofold. First, the author will illustrate strategies for calibrating a CDE system and explore the calibration performance. Although for the most part, the calibration procedure is based on Huff et al. (1997) and Hertel et al. (1991) except for a minor revision, the study provides the program written in GAMS in the Appendix so readers can easily use it for verification or research purposes. More importantly, the study examines through the calibration how accurate the targeted own-price and income elasticities of demand are matched. Since sometimes the calibrated elasticities could be quite far from the target numbers, to discern whether a projection or a policy impact could be overestimated or underestimated, the study argues that both a goodness-of-fit measure and an explicit comparison between targeted and calibrated elasticities are indispensable.

Next, the author presents the strategies for putting the CDE system into GTAP8inGAMS, a global CGE model written in GAMS and MPSGE using the GTAP 8 database (Rutherford, 2012). MPSGE is a subsystem of GAMS (Rutherford, 1999), and earlier it was sometimes thought that despite being a powerful tool that handles the calibration of CES functions automatically, MPSGE can only be applied to models with CES or LES utility functions (Konovalchuk, 2006; Hertel et al., 1991). The study shows that the potential of MPSGE applications is far beyond what was previously perceived. Finally, the revised GTAP8inGAMS with a CDE system is tested with income and price shocks to verify the model response is consistent to the calibrated elasticities.

² GTAP is the abbreviation for the Global Trade Analysis Project, and ENVISAGE stands for the Environmental Impact and Sustainability Applied General Equilibrium.

The rest of the paper is organized as follows: Section 2 briefly reviews the theories and settings of the CDE system, Section 3 presents the calibration, performance, and implementation of the CDE system, and Section 4 provides a conclusion.

2. THEORETICAL BACKGROUND

To understand what constitute a regular (i.e., valid) demand response, the section will briefly review the economic considerations for a regular demand system. A question that follows is: how can one evaluate the performance of a regular demand system in terms of representing the observed own-price and income demand elasticities? To explore this, the section will discuss a demand system's flexibilities in own-price and income demand elasticities calibration, introduce the settings of CDE system, and finally examine the implications of CDE regularity conditions on the calibration performance of the system.

2.1 Regularity and Flexibility of a Demand System

Let us denote a cost (or expenditure) function by $C(p, u)$ where p is a N -dimensional price vector and u is the utility. For C to be considered as well-behaved, $\partial C / \partial p$, which is the Hicksian demand vector $q(p, u)$, is nonnegative and homogeneous of degree zero in p , and $[\partial^2 C / \partial p_i \partial p_j]_{N \times N}$, which is the Slutsky matrix, is negative semi-definite (NSD).³ The intuition of a NSD Slutsky matrix is, for a given utility level u , when a good becomes more expensive, it will be replaced by other cheaper alternatives, and as a result, the cost increase with the new consumption bundle after the price increase will never exceed the cost increase when the bundle cannot be altered.

The Slutsky matrix $[\partial^2 C / \partial p_i \partial p_j]_{N \times N}$, or equivalently $[\partial q / \partial p]_{N \times N}$, is symmetric and each term of the matrix is:

$$\frac{\partial q_i(p, u)}{\partial p_j} = \frac{\partial x_i(p, w)}{\partial p_j} + \frac{\partial x_i(p, w)}{\partial w} x_j(p, w) \quad (1)$$

Equation (1) is the Slutsky equation, which decomposes the impacts of a price change on the uncompensated demand $x_i(p, w)$ into the income effect and substitution effect, where w is the income (or expenditure) level. With some algebra, the Slutsky equation can also be expressed as

$$\sigma_{ij}^c = \sigma_{ij}^m + \eta_i \theta_j \quad (2)$$

where σ_{ij}^c , σ_{ij}^m , η_i , and θ_j are compensated price elasticity of commodity i , uncompensated price elasticity of i , income elasticity of i , and expenditure share of j , respectively. If both sides of (2) are divided by θ_j , one can come up with a Slutsky matrix $[\sigma_{ij}]_{N \times N}$ in the form of Allen-Uzawa elasticity of substitution (AUES) (Allen and Hicks, 1934; Uzawa, 1962) with

$$\sigma_{ij} = \sigma_{ij}^m / \theta_j + \eta_i \quad (3)$$

It can be shown that $[\sigma_{ij}]_{N \times N}$ is also symmetric, and the matrix is NSD if and only if $[\partial q / \partial p]_{N \times N}$ is NSD. Therefore, a demand system is regular if and only if 1) the Slutsky matrix $[\sigma_{ij}]_{N \times N}$ is NSD; and 2) the Hicksian demand q is non-negative. For CGE modeling, it is

³ For example, see p.59 and p.933 in Mas-Colell et al. (1995).

necessary to ensure that the demand system is globally regular, i.e., it should remain regular everywhere in the domain of price. This is because the algorithm of the solver for finding equilibria may begin from an initial point of price and quantity combination that is far from the equilibrium levels, and in the process of solving the model, the algorithm might fail if the demand system is not globally regular, even the system is locally regular at the equilibrium points (Perroni and Rutherford, 1998).

Perroni and Rutherford (1995) defined a regular-flexible demand system as the one that is globally regular and can locally represent any valid configuration of compensated demands and the AUES matrix $[\sigma_{ij}]_{N \times N}$. While based on an inductive argument, Perroni and Rutherford proved that a demand system derived from a special version of the non-separable n-stage CES function is regular-flexible, in general, testing whether other demand systems are regular-flexible would need to identify the space of an AUES matrix first, which is beyond the scopes of their paper and the current research. This study, instead, will simply focus on, under a given expenditure share structure, the ability of a demand system in matching the observed own-price and income demand elasticities, which are usually of first-order importance in characterizing the model response, and are also the most ubiquitous data available for calibrating a demand system. In particular, this study will examine whether a global regular demand system under consideration is own-price and income flexible, i.e., if the system can be calibrated to $(\sigma_{ii}, \eta_i, \theta_i)$ consistent to any well-behaved cost function. Following this definition, for example, the demand system derived from a single-nest CES cost function is neither own-price nor income flexible. The settings of CDE and their implications on own-price and income flexibilities will be discussed below.

2.2 The CDE Demand System

Let us consider the expenditure function C with a price vector p and a Hicksian demand vector q , i.e., $c_0 = C(p_0, u) \equiv \{\min p_0 q_0 : f(q_0) \geq u\}$ where the subscript 0 denotes the benchmark condition. If the function is normalized by c_0 , it becomes $C(p_0/c_0, u) \equiv 1$. With this normalization, Hanoch (1975) proposes the expenditure function of a CDE demand system as follows:

$$C\left(\frac{p}{c_0}, u\right) = \sum_i \beta_i u^{e_i(1-\alpha_i)} \left(\frac{p_i}{c_0}\right)^{1-\alpha_i} \equiv 1 \quad (4)$$

where α_i and e_i are the substitution parameter and expansion parameter, respectively. In this setting, the utility u is only implicitly defined, and in general there is no reduced form representation for u . The Hicksian demand for commodity i based on this setting is:

$$q_i = \frac{\left[\beta_i u^{e_i(1-\alpha_i)} (1-\alpha_i) \left(\frac{p_i}{c_0}\right)^{-\alpha_i}\right]}{\sum_i \beta_i u^{e_i(1-\alpha_i)} (1-\alpha_i) \left(\frac{p_i}{c_0}\right)^{1-\alpha_i}} \quad (5)$$

For the CDE system, the substitution elasticity σ_{ij} in AUES form is presented in Equation (6), where the expenditure share is denoted by θ_i , and $\delta_{ij} = 1$ if $i = j$, otherwise $\delta_{ij} = 0$. The income elasticity of demand η_i is presented in Equation (7):

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_k \theta_k \alpha_k - \frac{\delta_{ij} \alpha_i}{\theta_i} \quad (6)$$

$$\eta_i = (\sum_k \theta_k e_k)^{-1} [e_i(1 - \alpha_i) + \sum_k \theta_k e_k \alpha_k] + (\alpha_i - \sum_k \theta_k \alpha_k) \quad (7)$$

It can be shown that the following aggregation conditions hold: the Cournot's aggregation $\sum_i \theta_i \sigma_{ij} = 0$ and the Engel's aggregation $\sum_i \theta_i \eta_i = 1$. Note that for each off-diagonal term, $\sigma_{ij} - \sigma_{ik} = \alpha_j - \alpha_k$ is invariant to i although σ_{ij} may vary, and therefore the system has a constant difference of (substitution) elasticities. The regularity condition for the system presented in Hanoch (1975) includes: $\beta_i \geq 0$; $e_i \geq 0$; $0 < \alpha_i < 1$ or $\alpha_i \geq 1 \forall i$ and $\alpha_i > 1$ for some $I \in i$. It is worth noting that with the regularity condition, each own-price elasticity of demand σ_{ii}^c is always negative. This is because from Equation (6) and $\sigma_{ii}^c = \sigma_{ii} \theta_i$, we have

$$\sigma_{ii}^c = -\alpha_i(1 - \theta_i)^2 - \theta_i \sum_{k|k \neq i} \theta_k \alpha_k \quad (8)$$

For a given vector of θ_i , the requirement that all α_i s should lie on the same side of one imposes a constraint in choosing the vector of α_i such that σ_{ii}^c can match the observed own-price demand elasticity. For instance, some sectors may have a very small expenditure share ($\theta_i \rightarrow 0$) and so for those sectors $\sigma_{ii}^c \rightarrow -\alpha_i$. However, if not all observed own-price elasticities of those sectors lie on the same side of one, it is impossible to match every single σ_{ii}^c with the targeted elasticity value no matter what regulatory condition on α_i is chosen. Therefore, the CDE system is not own-price flexible. Further, the requirement of $e_i \geq 0$ also suggests that some compromise has to be made in calibrating income elasticities of demand.

3. CALIBRATION, PERFORMANCE, AND IMPLEMENTATION

This section begins from the calibration approach of the CDE system, which is based on the three-step strategy presented in Huff et al. (1997) and Hertel et al. (1991) except for a minor revision. Next, while the CDE system is not own-price and income flexible, to understand under what circumstances the calibrated elasticities can better match the targeted elasticities, the section will examine the performance of CDE calibration both analytically and numerically. It will also demonstrate how to put the CDE system into GTAP8inGAMS and verify the model response is consistent to the calibrated elasticities.

