Incorporating Modern Trade Theory into CGE Models*

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ABSTRACT: We propose a parsimonious way to incorporate three workhorse models in the modern trade literature into the computable general equilibrium model (CGEs) GTAP. Furthermore we estimate (a part of) the parameters in the model structurally. We show that the Ethier-Krugman monopolistic competition model, and the Melitz firm heterogeneity model can be defined as an Armington model with generalized marginal costs, generalized trade costs, and a demand externality. As already known in the literature in both the Ethier-Krugman model and the Melitz model generalized marginal costs are a function of the amount of factor input bundles. In the Melitz model generalized marginal costs are also a function of the price of the factor input bundles. Lower factor prices raise the number of firms that can enter the market profitably (extensive margin), reducing generalized marginal costs of a representative firm. For the same reason the Melitz model features a demand externality: in a larger market more firms can enter. The Eaton-Kortum model deviates from the Armington model because aggregate industry prices do not vary by origin in this model. We implement the different models in the CGE model GTAP. We estimate the most important parameter in the different models, the trade cost elasticity, employing a structurally derived gravity equation. The simulations show that the largest welfare gains are generated in varying models, depending upon the type of trade liberalization and the number of sectors which are modelled as Ethier-Krugman or Melitz. Effects are largest in the Ethier-Krugman model under nationwide trade liberalization and all sectors modelled as either Ethier-Krugman, Melitz or Armington. With only some sectors modelled as Melitz or Ethier-Krugman, the largest welfare effects are produced in the Melitz model. We also provide a detailed and intuitive description of incorporation in the GEMPACK-code of the three different models.

Keywords: Firm Heterogeneity, CGE Model, Demand Externality
JEL codes: F12, F14

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ABSTRACT: We propose a way to incorporate the four workhorse models in the modern trade literature into computable general equilibrium models (CGEs). We show that the Ethier-Krugman monopolistic competition model, the Melitz firm heterogeneity model and the Eaton and Kortum model can be defined as an Armington model with generalized marginal costs, generalized trade costs and a demand externality. As already known in the literature in both the Ethier-Krugman model and the Melitz model generalized marginal costs are a function of the amount of factor input bundles. In the Melitz model generalized marginal costs are also a function of the price of the factor input bundles. Lower factor prices raise the number of firms that can enter the market profitably (extensive margin), reducing generalized marginal costs of a representative firm. For the same reason the Melitz model features a demand externality: in a larger market more firms can enter. We implement the different models in a CGE setting with multiple sectors, intermediate linkages, non-homothetic preferences and detailed data on trade costs. Calibrating the model to the empirically observable tariff elasticity we find that the largest welfare gains are generated in varying models, depending upon the type of trade liberalization and the number of sectors which are modelled as Ethier-Krugman or Melitz. Effects are largest in the Ethier-Krugman model under nationwide trade liberalization and all sectors modelled as either Ethier-Krugman, Melitz or Armington.With only some sectors modelled as Melitz or Ethier-Krugman, the largest welfare effects are produced in the Melitz model.

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1 Introduction

There is a lively debate in the recent trade literature about the value added of firm heterogeneity in trade models. Arkolakis, et al. (2012) show that the welfare gains from trade can be expressed with two sufficient statistics, the domestic spending share and the trade elasticity. This holds in the Armington model, the Ricardian Eaton-Kortum model, the equal firms monopolistic competition Ethier-Krugman model and the firm heterogeneity Melitz model. The only difference is the interpretation of the trade elasticity. In Armington and Ethier-Krugman the trade elasticity is determined by the substitution elasticity between varieties, whereas in Eaton-Kortum and Melitz it is determined by productivity dispersion. Melitz and Redding (2013) instead show that trade cost reductions generate larger welfare gains in the Melitz firm heterogeneity model than in the equivalent model with homogeneous firms, the Ethier-Krugman model.