3.1 Calibration

The calibration of a CDE system can be summarized by three steps below:

Step 1: *Calibrating the own-price elasticity of demand σ_{ii}^c*

Let us denote the targeted (or observed) own-price elasticity of demand by σ_{ii}^{ct} . The purpose of this step is to choose α_i so that the “distance” between the two vectors $[\sigma_{ii}^c]$ and $[\sigma_{ii}^{ct}]$ is minimized. Without explicitly considering the distance metric, Huff et al. (1997) minimizes $f(\sigma_{ii}^c) = \sum_i \sigma_{ii}^c [\ln(\sigma_{ii}^c / \sigma_{ii}^{ct}) - 1]$. However, since $(\partial^2 f / \partial (\sigma_{ii}^c)^2 = 1 / \sigma_{ii}^c < 0)$, f is concave in σ_{ii}^c , and, without considering any additional constraint, f will attain its maximum rather than minimum at $\sigma_{ii}^c = \sigma_{ii}^{ct}$ based on its first-order condition. Therefore, the minimization problem documented in Huff et al. seems to be erroneous. A quick fix to this would be to change the sign of the objective function so the function is convex in σ_{ii}^c , i.e., the objective function now

becomes $-\sum_i \sigma_{ii}^c [\ln(\sigma_{ii}^c / \sigma_{ii}^{ct}) - 1]$. In addition, this study also considers the objective function with alternative settings for the minimization problem:

$$\min_{\alpha_i} \sum_i \omega_i (\sigma_{ii}^c - \sigma_{ii}^{ct})^2 \text{ s. t. } \alpha_i \in (0, 1) \text{ or } \alpha_i \geq 1 \forall i \text{ and } \alpha_l > 1 \text{ for some } l \in i \quad (9)$$

where ω_i is the weight factor, and two settings considered here are $\omega_i = 1/N$ and $\omega_i = \theta_i$. The study will compare the performances of different settings in matching the targeted own-price demand elasticities.

Step 2: Calibrating the income elasticity of demand

Let us denote the targeted income elasticity of demand by η_i^t (η_i^t must satisfy the Engel's aggregation). Given α_i determined in the previous step, by choosing e_i , the goal is to calibrate η_i to η_i^t if possible. Similar to the idea of Step 1, the following problem is solved:

$$\min_{e_i | \alpha_i} \sum_i \omega_i (\eta_i - \eta_i^t)^2 \text{ s. t. } \sum_i \theta_i \eta_i = 1 \text{ and } (\eta_i - 1)(\eta_i^t - 1) > 0 \text{ for all } i \quad (10)$$

The condition $\sum_i \theta_i \eta_i = 1$ is to ensure the calibrated elasticities satisfy the Engel's aggregation, and following Huff et al. (1997), the second condition is to ensure the calibrated elasticities lie on the same side of one as the targeted values.

Step 3: Calibrating the scale coefficients holding the utility level equals one

With the calibrated α_i and e_i , and the normalization $u = 1$, $p_{0i} = 1$, and $q_{0i} = \theta_i$ (since $c_0 = \sum_i p_{0i} q_{0i} = 1$), the N scale parameters β_i can be solved by using (4) and (5):

$$\beta_i = \frac{q_{0i}}{1 - \alpha_i} / \sum_k \frac{q_{0k}}{1 - \alpha_k} \quad (11)$$

Because the calibration is done sequentially, how well the income elasticities of demand can be matched to targeted levels is also affected by the calibration of own-price demand elasticities. In the Appendix, the study provides the program for the three-step strategy. The program is written in GAMS, and each minimization problem in the program is formulated as a nonlinear programming (NLP) problem.

3.2 Performance

Before putting the system into a CGE model, two interesting questions are: under what circumstances the calibration becomes more accurate, and how well the targeted elasticities are represented? The analysis below will answer these questions.

Proposition 3.2.1:

The lower the expenditure share, the higher the influence of own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the higher the influence of other sectors' substitution parameters in determining the calibrated elasticity.

Proof:

Since $\sigma_{ii}^c = -\alpha_i(1 - \theta_i)^2 - \theta_i \sum_{k|k \neq i} \theta_k \alpha_k$, with a lower θ_i ($\theta_i \in (0,1)$), σ_{ii}^c depends more on the own-sector substitution parameter α_i , rather than the weighted average of other sectors' substitution parameters $\sum_{k|k \neq i} \theta_k \alpha_k$. In the extreme case with $\theta_i \rightarrow 0$, if the regularity condition is not violated, σ_{ii}^c can be matched to the targeted level σ_{ii}^{ct} by simply setting $\alpha_i = -\sigma_{ii}^{ct}$ since $\lim_{\theta_i \rightarrow 0} \sigma_{ii}^c = -\alpha_i$. On the other hand, with a higher θ_i , σ_{ii}^c depends more on the weighted average of other sectors' substitution parameters $\sum_{k|k \neq i} \theta_k \alpha_k$ rather than the own-sector substitution parameter α_i . In the extreme case with $\theta_i \rightarrow 1$, α_i has no control over σ_{ii}^c since $\lim_{\theta_i \rightarrow 1} \sigma_{ii}^c = -\sum_{k|k \neq i} \theta_k \alpha_k$.

Since the own-price elasticities of demand presented in GTAP 8 are between -1 and 0 , based on discussions above, considering the regularity condition with $\alpha_i \in (0,1)$ might produce more accurate calibration results for sectors with smaller expenditure shares. With a higher sectorial resolution, more commodities/sectors will have smaller expenditure shares, and thus having $\alpha_i \in (0,1)$ would make it possible for producing a better match between calibrated and target levels for each individual sector.

Proposition 3.2.2:

When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a higher level is less likely to violate $e_i \geq 0$, which is part of the regularity condition. On the other hand, when $\alpha_i \geq 1 \forall i$ and $\alpha_I > 1$ for some $I \in i$, calibrating the elasticity to a lower level is less likely to violate $e_i \geq 0$.

Proof:

From Equation (7), $e_i = \{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\} / (1 - \alpha_i)$. When $\alpha_i \in (0, 1)$, a positive numerator for the equation above is needed to ensure $e_i \geq 0$. Therefore, other things being equal, with a higher calibrated income elasticity of demand, the numerator is less likely to become negative. Similarly, for $\alpha_i \geq 1 \forall i$ and $\alpha_I > 1$, a lower calibrated η_i is less likely to violate $e_i \geq 0$.

If one considers $\alpha_i \in (0, 1)$, the second proposition suggests that matching the targeted income elasticities for the demand of agricultural products might be trickier since in general these products tend to have lower income elasticity values, and as a result the calibrated income elasticities for these products might end up with levels higher than the targeted numbers. Nevertheless, the values of α_i determined in Step 1 of the calibration procedure may also affect how well the targeted income elasticities of demand are met, as will be explored in the next proposition.

Proposition 3.2.3:

When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a targeted level is less likely to violate $e_i \geq 0$ with a smaller α_i . On the other hand, when $\alpha_i \geq 1 \forall i$ and $\alpha_I > 1$ for some $I \in i$, calibrating the elasticity to the targeted level is less likely to violate $e_i \geq 0$ with a larger α_i .

Proof:

This can be verified by $e_i = \{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\} / (1 - \alpha_i)$.

Continuing our previous example for commodities with low income elasticities of demand and with $\alpha_i \in (0, 1)$, while Proposition 3.2.2 says that for given values of α_i , it is harder to calibrate the income elasticity of demand to a lower value, Proposition 3.2.3 argues that if the calibrated α_i is small enough, it is still possible to calibrate the income elasticity of demand to the targeted (low) level.

Proposition 3.2.4:

Commodities with substitution parameters α_i close to one will have similar calibrated income elasticities of demand.

Proof:

From Equation (7), $\lim_{\alpha_i \rightarrow 1} \eta_i = \sum_k \theta_k e_k \alpha_k / \sum_k \theta_k e_k + 1 - \sum_k \theta_k \alpha_k = \lim_{\alpha_j \rightarrow 1} \eta_j$.

Proposition 3.2.4 shows that the calibrated α_i may work against the calibration for income elasticities of demand. For instance, if there are two commodities with α_i and α_j both approaching unity, according to the proposition, the calibrated income elasticities of demand η_i and η_j will be very close to each other, even their targeted values η_i^t and η_j^t are quite different.

In the following, the study presents two calibration exercises using the GTAP 8 database. In the first example, the 57 sectors of GTAP 8 are combined into 3 sectors: agriculture (AGRI), manufacture (MANU), and service (SERV), and in the second example, all 57 sectors of GTAP 8 are kept. For simplicity, both examples aggregate the 129 regions of GTAP 8 into a single region. Six settings are considered in each exercise:

- **LATW** (the low- α and θ -weighted setting): the considered regularity condition for α_i is $\alpha_i \in (0, 1)$, while the weight factor w_i of (9) (the objective function of own-price demand elasticity calibration) and (10) (the objective function for calibrating the income elasticity of demand) equals θ_i .
- **LAEW** (the low- α and equal-weighted setting): $\alpha_i \in (0, 1)$ but now an equal weight setting ($w_i = 1/N$) for calibrating both own-price and income elasticities of demand is considered.
- **HATW** (the high- α and θ -weighted setting): the regularity condition of α_i is: $\alpha_i \geq 1 \forall i$ and $\alpha_I > 1$ for some $I \in i$, while the considered weight w_i is θ_i .
- **HAEW** (the high- α and equal-weighted setting): similar to HATW except for an equal weight setting ($w_i = 1/N$).
- **HUFO** (the setting of Huff et al.): the original strategy documented in Huff et al. (1997), which minimizes $\sum_i \sigma_{ii}^c [\ln(\sigma_{ii}^c / \sigma_{ii}^{ct}) - 1]$ for calibrating own-price elasticities of demand.

The approach for calibrating income elasticities of demand is the same as LATW, i.e., solving for the minimization of (10) with $\alpha_i \in (0, 1)$ and the weight parameter θ_i .

- **HUFR** (the revised setting of Huff et al.): everything is the same as HUFO except for the objective function for own-price demand elasticities calibration, which now becomes $-\sum_i \sigma_{ii}^c [\ln(\sigma_{ii}^c / \sigma_{ii}^{ct}) - 1]$.

To assess the calibration performance for each type of elasticity, in addition to a one-by-one comparison between calibrated and targeted numbers for each commodity, it is informative to have an index for measuring how far the point of calibrated elasticities is from the point of targeted elasticities as follows:

$$d = \sqrt{\sum_{i=1}^N w_i \cdot (x_i - x_i^t)^2} \quad (12)$$

Depending on the type of elasticity evaluated, x_i in Equations (11) and (12) could be either the own-price elasticity of demand σ_{ii}^c or the income elasticity of demand η_i , while the superscript x_i^t denotes targeted value.

Table 1 shows that, with only three highly aggregated sectors, the expenditure share on each commodity is nontrivial. In this case, based on Proposition 3.2.1, the substitution parameter of that commodity will only play a limited role in determining the calibrated own-price demand elasticity, and this explains why in general, the calibrated own-price demand elasticities are quite off from their targeted values regardless of various settings.

Secondly, consistent to discussions following Proposition 3.2.1, with “low- α ” settings (LATW and LAEW), calibrated own-price demand elasticities for commodities with smaller expenditure shares are closer to their targeted values, compared to the case with “high- α ” settings (HATW and HAEW). For instance, under the LAEW setting, the calibrated own-price elasticity for AGRI (−0.469) is much closer to the targeted value (−0.429) than the calibrated elasticity (−0.891) under the HAEW setting.