Firm heterogeneity has not been incorporated in a comprehensive way in multisector CGE models. Most important work in this respect is Balistreri (2012), who have included firm heterogeneity in one sector in a CGE model with other sectors characterized by an Armington setup. Allowing for firm heterogeneity in all sectors might be useful for various reasons. First, it can shed light on the discussion about the value added of firm heterogeneity in trade models by exploring the differences in modelling outcomes with other models. Second, various realistic microeconomic features can be modelled like the distinction of welfare effects into an intensive and extensive margin effect. Third, CGE models contain a large degree of sectoral detail, but are sometimes somewhat outdated in terms of modelling setup. With the incorporation of firm heterogeneity in all sectors, this drawback would disappear.

In this paper we map out a parsimonious representation of firm heterogeneity enabling incorporation in multisector CGE models. In particular, we show that both the Ethier-Krugman and the Melitz model can be defined as an Armington model by generalizing the expressions for iceberg trade costs and for marginal costs and by allowing for a demand externality in the Melitz model. In Ethier-Krugman generalized marginal costs are a function of the number of input bundles leading to so-called variety scaling (Francois (2013)). Variety scaling also props up in the Melitz model, but on top of that generalized marginal costs are also a function of the price of input bundles. The reason is that the extensive margin relative and the compositional margin are affected by the price of input bundles. With a lower price of input bundles more firms can sell profitably to the different destination markets generating a positive effect through the extensive margin (more varieties) and a negative effect through the compositional margin (lower average productivity because of the survival of the least productive firms as well). For the same reason there is a demand externality in the Melitz model: in a larger market with a higher price index more firms can survive, raising the extensive margin relative to the compositional margin. Generalized iceberg trade costs are a function of fixed and iceberg trade costs and of tariffs. We show theoretically that the Ethier-Krugman model is a special version of the Melitz model if the firm size distribution becomes granular. Granularity corresponds with a trade elasticity in Melitz equal to the substitution elasticity minus one. The reason is that under granularity the destination-varying component of the extensive margin cancels out against the compositional margin leaving only the intensive margin and the number of entrants-component of the extensive margin, the two channels also operative in Ethier-Krugman.

ADD ON MULTISECTOR RESULTS

Our results on the ambiguity of the welfare effects in the different models seem to be at odds with Melitz and Redding (2013) who argue that welfare is always larger under firm heterogeneity than under firm homogeneity, given the additional flexibility of the economy under firm heterogeneity. To compare our results with Melitz and Redding (2013), we replicate the numerical simulations in Melitz and Redding (2013). These authors consider a symmetric two-country single-sector model without intermediate linkages, showing that in this setting lower trade costs (smaller trade costs) lead to larger welfare gains (smaller welfare losses) under firm heterogeneity than under firm homogeneity. There are two differences between our multi-sector comparisons of the Melitz firm heterogeneity model and the Ethier-Krugman firm homogeneity model and the comparison in Melitz and Redding (2013). First, the firm heterogeneity model used in the numerics in Melitz and Redding (2013) differs from the plain Ethier-Krugman model, because there are fixed export costs and only a fraction of firms producing for the domestic market exports. We work instead with the plain Ethier-Krugman model with only one fixed cost of production and all firms selling in all markets. Second, we calibrate the parameters of the model such that the empirically observable tariff elasticity is equal in the firm heterogeneity and frm homogeneity models, whereas Melitz and Redding (2013) set the substitution elasticty equal in the two models implying different tariff and trade elasticities in the two models. Since the two models are structurally different, we think that the structural parameters like the substitution elasticity do not have to be equal in a comparison of the two models. Instead, we propose to set the implied empirically observable parameters identical. Simulations with the symmetric two-country model show that the different parameterization of the two models leads to the different conclusions on the welfare effect-ranking of the two models. Calibrating as in Melitz and Redding (2013) to the structural substitution elasticity, the welfare gains (losses) from lower (higher) trade costs are larger (smaller) in the firm heterogeneity model than in the firm homogeneity model. Calibrating instead to the empirically observable tariff elasticity (or trade elasticity) the welfare gains (losses) from lower (higher) trade costs instead are larger (smaller) in the firm homogeneity model than in the firm heterogeneity model.

Further simulations with the two-country model show that the effect of changes in the empirical and structural parameters have a different impact on the welfare effects of changes in trade costs, depending on whether trade costs are calibrated such that baseline import shares are equal for different parameter values or whether trade costs themselves are equal for different parameter values. This exercise enables us to compare our results with the findings in di Giovanni and Levchenko (2012) on the effect of granularity on the welfare effects of trade liberalization. Furthermore, we compare our results with Fan et al. (2013) and Ossa (2015).

Costinot and Rodriguez-Clare (2013) is close to our work. They compare the welfare effects of trade and trade liberalization in the different trade models in different setups. They show that the expression for the price index in the most general model, the firm heterogeneity model, nests the expressions in the Armington and Ethier-Krugman model. Their exposition is different in several respects. First, they concentrate on welfare and thus only derive an expression for the price index. Second, they do not write the different models as special versions of an Armington economy with generalized marginal costs, generalized trade costs and a demand side externality. By doing just that, we can incorporate the different models into a model suitable to analyze detailed trade policy experiments and their effect on a range of outcome variables and not only on welfare. Third, they limit their analysis mostly to a single sector setting, whereas we incorporate the different models into a multisector model.

2 Model

2.1 General Setup

Consider an economy with J countries. There are three groups of agents ag with demand for goods in sector r, private households p, government g and firms f. The group of agents ag in country j has demand q_j^{ag} with CES preferences over quantities of domestic and imported representative goods $q_j^{d,ag}$ and $q_j^{m,ag}$. We omit sector r subscripts as well as the derivation of demand for sector r goods and take this demand as our starting point:[1]

$$q_j^{ag} = \left(\left(e_j^d q_j^{d,ag} \right)^{\frac{\sigma-1}{\sigma}} + \left(e_j^m q_j^{m,ag} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

Quantities of imported and domestic varieties can be summed up to give total importer and domestic demand, q_j^s with $s = d, m$:

$$q_j^s = \sum_{ag \in \{p,g,f\}} q_j^{s,ag} \tag{2}$$

e_j^s is a demand side externality playing a role in the firm heterogeneity version of the model. The demand externality is identical for the different groups of agents. The reason is that upon paying fixed export costs for a destination country firms can serve all three groups of agents in the destination country and the zero cutoff profit condition is thus formulated over all three groups

[1] Derivations and expressions for sectoral demand for the three groups of agents can be found in Hertel (1997) and also in Bekkers, et al. (2015). We also refer the reader to these publications for a description of the general structure of the GTAP-model. We describe here only the parts of the model that are changed by extending the model with monopolistic competition and firm heterogeneity.

together. The externality is source-specific with the source domestic or importer, $s = d, m$. The reason is that we want to allow for different destination-specific taxes for imported goods and domestic goods.

Demand for $q_j^{s,ag}$ can be written as:

$$q_j^{s,ag} = \left(e_j^s\right)^{\sigma-1} \left(\frac{ta_j^{s,ag} p_j^s}{P_j^{ag}}\right)^{-\sigma} q_j^{ag} \tag{3}$$

$ta_j^{s,ag}$ is a group-importer specific import tariff, expressed in power terms. P_j^{ag} and p_j^s are respectively the price indices corresponding to q_j^{ag} and q_j^s defined below. For domestic goods equations (1)-(3) are the final equations generating total domestic demand q_j^d, but for imported goods, demand q_j^m consists of demand for goods from different sources i, q_{ij}:

$$q_j^m = \left(\sum_{i \neq j} \left(q_{ij}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

Solving for demand from source i, q_{ij}, gives:

$$q_{ij} = \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} q_j^m \tag{5}$$

p_{ij} is the price of the representative good traded from i to j. The different prices are defined as follows:

$$P_j^{ag} = \left(\left(\frac{ta_j^{d,ag} p_j^d}{e_j^d}\right)^{1-\sigma} + \left(\frac{ta_j^{m,ag} p_j^m}{e_j^m}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{6}$$

$$p_j^d = c_j b_j p_{Z_j} \tag{7}$$

$$p_j^m = \left(\sum_{i \neq j} \left(p_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{8}$$

$$p_{ij} = ta_{ij} t_{ij} c_i \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{9}$$