Thirdly, since the “theta-weighted” settings (LATW and HATW) put a highest weight on SERV in the objective function, the calibrated own-price elasticity for SERV (e.g. −0.342 with LATW) will be somewhat closer to the targeted value (−0.766) compared to results with “equal-weighted” settings (e.g. −0.326 with LAEW)—at the expense of worse calibration performance for commodities with lower expenditure shares.⁴ Under the HUFO setting, the calibration for own-price demand elasticities is least satisfactory, as minimizing a concave function results in boundary solutions for the substitution parameters and own-price demand elasticities. With the concavity issue of HUFO fixed, HUFR is able to produce results close to those “low- α settings” especially LAEW.

Finally, Table 1 shows that in this 3-sector example, the performances of calibrating own-price demand elasticities are mediocre under all settings. In particular, except for HUFO, the calibrated income elasticity levels for AGRI under various settings are close to one—much higher than the targeted value (0.730). This is because matching a low η_i^t without a smaller α_i

⁴ Similar observations hold for the results between HATW and HAEW.

turns out to be tricky, as explained by Propositions 3.2.2 and 3.2.3. In addition, with the “high- α ” settings, the calibrated income elasticities for MANU are less accurate (0.913 with HATW and 0.921 with HATW as opposed to the target level 1.000), and the calibrated values are essentially the same as those for AGRI. This is because the calibrated α_i for AGRI and MANU are both close to one, and from Proposition 3.2.4, the calibrated income elasticities for the two commodities will be similar. Besides, HUFO accidentally produces a perfect match in income elasticities of demand for a wrong reason—the result is based on a faulty own-price demand calibration that generates almost-zero substitution parameters, which favor the calibration for income elasticities of demand according to Proposition 3.2.3. Results from HUFR are similar to LAEW as they share similar calibrated substitution parameters.

Table 1. Performance of the CDE calibration: The 3-sector Case

Setting	Weight		LATW	LAEW	HATW	HAEW	HUFO	HUFR
Sector	Range		θ_i	$1/N$	θ_i	$1/N$		
	θ_i	σ_{ii}^{ct}	$\alpha_i \in (0,1)$	$\alpha_i \in (0,1)$	$\alpha_i \geq 1$	$\alpha_i \geq 1$	$\alpha_i \in (0,1)$	$\alpha_i \in (0,1)$
AGRI	0.118	-0.429	-0.642	-0.469	-0.992	-0.891	-0.001	-0.464
MANU	0.248	-0.665	-0.742	-0.736	-0.982	-0.772	-0.001	-0.736
SERV	0.634	-0.766	-0.342	-0.326	-0.562	-0.382	-0.0004	-0.325
		Distance: EW	0.277	0.258	0.391	0.352	0.635	0.258
		Distance: TW	0.347	0.353	0.298	0.348	0.709	0.353
		η_i^t	η_i					
AGRI		0.730	0.992	0.992	0.913	0.921	0.730	0.992
MANU		1.000	1.000	1.000	0.913	0.921	1.000	1.000
SERV		1.050	1.001	1.001	1.050	1.045	1.050	1.001
		Distance: EW	0.154	0.154	0.117	0.119	0.000	0.154
		Distance: TW	0.098	0.098	0.076	0.076	0.000	0.098
		α_i						
AGRI			0.692	0.469	1.000	1.000	0.001	0.463
MANU			0.999	0.999	1.000	1.000	0.001	0.999
SERV			0.999	0.999	2.465	1.124	0.001	0.999
		e_i						
AGRI			35.022	1.654	1.000	1.000	1.000	1.630
MANU			0.000	0.000	0.000	0.000	1.370	0.000
SERV			50.277	2.343	0.251	0.000	1.439	2.308
		β_i						
AGRI			4.329E-04	2.515E-04	3.221E-01	3.221E-01	1.178E-01	2.487E-04
MANU			2.809E-01	2.809E-01	6.779E-01	6.779E-01	2.479E-01	2.809E-01
SERV			7.187E-01	7.188E-01	1.184E-06	1.396E-05	6.343E-01	7.188E-01

For the distance measure, “EW” and “TW” use the setting $w_i = 1/N$ and $w_i = \theta_i$ in Equation (12), respectively.

In contrast to the highly aggregated three-sector example discussed previously, **Table 2** and **Table 3** present the calibration performance for the own-price and income elasticities of demand under the 57-sector setting, respectively. Table 2 shows that, with the higher sectorial resolution, the expenditure shares become much smaller than the three-sector case, with the largest share being the expenditure on the commodity (service) of trade (17.19%)—a much smaller number compared to the share of SERV in the three-sector example (63.43%).⁵

⁵ The sector SERV in the three-sector example includes the trade sector of GTAP 8, which includes: all retail sales, wholesale trade and commission trade, hotels and restaurants, repairs of motor vehicles and personal and household goods, and retail sale of automotive fuel. See GTAP (2015).

The results in Table 2 reveal that with the “low- α settings” (LATW and LAEW), the calibrated own-price elasticities of demand match the targeted values perfectly! The reason is because the much smaller expenditure share of each sector makes that sector’s substitution parameter α_i become the dominant factor in determining the desired elasticity level as $\lim_{\theta_{ii} \rightarrow 0} \sigma_{ii}^c = -\alpha_i$, as discussed in Proposition 3.2.1. In addition, since the own-price demand elasticities presented in GTAP 8 are between -1 and 0 , the setting with $\alpha_i \in (0, 1)$ in LATW and LAEW is consistent to the underlying data and is therefore less likely to become a binding constraint. The comparison between **Table 3** and Table 2 reveals that for most sectors, the substitution parameters are very close to the calibrated own-price demand elasticities, except for the trade sector where the expenditure share is much larger than others.

Since with the higher sectorial resolution, $-\alpha_i$ will be close to the own-price demand elasticity σ_{ii}^c , the requirement of $\alpha_i \geq 1$ in the “high- α settings” (HATW and HAEW) is not coherent to the targeted elasticity levels, and this explains why in Table 2, results for HATW and HAEW are less satisfactory. Table 3 also demonstrates that the HUFO setting results in corner solutions for the substitution parameters, and that is why with this setting the calibrated own-price demand elasticities shown in Table 2 tend to approach either 0 or -1 . On the contrary, with the concavity issue resolved, HUFR also allows a perfect match between calibrated and targeted points.

Table 4 presents results for the income elasticity calibration. Similar to the own-price demand elasticity calibration, LATW, LAEW, and HUFR are able to match the targeted income demand elasticity perfectly. In particular, the table shows that two-third of the sectors have targeted income elasticities no less than 0.9 , and more than half of these sectors have calibrated substitution parameters no greater than 0.74 . Propositions 3.2.2 and 3.2.3 explain why the income elasticities calibration may produce a perfect match. Let us take the sector “crop – not else classified” for instance. Even Table 4 shows that it has the lowest targeted income demand elasticity (0.27), which may be trickier to match based on Proposition 3.2.2, the substitution parameter of this sector (see Table 3) is close to pretty small (0.07) and almost hit the lower bound, which works in favor of a perfect matching according to Proposition 3.2.3. For other settings the results are much less satisfactory—with HATW and HAEW all calibrated substitution parameters share the same value (1.000), and because of that, the calibrated income demand elasticities are the same for all commodities (see Table 4), which can be explained by Proposition 3.2.4, and HUFO is again plagued by the concavity issue that results in a corner solution point for the substitution parameters. More precisely, a calibrated substitution parameter α_i that hits its upper bound clearly works against matching the calibrated income demand elasticity to its targeted level, as suggested by Proposition 3.2.3.

The two exercises with different sectorial resolution suggest that the CDE system can be calibrated to the targeted elasticities more accurately when there are more sectors, lower own-price demand elasticities (so the substitution parameters are lower, which helps the calibration for income demand elasticities), and higher targeted income demand elasticities. However, higher sectorial resolution, of course, will pose other challenges to CGE applications—more data

are needed to parameterize the model, and solving it becomes much more computer-resource-intensive. Lastly, for demonstration purposes, while results from HUFO are presented for the two exercises, HUFO is an erroneous setting and will not be discussed in the following CGE application.

Table 2. Performance of the CDE calibration (Own-price elasticity of demand): The 57-sector Case

Setting			LATW	LAEW	HATW	HAEW	HUFO	HUFR
	Weight		θ_i	$1/N$	θ_i	$1/N$		
Sector	Range		(0,1)	(0,1)	$\alpha_i \geq 1$	$\alpha_i \geq 1$	(0,1)	(0,1)
	θ_i	σ_{ii}^{ct}	σ_{ii}^c					
air transport	0.0068	-0.651	-0.651	-0.651	-0.993	-0.993	-0.001	-0.651
beverages and tobacco products	0.0264	-0.559	-0.559	-0.559	-0.974	-0.974	-0.002	-0.559
sugar cane - sugar beet	0.0002	-0.091	-0.091	-0.091	-1.000	-1.000	-0.999	-0.091
communication	0.0258	-0.694	-0.694	-0.694	-0.974	-0.974	-0.002	-0.694
bo meat products	0.0077	-0.518	-0.518	-0.518	-0.992	-0.992	-0.001	-0.518
construction	0.0034	-0.648	-0.648	-0.648	-0.997	-0.997	-0.001	-0.648
coal	0.0002	-0.416	-0.416	-0.416	-1.000	-1.000	-0.999	-0.416
chemical - rubber - plastic products	0.0279	-0.686	-0.686	-0.686	-0.972	-0.972	-0.002	-0.686
bo horses	0.0008	-0.275	-0.275	-0.275	-0.999	-0.999	-0.997	-0.275
ownership of dwellings	0.0930	-0.769	-0.769	-0.769	-0.907	-0.907	-0.005	-0.769
electronic equipment	0.0120	-0.694	-0.694	-0.694	-0.988	-0.988	-0.002	-0.694
electricity	0.0190	-0.656	-0.656	-0.656	-0.981	-0.981	-0.002	-0.656
metal products	0.0030	-0.677	-0.677	-0.677	-0.997	-0.997	-0.001	-0.677
forestry	0.0010	-0.485	-0.485	-0.485	-0.999	-0.999	-0.997	-0.485
fishing	0.0029	-0.351	-0.351	-0.351	-0.997	-0.997	-0.993	-0.351
gas	0.0008	-0.680	-0.680	-0.680	-0.999	-0.999	-0.001	-0.680
gas manufacture - distribution	0.0027	-0.698	-0.698	-0.698	-0.997	-0.997	-0.001	-0.698
cereal grains nec	0.0015	-0.112	-0.112	-0.112	-0.998	-0.998	-0.996	-0.112
ferrous metals	0.0003	-0.609	-0.608	-0.609	-1.000	-1.000	-0.001	-0.609
insurance	0.0259	-0.759	-0.759	-0.759	-0.974	-0.974	-0.002	-0.759
leather products	0.0074	-0.604	-0.604	-0.604	-0.993	-0.993	-0.001	-0.604
wood products	0.0033	-0.696	-0.696	-0.696	-0.997	-0.997	-0.001	-0.696
dairy products	0.0132	-0.507	-0.507	-0.507	-0.987	-0.987	-0.002	-0.507
motor vehicles and parts	0.0328	-0.741	-0.741	-0.741	-0.967	-0.967	-0.002	-0.741
metals nec	0.0003	-0.672	-0.672	-0.672	-1.000	-1.000	-0.001	-0.672
mineral products nec	0.0028	-0.650	-0.650	-0.650	-0.997	-0.997	-0.001	-0.650
animal products nec	0.0044	-0.316	-0.316	-0.316	-0.996	-0.996	-0.990	-0.316
business services nec	0.0699	-0.809	-0.809	-0.809	-0.930	-0.930	-0.004	-0.809
crops nec	0.0024	-0.071	-0.071	-0.071	-0.998	-0.998	-0.994	-0.071
food products nec	0.0389	-0.551	-0.551	-0.551	-0.961	-0.961	-0.003	-0.551
financial services nec	0.0350	-0.802	-0.802	-0.802	-0.965	-0.965	-0.002	-0.802
oil	0.0000	-0.547	-0.547	-0.547	-1.000	-1.000	-0.001	-0.547
machinery and equipment nec	0.0163	-0.721	-0.721	-0.721	-0.984	-0.984	-0.002	-0.721
manufactures nec	0.0162	-0.724	-0.724	-0.724	-0.984	-0.984	-0.002	-0.724
minerals nec	0.0001	-0.563	-0.563	-0.563	-1.000	-1.000	-0.001	-0.563
meat products	0.0104	-0.512	-0.512	-0.512	-0.990	-0.990	-0.001	-0.512
oil seeds	0.0005	-0.101	-0.101	-0.101	-0.999	-0.999	-0.998	-0.101
public admin & defence - edu- health	0.1051	-0.780	-0.780	-0.780	-0.895	-0.895	-0.005	-0.780
transport equipment nec	0.0046	-0.668	-0.668	-0.668	-0.995	-0.995	-0.001	-0.668
transport nec	0.0325	-0.614	-0.614	-0.614	-0.967	-0.967	-0.002	-0.614
petroleum - coal products	0.0274	-0.646	-0.646	-0.646	-0.973	-0.973	-0.002	-0.646
processed rice	0.0035	-0.106	-0.106	-0.106	-0.997	-0.997	-0.992	-0.106
paddy rice	0.0003	-0.129	-0.129	-0.129	-1.000	-1.000	-0.998	-0.129
plant-based fibers	0.0005	-0.419	-0.419	-0.419	-0.999	-0.999	-0.998	-0.419
paper products - publishing	0.0115	-0.738	-0.738	-0.738	-0.988	-0.988	-0.001	-0.738
raw milk	0.0030	-0.276	-0.276	-0.276	-0.997	-0.997	-0.993	-0.276
recreational and other services	0.0666	-0.760	-0.760	-0.760	-0.933	-0.933	-0.004	-0.760
sugar	0.0025	-0.364	-0.364	-0.364	-0.998	-0.998	-0.994	-0.364
textiles	0.0096	-0.595	-0.595	-0.595	-0.990	-0.990	-0.001	-0.595
trade	0.1719	-0.779	-0.779	-0.779	-0.828	-0.828	-0.008	-0.779
vegetables - fruit - nuts	0.0145	-0.115	-0.115	-0.115	-0.986	-0.986	-0.971	-0.115
vegetable oils and fats	0.0037	-0.312	-0.312	-0.312	-0.996	-0.996	-0.992	-0.312
wearing apparel	0.0202	-0.640	-0.640	-0.640	-0.980	-0.980	-0.002	-0.640
wheat	0.0011	-0.085	-0.085	-0.085	-0.999	-0.999	-0.997	-0.085
wool - silk-worm cocoons	0.0002	-0.265	-0.265	-0.265	-1.000	-1.000	-0.999	-0.265
water transport	0.0017	-0.576	-0.576	-0.576	-0.998	-0.998	-0.001	-0.576
water	0.0043	-0.696	-0.696	-0.696	-0.996	-0.996	-0.001	-0.696
Distance: EW			0.000	0.000	0.511	0.511	0.700	0.000
Distance: TW			0.000	0.000	0.286	0.286	0.727	

* For the distance measure, “EW” and “TW” use the setting $w_i = 1/N$ and $w_i = \theta_i$ in Equation (12), respectively.

Table 3. Substitution parameters in the CDE calibration: The 57-sector Case

Setting	LATW	LAEW	HATW	HAEW	HUFO	HUFR
	θ_i	$1/N$	θ_i	$1/N$		
Sector	$\alpha_i \in (0,1)$	$\alpha_i \in (0,1)$	$\alpha_i \geq 1$	$\alpha_i \geq 1$	$\alpha_i \in (0,1)$	$\alpha_i \in (0,1)$
	α_i					
air transport	0.654	0.654	1.000	1.000	0.001	0.654
beverages and tobacco products	0.569	0.569	1.000	1.000	0.001	0.569
sugar cane - sugar beet	0.091	0.091	1.000	1.000	0.999	0.091
communication	0.711	0.711	1.000	1.000	0.001	0.711
bo meat products	0.520	0.520	1.000	1.000	0.001	0.520
construction	0.650	0.650	1.000	1.000	0.001	0.650
coal	0.416	0.416	1.000	1.000	0.999	0.416
chemical - rubber - plastic products	0.704	0.704	1.000	1.000	0.001	0.704
bo horses	0.275	0.275	1.000	1.000	0.999	0.275
ownership of dwellings	0.857	0.857	1.000	1.000	0.001	0.857
electronic equipment	0.701	0.701	1.000	1.000	0.001	0.701
electricity	0.667	0.667	1.000	1.000	0.001	0.667
metal products	0.679	0.679	1.000	1.000	0.001	0.679
forestry	0.485	0.485	1.000	1.000	0.999	0.485
fishing	0.350	0.350	1.000	1.000	0.999	0.350
gas	0.681	0.681	1.000	1.000	0.001	0.681
gas manufacture - distribution	0.700	0.700	1.000	1.000	0.001	0.700
cereal grains nec	0.112	0.112	1.000	1.000	0.999	0.112
ferrous metals	0.609	0.609	1.000	1.000	0.001	0.609
insurance	0.779	0.779	1.000	1.000	0.001	0.779
leather products	0.608	0.608	1.000	1.000	0.001	0.608
wood products	0.698	0.698	1.000	1.000	0.001	0.698
dairy products	0.511	0.511	1.000	1.000	0.001	0.511
motor vehicles and parts	0.766	0.766	1.000	1.000	0.001	0.766
metals nec	0.672	0.672	1.000	1.000	0.001	0.672
mineral products nec	0.651	0.651	1.000	1.000	0.001	0.651
animal products nec	0.315	0.315	1.000	1.000	0.999	0.315
business services nec	0.878	0.878	1.000	1.000	0.001	0.878
crops nec	0.070	0.070	1.000	1.000	0.999	0.070
food products nec	0.565	0.565	1.000	1.000	0.001	0.565
financial services nec	0.833	0.833	1.000	1.000	0.001	0.833
oil	0.547	0.547	1.000	1.000	0.001	0.547
machinery and equipment nec	0.732	0.732	1.000	1.000	0.001	0.732
manufactures nec	0.735	0.735	1.000	1.000	0.001	0.735
minerals nec	0.563	0.563	1.000	1.000	0.001	0.563
meat products	0.515	0.515	1.000	1.000	0.001	0.515
oil seeds	0.101	0.101	1.000	1.000	0.999	0.101
public admin & defence - education- health	0.886	0.886	1.000	1.000	0.001	0.886
transport equipment nec	0.670	0.670	1.000	1.000	0.001	0.670
transport nec	0.630	0.630	1.000	1.000	0.001	0.630
petroleum - coal products	0.661	0.661	1.000	1.000	0.001	0.661
processed rice	0.104	0.104	1.000	1.000	0.999	0.104
paddy rice	0.129	0.129	1.000	1.000	0.999	0.129
plant-based fibers	0.419	0.419	1.000	1.000	0.999	0.419
paper products - publishing	0.747	0.747	1.000	1.000	0.001	0.747
raw milk	0.275	0.275	1.000	1.000	0.999	0.275
recreational and other services	0.818	0.818	1.000	1.000	0.001	0.818
sugar	0.364	0.364	1.000	1.000	0.999	0.364
textiles	0.600	0.600	1.000	1.000	0.001	0.600
trade	0.986	0.986	1.000	1.000	0.001	0.986
vegetables - fruit - nuts	0.107	0.107	1.000	1.000	0.999	0.107
vegetable oils and fats	0.312	0.312	1.000	1.000	0.999	0.312
wearing apparel	0.651	0.651	1.000	1.000	0.001	0.651
wheat	0.084	0.084	1.000	1.000	0.999	0.084
wool - silk-worm cocoons	0.265	0.265	1.000	1.000	0.999	0.265
water transport	0.576	0.576	1.000	1.000	0.001	0.576
water	0.699	0.699	1.000	1.000	0.001	0.699

Table 4. Performance of the CDE calibration (Income elasticity of demand): The 57-sector Case

Setting	Weight		LATW	LAEW	HATW	HAEW	HUFO	HUFR
Sector	θ_i	η_i^i	θ_i	$1/N$	θ_i	$1/N$	$(0,1)$	$(0,1)$
	Range		(0,1)	(0,1)	$\alpha_i \geq 1$	$\alpha_i \geq 1$		
air transport	0.0068	0.995	0.995	0.995	1.000	1.000	0.976	0.995
beverages and tobacco products	0.0264	0.832	0.832	0.832	1.000	1.000	0.813	0.832
sugar cane - sugar beet	0.0002	0.370	0.370	0.370	1.000	1.000	0.980	0.370
communication	0.0258	0.991	0.991	0.991	1.000	1.000	0.972	0.991
bo meat products	0.0077	0.809	0.809	0.809	1.000	1.000	0.790	0.809
construction	0.0034	1.036	1.036	1.036	1.000	1.000	1.017	1.036
coal	0.0002	1.057	1.057	1.057	1.000	1.000	1.000	1.057
chemical - rubber - plastic products	0.0279	1.044	1.044	1.044	1.000	1.000	1.025	1.044
bo horses	0.0008	0.877	0.877	0.877	1.000	1.000	0.980	0.877
ownership of dwellings	0.0930	1.034	1.034	1.034	1.000	1.000	1.015	1.034
electronic equipment	0.0120	1.040	1.040	1.040	1.000	1.000	1.021	1.040
electricity	0.0190	1.039	1.039	1.039	1.000	1.000	1.020	1.039
metal products	0.0030	1.046	1.046	1.046	1.000	1.000	1.027	1.046
forestry	0.0010	1.057	1.057	1.057	1.000	1.000	1.000	1.057
fishing	0.0029	0.838	0.838	0.838	1.000	1.000	0.980	0.838
gas	0.0008	1.036	1.036	1.036	1.000	1.000	1.016	1.036
gas manufacture - distribution	0.0027	1.030	1.030	1.030	1.000	1.000	1.011	1.030
cereal grains nec	0.0015	0.437	0.437	0.437	1.000	1.000	0.980	0.437
ferrous metals	0.0003	1.086	1.086	1.086	1.000	1.000	1.067	1.086
insurance	0.0259	1.018	1.018	1.018	1.000	1.000	1.000	1.018
leather products	0.0074	0.952	0.952	0.952	1.000	1.000	0.933	0.952
wood products	0.0033	1.043	1.043	1.043	1.000	1.000	1.024	1.043
dairy products	0.0132	0.820	0.820	0.820	1.000	1.000	0.801	0.820
motor vehicles and parts	0.0328	1.035	1.035	1.035	1.000	1.000	1.016	1.035
metals nec	0.0003	1.065	1.065	1.065	1.000	1.000	1.046	1.065
mineral products nec	0.0028	1.048	1.048	1.048	1.000	1.000	1.029	1.048
animal products nec	0.0044	0.824	0.824	0.824	1.000	1.000	0.980	0.824
business services nec	0.0699	1.119	1.119	1.119	1.000	1.000	1.100	1.119
crops nec	0.0024	0.270	0.270	0.270	1.000	1.000	0.980	0.270
food products nec	0.0389	0.825	0.825	0.825	1.000	1.000	0.806	0.825
financial services nec	0.0350	1.118	1.118	1.118	1.000	1.000	1.099	1.118
oil	0.0000	1.015	1.015	1.015	1.000	1.000	1.014	1.015
machinery and equipment nec	0.0163	1.036	1.036	1.036	1.000	1.000	1.017	1.036
manufactures nec	0.0162	1.036	1.036	1.036	1.000	1.000	1.017	1.036
minerals nec	0.0001	1.078	1.078	1.078	1.000	1.000	1.058	1.078
meat products	0.0104	0.818	0.818	0.818	1.000	1.000	0.799	0.818
oil seeds	0.0005	0.460	0.460	0.460	1.000	1.000	0.980	0.460
public admin & defence - edu- health	0.1051	1.031	1.031	1.031	1.000	1.000	1.012	1.031
transport equipment nec	0.0046	0.994	0.994	0.994	1.000	1.000	0.975	0.994
transport nec	0.0325	0.998	0.998	0.998	1.000	1.000	0.979	0.998
petroleum - coal products	0.0274	0.997	0.997	0.997	1.000	1.000	0.977	0.997
processed rice	0.0035	0.385	0.385	0.385	1.000	1.000	0.980	0.385
paddy rice	0.0003	0.566	0.566	0.566	1.000	1.000	0.980	0.566
plant-based fibers	0.0005	0.948	0.948	0.948	1.000	1.000	0.980	0.948
paper products - publishing	0.0115	1.034	1.034	1.034	1.000	1.000	1.014	1.034
raw milk	0.0030	0.837	0.837	0.837	1.000	1.000	0.980	0.837
recreational and other services	0.0666	1.032	1.032	1.032	1.000	1.000	1.013	1.032
sugar	0.0025	0.762	0.762	0.762	1.000	1.000	0.980	0.762
textiles	0.0096	0.959	0.959	0.959	1.000	1.000	0.940	0.959
trade	0.1719	1.062	1.062	1.062	1.000	1.000	1.043	1.062
vegetables - fruit - nuts	0.0145	0.328	0.328	0.328	1.000	1.000	0.980	0.328
vegetable oils and fats	0.0037	0.743	0.743	0.743	1.000	1.000	0.980	0.743
wearing apparel	0.0202	0.957	0.957	0.957	1.000	1.000	0.938	0.957
wheat	0.0011	0.283	0.283	0.283	1.000	1.000	0.980	0.283
wool - silk-worm cocoons	0.0002	0.944	0.944	0.944	1.000	1.000	0.980	0.944
water transport	0.0017	1.006	1.006	1.006	1.000	1.000	1.000	1.006
water	0.0043	1.029	1.029	1.029	1.000	1.000	1.010	1.029
Distance: EW			0.000	0.000	0.249	0.249	0.232	0.000
Distance: TW			0.000	0.000	0.130	0.130	0.104	

* For the distance measure, “EW” and “TW” use the setting $w_i = 1/N$ and $w_i = \theta_i$ in Equation (12), respectively.

3.3 Implementation

With the calibrated parameters, the study demonstrates how to put the CDE system into the multi-region and multi-sector CGE model of GTAP8inGAMS. The original model is constructed based on CES technologies for both production and final consumption. It includes a series of mixed complementary problems (MCP) (Mathiesen, 1985; Rutherford, 1995; Ferris and Peng, 1997) written in MPSGE, a subsystem of GAMS (Rutherford, 1999). To implement the CDE system, the CES expenditure function is dropped, and by declaring auxiliary variables and equations in MPSGE to formulate relevant MCP problems, three sets of conditions below are incorporated into the revised model:

- **The equation for total expenditure.** The total expenditure c for purchasing one unit of utility (Equation (4)) is added into the model to form a MCP problem with a complementarity variable c . Note that in Equation (4), c is only implicitly defined. The purpose of this problem is to determine c jointly with other conditions. As previously mentioned, in the benchmark, both the utility level and price indices of commodities are normalized to unity.
- **The equation for final demand.** The equation for final demand (Equation (5)) is coupled with its complementarity variable, the activity level of final demand, to form a MCP problem. The problem is incorporated into the model to solve for the final demand of each commodity.
- **The zero profit condition for utility.** Let us denote the marginal cost and marginal revenue of utility (i.e., price of utility) by mcu and pu , respectively.⁶ The zero profit condition of utility and the activity level of utility compose another MCP problem:

$$mcu \geq pu; u \geq 0; (mcu - pu) \cdot u = 0; mcu = \frac{c \sum_i \beta_i e_i (1 - \alpha_i) u^{e_i(1 - \alpha_i) - 1} \left(\frac{p_i}{c}\right)^{1 - \alpha_i}}{\sum_i \beta_i (1 - \alpha_i) u^{e_i(1 - \alpha_i)} \left(\frac{p_i}{c}\right)^{1 - \alpha_i}} \quad (13)$$

Condition (13) states that in equilibrium, if the supply of utility u is positive, the marginal cost of utility mcu must equal the marginal revenue pu , and if mcu is higher than pu in equilibrium, u must be zero.

With the commodity price being a complementarity variable, the market clearing condition of each commodity is also formulated as a MCP problem by comparing the commodity supply (determined by its zero profit condition) with the final demand shown above plus the intermediate demand derived from a CES cost function as the original GTAP8inGAMS. Similarly, with the price of utility being the complementarity variable, the supply of utility combined with the demand for utility ($income/pu$) make up the MCP problem for the market clearing condition of utility. The model code is provided in the Appendix, and interested readers may refer to Rutherford (1999) and Markusen (2013) for details of MPSGE. Let us now consider a 2-region (USA and the rest of the world (ROW)) and 11-sector setting. The sectorial setting presented in **Table 5** is similar to that of the MIT EPPA6 model (Chen et al., 2015)

⁶ mcu in Condition (13) can be derived by taking the total derivative of Equation (4) with respect to u and c at a given commodity price vector.

except for the fossil fuel sectors of the EPPA6, which are now aggregated into a single one for simplicity.⁷

Table 5. Sectors in this study

Sector	Details
CROP	Agriculture – Crops
LIVE	Agriculture – Livestock
FORS	Agriculture – Forestry
FOOD	Food Products
FOSL	Fossil Fuels
ELEC	Electricity
EINT	Energy-Intensive Industries
OTHR	Other Industries
DWE	Ownership of Dwellings
SERV	Services
TRAN	Transport

Table 6 presents the calibration performance under the following three strategies: LAEW, HAEW, and HUFR. The θ -weighted settings are dropped since they tend to significantly reduce the accuracy of calibrated elasticities for commodities with smaller expenditure shares, but only marginally improve the elasticity matches for higher expenditure commodities. Table 6 shows that for both regions, results under LAEW and HUFR are both better than HAEW and are very close to each other. One can also find that under LAEW and HUFR, almost all targeted elasticity values (σ_{ii}^{ct} and η_i^t) are well-matched, except for the own-price elasticities for SERV demands in both regions. The study chooses results based on LAEW to calibrate the CDE system in the general equilibrium model, as LAEW produces a slightly better match for the own-price demand elasticity of food sector for the U.S., the focused sector and region of this exercise. Finally, all primary factors are aggregated into a single one and the price for the aggregated primary factor in the U.S. is chosen as the numeraire. These treatments facilitate the analysis of income effect.

⁷ For the US, the base year final consumption levels of coal, gas, crude oil, and refined oil products, which are separately identified in EPPA6, are 0.0007, 30.7722, 0.00002, and 191.5950 billion US\$, respectively. For the CDE application, aggregating these commodities/sectors into a single one avoids numerical issues caused by the extremely uneven final consumption distribution of these commodities.

Table 6. Performance of the CDE calibration: The 11-sector Case

Setting			LAEW	HAEW	HUFR		LAEW	HAEW	HUFR
		Weight	1/N	1/N			1/N	1/N	
Sector		Range	(0,1)	$\alpha_i \geq 1$	(0,1)		(0,1)	$\alpha_i \geq 1$	(0,1)
Region: USA		θ_i	σ_{ii}^{ct}	σ_{ii}^c		η_i^t	η_i		
crop	0.005	-0.008	-0.010	-0.999	-0.008	0.014	0.018	1.000	0.017
live	0.001	-0.738	-0.739	-1.000	-0.739	0.899	0.899	1.000	0.899
fors	0.000	-0.831	-0.831	-1.000	-0.831	1.010	1.010	1.000	1.010
food	0.054	-0.747	-0.765	-0.989	-0.770	0.906	0.907	1.000	0.907
eint	0.035	-0.833	-0.844	-0.993	-0.849	1.010	1.011	1.000	1.011
fosl	0.022	-0.818	-0.824	-0.995	-0.827	0.991	0.991	1.000	0.991
elec	0.013	-0.828	-0.832	-0.997	-0.834	1.005	1.005	1.000	1.005
othr	0.101	-0.825	-0.862	-0.980	-0.879	1.001	1.001	1.000	1.001
tran	0.015	-0.815	-0.819	-0.997	-0.821	0.989	0.989	1.000	0.989
serv	0.610	-0.855	-0.869	-0.988	-0.371	1.014	1.014	1.000	1.013
dwe	0.142	-0.849	-0.852	-0.971	-0.853	1.010	1.012	1.000	1.013
Distance: EW			0.130	0.312	0.130		0.001	0.266	0.001
Region: ROW		θ_i	σ_{ii}^{ct}	σ_{ii}^c		η_i^t	η_i		
crop	0.028	-0.124	-0.127	-0.972	-0.125	0.376	0.376	1.000	0.376
live	0.016	-0.298	-0.300	-0.984	-0.299	0.835	0.835	1.000	0.835
fors	0.001	-0.432	-0.432	-0.999	-0.432	1.065	1.065	1.000	1.065
food	0.128	-0.472	-0.493	-0.872	-0.489	0.788	0.788	1.000	0.788
eint	0.050	-0.655	-0.661	-0.950	-0.662	1.052	1.052	1.000	1.052
fosl	0.035	-0.605	-0.609	-0.965	-0.609	1.004	1.004	1.000	1.004
elec	0.021	-0.610	-0.613	-0.979	-0.613	1.048	1.048	1.000	1.048
othr	0.143	-0.650	-0.673	-0.857	-0.676	1.015	1.015	1.000	1.015
tran	0.052	-0.595	-0.602	-0.948	-0.602	0.999	0.999	1.000	0.999
serv	0.453	-0.733	-0.455	-0.547	-0.455	1.082	1.082	1.000	1.082
dwe	0.072	-0.703	-0.713	-0.928	-0.715	1.055	1.055	1.000	1.055
Distance: EW			0.075	0.403	0.075		0.000	0.185	0.000

For the distance measure, “EW” is the setting with $w_i = 1/N$ in Equation (12).

With the CDE system in the CGE model, the study will test under given price or income shocks, if the model behaviors are consistent to the underlying calibrated elasticities. Taking the shock on food price in the U.S. as an example, the first exercise changes the cost of final consumption for food in the U.S. exogenously to create the considered price shock.⁸ The goal is to calculate the uncompensated (Marshallian) average own-price elasticity of food demand based on the model response, and see if the realized elasticity levels are consistent to their calibrated counterparts.

It is worth noting that while the targeted own-price demand elasticity is $\sigma_{ii}^{ct} = -0.747$, the calibrated own-price demand elasticity for food $\sigma_{ii}^c = -0.765$, which again an evidence that shows the CDE system is not own-price flexible—although in this particular example the calibrated value is only 2.4% off from the targeted one. Also, since it is more convenient to derive an uncompensated average elasticity from outputs of the CGE model under a nontrivial price shock, for comparison purposes, the study will convert the calibrated elasticity σ_{ii}^c , which is an input to the CGE model and is a compensated point elasticity, into an uncompensated average elasticity.

The calibrated uncompensated own-price elasticity for food, $\sigma_{ii}^m = -0.814$ (a point elasticity), can be derived from σ_{ii}^c , η_i , and θ_i based on the Slutsky equation (see Equation (2)). Let us consider the quantity index $\tilde{q}_i = q_i/\theta_i$ with the benchmark level $\tilde{q}_{0i} = 1$ since $q_{0i} = \theta_i$

⁸ For instance, in the revised GTAP8inGAMS model, a 10% increase in food price is achieved by multiplying both vdfm(“food”, c, “usa”) and vifm(“food”, c, “usa”) by 1.1.

(see Step 3 in Section 3.1). Because the percentage change in $\tilde{q}_i$ is equivalent to the percentage change in q_i , $\tilde{q}_i$ can replace q_i in deriving the average uncompensated (Marshallian) elasticity σ_{ii}^{ma} —with both price and quantity indices normalized to unity, σ_{ii}^{ma} can be expressed as:

$$\sigma_{ii}^{ma} = \frac{p_i^{\sigma_{ii}^m} - 1}{p_i - 1} \cdot \frac{p_{i+1}}{p_i^{\sigma_{ii}^m} + 1} ; p_i \text{ is the after-shock price level} \quad (14)$$

With various food price shocks, the values for σ_{ii}^{ma} (the calibrated average Marshallian elasticity) and the realized average elasticity levels σ_{ii}^{mar} (derived from the model output) are both presented in **Figure 1**. Note that with the exogenous food price shocks, the new equilibrium of the model may also observe changes in prices of other commodities relative to their no-shock levels, and this will in turn affect the equilibrium food consumption level due to the existence of cross-price elasticities of food demand. Similarly, the exogenous price shock may also incur an income effect as reflected by the change in total (final) expenditure level. Therefore, to calculate σ_{ii}^{mar} , the observed food consumption index $\tilde{q}_i$ is adjusted such that it is net of the cross-price and income effects. The result in Figure 1 shows that, as expected, the larger the price shock, the more the average elasticity deviates from the point elasticity $\sigma_{ii}^m = -0.814$, which is the calibrated level without any price shock in the figure. More importantly, Figure 1 verifies that the uncompensated average food demand elasticity calculated from the model output replicates its calibrated counterpart.



Figure 1. Own-price elasticity of food demand in the U.S.: calibrated vs. realized.

In the following exercise, the study examines the model response under various income shocks in the U.S. The shocks are created by changing the quantity of the aggregated primary factor of the U.S., which is just the real GDP level of the U.S. Since GDP is not only spent on private consumption, to calculate the income elasticities of various commodities based on the model response, instead of using the percentage change in GDP as the denominator of the

elasticity, one needs to use the percentage change in the portion of income dedicated to private consumption, or equivalently, the percentage change in total expenditure on private consumption. Similar to the price shock example, the larger the income shock, the farther the deviation of the average income elasticity η_i^a from the point income elasticity η_i . Following the same logic as Equation (14), the average income elasticity can be written as:

$$\eta_i^a = \frac{c^{\eta_i-1}}{c-1} \cdot \frac{c+1}{c^{\eta_i+1}} ; c \text{ is the after-shock income level} \quad (15)$$

Under various levels of income shock, Equation (15) is used to convert the calibrated point elasticity into the calibrated average elasticity, which serves as the benchmark for the comparison between the realized average elasticity from model outputs and the calibrated level the model is given. Finally, as the previous example, the new equilibrium with an income shock, in general, will accompany changes in price levels of various commodities, which means that the observed consumption levels will be contaminated by changes in prices, although these changes are usually small. The study accounts for this price effect and removes it out of the observed consumption levels, and then for each commodity, use the percentage change of the adjusted consumption level as the numerator of the income elasticity. The results in **Figure 2** demonstrate that for the final consumption of food, the realized average income elasticity levels again replicate their calibrated counterparts. The two exercises presented here can be extended to other sectors. For instance, the comparison between the calibrated and realized average income elasticities for other sectors are presented in **Figure A1** in the Appendix.

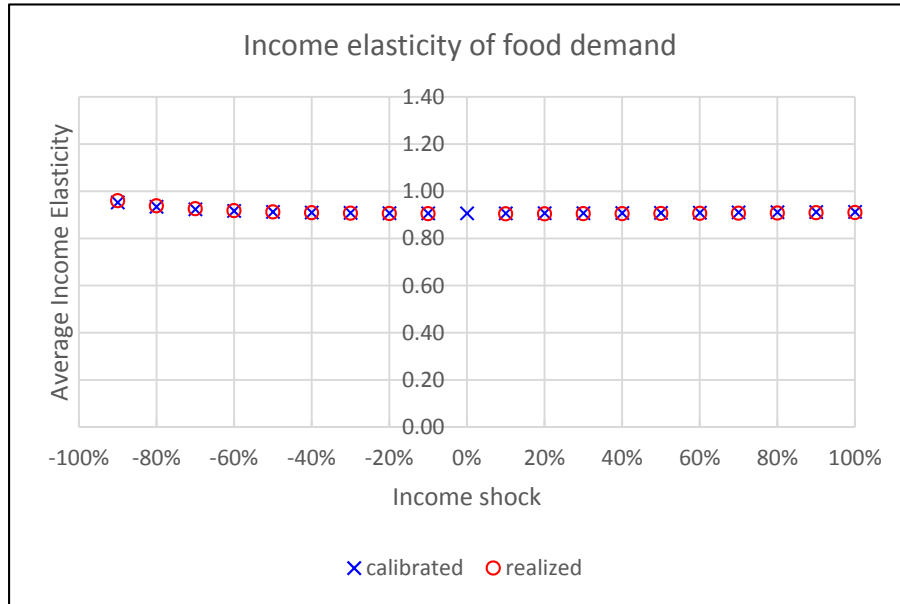


Figure 2. Income elasticity of food demand in the U.S.: calibrated vs. realized.

4. CONCLUSION

This is the first paper to explore for a CDE demand system, under what circumstances the calibrated own-price and income elasticities of demand can be matched to their targeted levels more accurately. It finds that while the system is neither own-price nor income flexible, the calibration performance improves with a higher sectorial resolution, lower targeted own-price demand elasticities, and higher targeted income demand elasticities. In any case, to understand the extent the elasticity targets are correctly represented in a CGE model, it is crucial to disclose how well the calibrated elasticities match their targeted counterparts. In addition, using GTAP8inGAMS, the study also provides the first example of incorporating the CDE system into a global CGE model written in MPSGE, which has not been presented before. Finally, price and income shocks are imposed on the revised GTAP8inGAMS with the CDE system, and the model responses successfully replicate the calibrated elasticities of the demand system.

Future studies may inspect if other CGE applications with the CDE demand can produce results consistent to the calibrated elasticities, or they may investigate the flexibility and calibration performance of other demand systems, as these issues are still rarely studied so far but are essential for reasons discussed in this research.

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APPENDIX

The CDE calibration program⁹

```

                                cdecalib
$title Calibrate a CDE Demand System using GTAP data
$ontext
Consider the following implicit function that defines the utility u:
G(z,u) = sum(i,BETA(i)*u**(e(i)*(1-ALPHA(i)))*z(i)**(1-ALPHA(i))) = 1 where z(i) = p0(i)/c0
p0 = benchmark price index, and c0 is the benchmark expenditure level

Allen partial elasticities of substitution:
sigma(i,j) = ALPHA(i)+ALPHA(j)-sum(k,theta(k)*ALPHA(k))-delta(i,j)*ALPHA(i)/theta(i);

Income elasticities:
eta(i) = (sum(k,theta(k)*e(k))**(-1))*(e(i)*(1-ALPHA(i))+sum(k,theta(k)*e(k)*ALPHA(k)))
        + (ALPHA(i)-sum(k,theta(k)*ALPHA(k)));

The upper and lower bounds for ALPHA, e, and BETA come from the CDE regulatory conditions:
substitution coefficient ALPHA(i) in (0,1) for all i (or ALPHA > 1 for all i)
expansion parameter e(i) >= 0
scale parameter BETA(i) >= 0
References: Takeda (2012); Hertel et al. (1997); Hanoch (1975)
Equations must be declared with static sets, but they can be assigned with dynamic sets.
YHC: 11/26/2014
$offtext

$if not set ds $set ds eppaf_02
$if not set datadir $set datadir .\input\
$if not set wt $set wt 1
$include gtap8data

set      info      Information about this calibration /
        ds          "%ds%",
        datadir     "%datadir%",
        workdir     "%gams.workdir%"
        date        "%system.date%"
        time        "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
z(i,r)      normalized price
theta(i,r)  value share in final demand
vafm(i,r)   Aggregate final demand,
delta(i,j,r) diagonal-one off-diagonal-zero
sigma(i,j,r) Allen partial elasticity of substitution
epsilon_(i,r) targeted own-price elasticity of demand
eta_(i,r)   targeted income elasticity of demand
p0(i,r)     benchmark price index
q0(i,r)     benchmark consumption level
c0(r)       expenditure level
mc0(r)      marginal cost when u is one
weight(i,r) weight for the square distance
beta(i,r)   scale coefficient

;

vafm(i,r) = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up.";

epsilon_(i,r) = epsilon(i,r);
eta_(i,r) = eta(i,r);

p0(i,r) = 1;
q0(i,r) = theta(i,r)/p0(i,r);
c0(r) = sum(i,p0(i,r)*q0(i,r));

```

Page 1

⁹ The program “cdecalib.gms” written in GAMS implements the three-step procedure for calibrating the CDE system. To run it, one needs: 1) the GTAP 8 data in the gdx format (created by GTAP8inGAMS) with desired resolutions for regions, sectors, and production factors; and 2) the subroutine “gtap8data.gms,” which is presented in GTAP8inGAMS, that reads data for parameters needed in the calibration program. To calibrate the system using the database “eppaf_02.gdx,” under the DOS command prompt, type “gams cdecalib”. The environment variable “ds” can overwrite the default database setting (eppaf_02.gdx), and another environment variable “wt” with a default value of 1 controls the weight ω_i presented in Section 3.1 ($wt=0 \leftrightarrow \omega_i=\theta_i$; $wt=1 \leftrightarrow \omega_i=1/n$).

```

                                cdecalib
delta(i,j,r)                  = 0;
delta(i,j,r)$sameas(i,j) = 1;
weight(i,r)                   = theta(i,r)$(%wt% eq 0) + (1/card(j))$(%wt% ne 0);

*      Finish reading data
*      -----

variables
ALPHA(i,r)    substitution coefficient
V(i,r)        own-price elasticity of demand
E(i,r)        expansion coefficient
ETAV(i,r)     income elasticity of demand
*BETA(i,r)    scale coefficient
OBJONE        objective value for own-price elasticity calibration
OBJTWO        objective value for income elasticity calibration
OBJTHR        objective value for the dummy
OBJFOR        objective value for the dummy
U(r)          utility
;

*      The equation "e_engel" deals with the case where eta from data doesn't satisfy the Engel aggregation

equations
e_v(i,r)      for v
e_eta(i,r)    for ETAV
e_objone      for OBJONE
e_objtwo      for OBJTWO
e_objthr      for OBJTHR
e_objfor      for OBJFOR
e_engel(r)    Engel aggregation
e_etaside(i,r) ensure eta & eta_ lies on the same side of one
e_exp(r)      expenditure function
e_dfn(i,r)    compensated demand
;

*      Step 1: Calibrating to the own-price elasticity of demand (only an approximation due to regulatory
conditions)

e_v(i,rr) ..
V(i,rr)$theta(i,rr) =e= theta(i,rr)*(2*ALPHA(i,rr)-sum(k,theta(k,rr)*ALPHA(k,rr)))-ALPHA(i,rr);

e_objone ..
OBJONE =e= sum((i,rr),weight(i,rr)*(V(i,rr)-epsilon_(i,rr))*(V(i,rr)-epsilon_(i,rr)));

model demandelas / e_v, e_objone /;

loop(r,
rr(r)      = yes;
ALPHA.L(i,rr) = 0.5;
ALPHA.UP(i,rr) = 0.999;
ALPHA.LO(i,rr) = 0.0001;

V.L(i,rr)   = epsilon_(i,rr);
OBJONE.L    = 0;
solve demandelas using nlp minimizing OBJONE;
sigma(i,j,r)$theta(i,r) =
ALPHA.L(i,r)+ALPHA.L(j,r)-sum(k,theta(k,r)*ALPHA.L(k,r))-delta(i,j,r)*ALPHA.L(i,r)/theta(i,r);
rr(r)      = no;
);

*      Step 2: Calibrating the income elasticity of demand

e_eta(i,rr) ..
ETAV(i,rr) =e=
(1/sum(k,theta(k,rr)*E(k,rr)))*(E(i,rr)*(1-ALPHA.L(i,rr))+sum(k,theta(k,rr)*E(k,rr)*ALPHA.L(k,rr))
+ (ALPHA.L(i,rr)-sum(k,theta(k,rr)*ALPHA.L(k,rr)));

```

Page 2

```

cdecalib

e_objtwo ..
OBJTWO =e= sum((i,rr),weight(i,rr)*(ETAV(i,rr)-eta_(i,rr))*(ETAV(i,rr)-eta_(i,rr)));

e_engel(rr) ..
sum(i,theta(i,rr)*ETAV(i,rr)) =e= 1;

e_etaside(i,rr) ..
(ETAV(i,rr)-1)*(eta_(i,rr)-1) =g= 0;

model incomeelas /e_objtwo, e_engel, e_eta, e_etaside/;

loop(r,
  rr(r)      = yes;
  E.L0(i,rr) = 0.0;
  E.L(i,rr)  = 1;
  ETAV.L(i,rr) = eta_(i,r);

  OBJTWO.L   = 0;
  solve incomeelas using nlp minimizing OBJTWO;
  rr(r)      = no;
);

*      Step 3: Calibrating the scale coefficient BETA holding the utility level equals one

beta(i,r) = (q0(i,r)/(1-ALPHA.L(i,r)))/sum(j,q0(j,r)/(1-ALPHA.L(j,r)));
U.FX(r) = 1;

parameter  epsilonv00(i,r)  EPSILONV solved by the CDE calibration routine,
          etav00(i,r)      ETAV solved by the CDE calibration routine
          alpha00(i,r)     ALPHA solved by the CDE calibration routine
          e00(i,r)         E solved by the CDE calibration routine
          u00(r)           U solved by the CDE calibration routine
          beta00(i,r)      beta solved by the CDE calibration routine
          mc00(r)          Marginal cost
;

epsilonv00(i,r) = V.L(i,r);
etav00(i,r)     = ETAV.L(i,r);
alpha00(i,r)    = ALPHA.L(i,r);
e00(i,r)        = E.L(i,r);
u00(r)          = U.L(r);
beta00(i,r)     = beta(i,r);

mc00(r) =

c0(r)*sum(i,beta(i,r)*E.L(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-ALPHA.L(i,r))-1))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r))))

/sum(i,beta(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-ALPHA.L(i,r)))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r))));

parameter data model output;
data(i,r,"theta")      = theta(i,r);
data(i,r,"epsilon")    = epsilon_(i,r);
data(i,r,"epsilonv00") = epsilonv00(i,r);
data(i,r,"eta")        = eta_(i,r);
data(i,r,"etav")       = ETAV.L(i,r);
data(i,r,"alpha")      = ALPHA.L(i,r);
data(i,r,"e")          = E.L(i,r);
data(i,r,"beta")       = beta(i,r);
data("mc",r,"mc")      = mc00(r);
data(i,r,"weight")     = weight(i,r);

execute_unload ".\output\cdecalib_%ds%_wt=%wt%.gdx";

```

The revised CGE model with the CDE demand system for GTAP8inGAMS¹⁰

```

                                mrtmge_cde
$title  Read GTAP8 Basedata and Replicate the Benchmark in MPSGE

* The following pre-assignment for ds will be used in a $gdxin command in gtap8data.gms
*if not set ds $set ds iea_001
*if not set ds $set ds eppaf_02

$if not set wt $set wt 1

* Sets, parameters declarations and assignments are done in gtap8data.gms (YHC: 20120614)
$include ..\build\gtap8data

set c(g) private consumption /c/;
set e(g) exogenous consumption /g, i/;

parameters
  esub(g)      Top-level elasticity in demand /C 1/
  vcm(i,c,r)   Tax included Armington good i for private consumption,
  data(*,*,*)  Output from cdecilib,
  cde          CDE calibration,
  chkd(i,r)    Check final expenditure D,
  priexp(r)    Total private expenditure;

* Aggregate final demand (Armington good)
vcm(i,c,r) = vdfm(i,c,r)*(1+rtfd0(i,c,r))+vifm(i,c,r)*(1+rtfi0(i,c,r));

* Read the CDE coefficients
execute_load "..\input\cdecilib_%ds%_wt%_gdx" data = data;
cde(i,r,"alpha") = data(i,r,"alpha");
cde(i,r,"e")      = data(i,r,"e");
cde(i,r,"beta")   = data(i,r,"beta");
cde("utility",r,"u") = 1;
cde("mc",r,"mc")  = data("mc",r,"mc");

cde(i,r,"alpha")$(cde(i,r,"alpha") eq eps) = 0;
cde(i,r,"e")$(cde(i,r,"e") eq eps)         = 0;
cde(i,r,"beta")$(cde(i,r,"beta") eq eps)    = 0;
cde("utility",r,"u")$(cde("utility",r,"u") eq eps) = 0;
cde("mc",r,"mc")$(cde("mc",r,"mc") eq eps)  = 0;

$ontext
$model:gtap8

$sectors:
  y(g,r)$ (not c(g) and vom(g,r)) ! Supply
  m(i,r)$vim(i,r)                 ! Imports
  yt(j)$vtw(j)                    ! Transportation services
  ft(f,r)$ (sf(f) and evom(f,r)) ! Specific factor transformation
  yc(i,c,r)$vcm(i,c,r)            ! Private consumption by commodity

$commodities:
  p(g,r)$vom(g,r)                 ! Domestic output price
  pm(j,r)$vim(j,r)                ! Import price
  pt(j)$vtw(j)                    ! Transportation services
  pf(f,r)$evom(f,r)               ! Primary factors rent
  ps(f,g,r)$ (sf(f) and vfm(f,g,r)) ! Sector-specific primary factors
  pc(i,c,r)$vcm(i,c,r)            ! Private consumption price

$consumers:
  ra(r)                           ! Representative agent

$auxiliary:
  TC(r)                           ! Expenditure for the CDE system
  U(r)                            ! Activity level of Utility
  D(i,r)$vcm(i,"c",r)             ! Activity level of final consumption

* Sectoral output

```

Page 1

¹⁰ This program “mrtmge_cde.gms” written in MPSGE is the CGE model with a CDE system. It needs to be placed inside the subdirectory “model” of GTAP8inGAMS. To run it, under the DOS command prompt, type “gams mrtmge_cde”. The environment variables “ds” can overwrite the default database setting (eppaf_02.gdx), “wt” with a default value of 1 controls the weight ω_i presented in Section 3.1 ($wt=0 \leftrightarrow \omega_i=\theta_i$; $wt=1 \leftrightarrow \omega_i=1/n$), and “step” with a default value of 0 controls the shock of the counterfactual simulation.

```

mrtmge_cde
$prod:y(j,r)$vom(j,r) s:esub(j) i.tl:esubd(i) va:esubva(j)
o:p(j,r) q:vom(j,r) a:ra(r) t:rto(j,r)
i:p(i,r) q:vdfm(i,j,r) p:(1+rtfd0(i,j,r)) i.tl: a:ra(r) t:rtfd(i,j,r)
i:pm(i,r) q:vifm(i,j,r) p:(1+rtfi0(i,j,r)) i.tl: a:ra(r) t:rtfi(i,j,r)
i:ps(sf,j,r) q:vfm(sf,j,r) p:(1+rtf0(sf,j,r)) va: a:ra(r) t:rtf(sf,j,r)
i:pf(mf,r) q:vfm(mf,j,r) p:(1+rtf0(mf,j,r)) va: a:ra(r) t:rtf(mf,j,r)

* Government consumption and investment (exogenous consumption)
$prod:y(e,r)$vom(e,r) s:esub(e) i.tl:esubd(i)
o:p(e,r) q:vom(e,r) a:ra(r) t:rto(e,r)
i:p(i,r) q:vdfm(i,e,r) p:(1+rtfd0(i,e,r)) i.tl: a:ra(r) t:rtfd(i,e,r)
i:pm(i,r) q:vifm(i,e,r) p:(1+rtfi0(i,e,r)) i.tl: a:ra(r) t:rtfi(i,e,r)

* Private consumption: new
* Level 1: Armington good of commodity i
$prod:yc(i,c,r)$vcm(i,c,r) s:esubd(i)
o:pc(i,c,r) q:vcm(i,c,r) a:ra(r) t:rto(c,r)
i:p(i,r) q:vdfm(i,c,r) p:(1+rtfd0(i,c,r)) a:ra(r) t:rtfd(i,c,r)
i:pm(i,r) q:vifm(i,c,r) p:(1+rtfi0(i,c,r)) a:ra(r) t:rtfi(i,c,r)

* Level 2: Aggregate various goods to a single consumption good c. This is where we need to work on for CDE.
* Let's temporarily remove the declaration of y(c,r), and move the sources and sinks in this block to demand
block.
* This strategy is similar to linking the top-down and bottom-up.
* Now this is moved to the representative agent block.

$prod:yt(j)$vtw(j) s:l
o:pt(j) q:vtw(j)
i:p(j,r) q:vst(j,r)

$prod:m(i,r)$vim(i,r) s:esubm(i) s.tl:0
o:pm(i,r) q:vim(i,r)
i:p(i,s) q:vxmd(i,s,r) p:pvxmd(i,s,r) s.tl: a:ra(s) t:(-rtxs(i,s,r)) a:ra(r)
t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
i:pt(j)#(s) q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl: a:ra(r) t:rtms(i,s,r)

$prod:ft(sf,r)$vom(sf,r) t:etrae(sf)
o:ps(sf,j,r) q:vfm(sf,j,r)
i:pf(sf,r) q:evom(sf,r)

$demand:ra(r)
d:p("c",r) q:vom("c",r)
e:p("c",rnum) q:vb(r)
e:p("g",r) q:(-vom("g",r))
e:p("i",r) q:(-vom("i",r))
e:pf(f,r) q:evom(f,r)
e:p(c,r) q:vom(c,r) r:U(r)
e:pc(i,c,r) q:(-vcm(i,c,r)) r:D(i,r)

$constraint:TC(r)
sum(i,cde(i,r,"beta")*(cde("utility",r,"u")*U(r))**((cde(i,r,"e")*(1-cde(i,r,"alpha")))*
(PC(i,"c",r)/TC(r))**((1-cde(i,r,"alpha")))) =e= 1;

$constraint:U(r)
TC(r)*sum(i,cde(i,r,"beta")*(cde(i,r,"e")*(1-cde(i,r,"alpha"))*(U(r)**((cde(i,r,"e")*(1-cde(i,r,"alpha"))-1))*
(PC(i,"c",r)/TC(r))**((1-cde(i,r,"alpha"))))
=e=
data("mc",r,"mc")*P("c",r)*sum(i,cde(i,r,"beta")*(1-cde(i,r,"alpha"))*(U(r)**((cde(i,r,"e")*(1-cde(i,r,"alpha")
))))*(PC(i,"c",r)/TC(r))**((1-cde(i,r,"alpha"))));

$constraint:D(i,r)$vcm(i,"c",r))
vcm(i,"c",r)/vom("c",r)*D(i,r)*sum(j,cde(j,r,"beta")*(U(r)**((1-cde(j,r,"alpha"))*(cde(j,r,"e"))*(1-cde(j,r,"
alpha"))*(PC(j,"c",r)/TC(r))**((1-cde(j,r,"alpha"))))
=e=

```

```

mrtmge_cde
(cde(i,r,"beta"))*(U(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e"))*(1-cde(i,r,"alpha"))*(pc(i,"c",r)/TC(r))**(-cde(
i,r,"alpha"))));

$offtext

$sysinclude mpsgeset gtap8

TC.L(r) = 1;
TC.L0(r) = 0.000001;
U.L(r) = 1;
U.L0(r) = 0.000001;
D.L(i,r) = 1;
D.L0(i,r) = 0.000001;
P.FX("c","usa") = 1;
*PF.FX("primary","usa") = 1;

gtap8.workspace = 128;
gtap8.iterlim = 0;
gtap8.optfile = 1;
$include gtap8.gen
solve gtap8 using mcp;

chkd(i,r) = vcm(i,"c",r)/vom("c",r)*D.L(i,r)
-
(cde(i,r,"beta"))*(U.L(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e"))*(1-cde(i,r,"alpha"))*(PC.L(i,"c",r)/TC.L(r))**
(-cde(i,r,"alpha"))))

/sum(j,cde(j,r,"beta"))*(U.L(r)**((1-cde(j,r,"alpha"))*cde(j,r,"e"))*(1-cde(j,r,"alpha"))*PC.L(j,"c",r)**(1-c
de(j,r,"alpha"))*TC.L(r)*cde(i,r,"alpha")));

execute_unload ".\output\mrtmge_cde_ref_ds=%ds%.gdx";

*$exit

$if not set step $set step 0
set t time period t /1*1/
parameter counter, step;
counter = 1;
step = %step%;

loop(t,

* Consumer's price shock:
*vdfm("food",c,"usa") = vdfm("food",c,"usa")*(counter+ord(t)*step);
*vifm("food",c,"usa") = vifm("food",c,"usa")*(counter+ord(t)*step);

* Endowment shock:
evom(f,"usa") = evom(f,"usa")*(counter+ord(t)*step);

counter = counter + ord(t)*step;

gtap8.iterlim = 50000;
gtap8.workspace = 50;
gtap8.savepoint = 1;
$if exist "GTAP8_p.gdx" execute_loadpoint 'GTAP8_p.gdx';
$include gtap8.gen
solve gtap8 using mcp;
priexp(r) = sum(i,PC.L(i,"c",r)*D.L(i,r)*vcm(i,"c",r));

*execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_priceshock=%step%.gdx";
*execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_incomeshock=%step%.gdx";
*execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_incomeshock.gdx";
execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_incomeshock=%step%_priexp=%pri%.gdx";

);

```

Income elasticities of demand: calibrated vs. realized

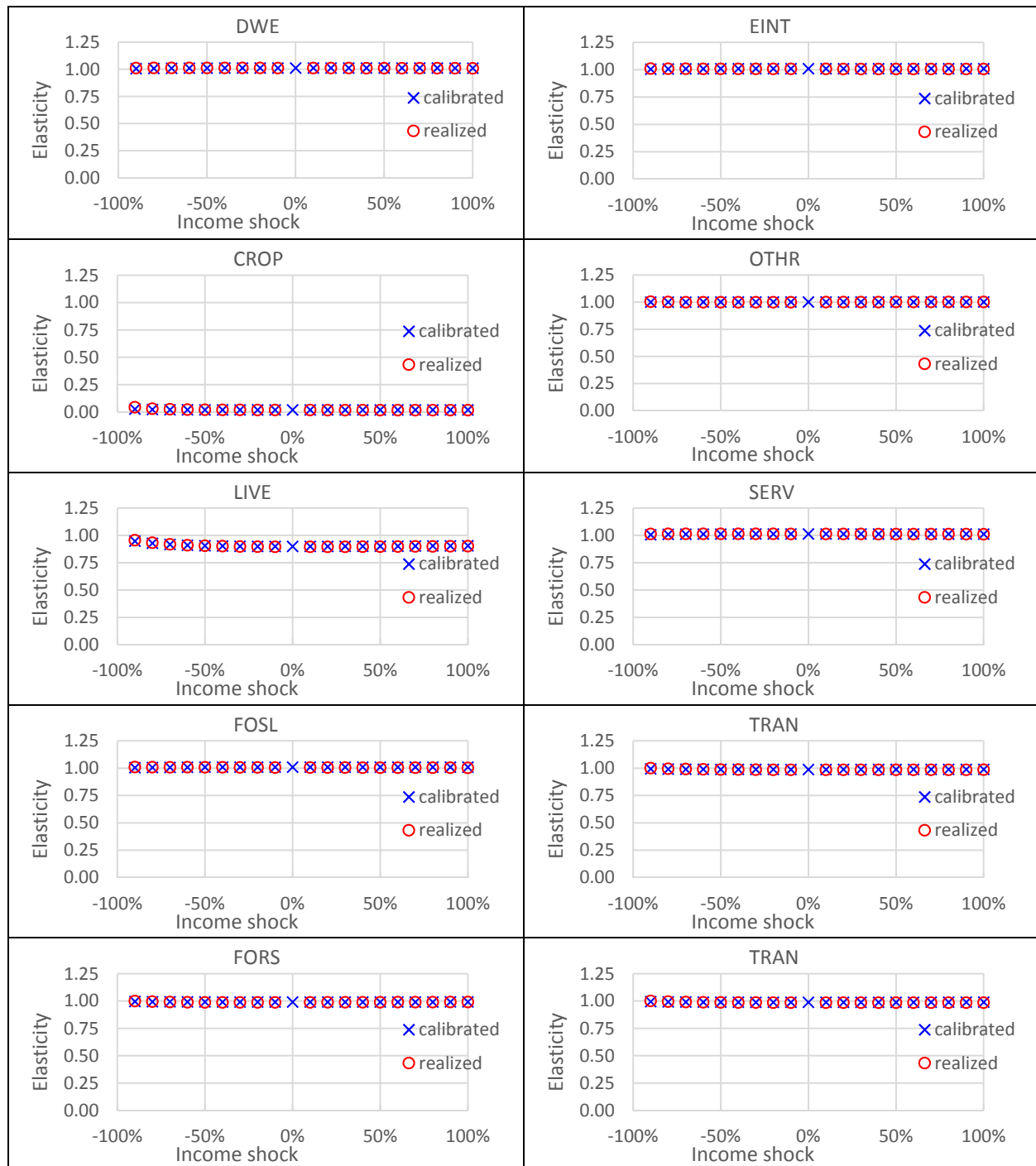


Figure A1. Income elasticities of demand for other sectors: calibrated vs. realized.