The price of the representative good, p_{ij}, in equation (9) is equal to cif-price calculated as the sum of the marginal cost times the price of input bundles in the exporting country, $b_i p_{Z_i}$, times the export subsidy applied to the fob-price plus the price of transport services p_{ij}^{tr} divided by a transport services technology shifter a_{ij}^{tr}, multiplied by generalized marginal costs in the exporting country, c_i, generalized iceberg trade costs t_{ij} and bilateral ad valorem tariffs, ta_{ij}, both expressed in power terms. Firms spend a fixed quantity share of sales on transport services. Technically, the cif-quantity traded o_{ij}^{cif} is a Leontief function of the quanity in fob-terms o_{ij}^{fob} and transport services tr_{ij}. The implication is that transport services work as a per unit trade cost and appear thus as an additive term to the fob price $te_{ij}b_i p_{Z_i}$. Equation (9) makes clear that the costs for transport services could be rewritten as ad valorem trade costs if the input bundles used in transport services would be identical to regular input bundles, since this would imply $p_{ij}^{tr} = p_{Z_i}$. So the reason that the costs for transport services operate as a per unit trade cost is that different input bundles are used.

The Armington model, the Krugman/Ethier model and the Melitz model can all be seen as special versions of the above structure, depending upon how the demand externality e_j^s in equation (1), generalized iceberg trade costs t_{ij}, and generalized marginal cost c_i in equation (9) are specified. In the subsections below we describe the main features of the different models, give the expressions for c_i, t_{ij}, e_j^s and provide the intuition of these expressions. In the appendix we give formal proofs that with the choices for c_i, t_{ij}, e_j^s the general setup-model is equivalent to the different models.

2.2 Armington Economy

Perfectly competitive firms in country i produce homogeneous country i varieties with marginal cost b_i. So, input bundles Z_i can be transformed into output x_i according to $x_i = \frac{Z_i}{b_i}$. With marginal cost pricing the price of output in country i, p_i^x, is given by $p_i^x = b_i p_{Z_i}$. Firms face iceberg trade cost τ_{ij}. There is no demand externality in the Armington economy, so $e_j^s = 1$. Therefore, the Armington economy is characterized by equations (1)-(9) with the following expressions for c_i, t_{ij} and e_j^s:

$$c_i = 1 \tag{10}$$

$$t_{ij} = \tau_{ij} \tag{11}$$

$$e_j^s = 1 \tag{12}$$

2.3 Ethier-Krugman Economy

In the Ethier-Krugman economy, preferences are characterized by love for variety over varieties ω produced in different countries. Utility q_j^{ag} can thus be defined over physical quantities (output) $o(\omega)$ of varieties $\omega \in \Omega_{ij}$ shipped from all exporters i:

$$q_j^{ag} = \left(\sum_{i=1}^{J} \int\limits_{\omega \in \Omega_{ij}} o^{ag}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{13}$$

The corresponding price index is defined over the prices of physical quantities of the varieties, $p^o(\omega)$:

$$P_j^{ag} = \left(\sum_{i=1}^{J} \int\limits_{\omega \in \Omega_{ij}} p^{ag,o}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{14}$$

Firms in country i produce with an identical increasing returns to scale technology with fixed cost a_i and marginal cost b_i implying that each firm produces a unique variety. Increasing returns in combination with love for variety implies also that a larger number of input bundles leads to a more than proportional increase in utility since the number of varieties is larger. To capture this externality, generalized marginal costs c_i are falling in the number of varieties N_i and thus in the amount of input bundles Z_i. Employing the expressions for markup pricing, the free entry condition and factor market closure, c_i can be expressed as follows:[2]

$$c_i = \gamma_{ek} \left(\frac{\widetilde{Z}_i}{a_i} \right)^{\frac{1}{1-\sigma}} \tag{15}$$

γ_{ek} is a function of the substitution elasticity σ:

$$\gamma_{ek} = \frac{\sigma-1}{\sigma} \sigma^{\frac{1}{1-\sigma}} \tag{16}$$

And $\widetilde{Z}_i$ is a function of the number of input bundles, but also of the transport services and export subsidies paid.

$$\widetilde{Z}_i = Z_i - \frac{\sigma-1}{\sigma} \left(\sum_{j=1}^{J} \frac{N_i \overline{r}_{ij}}{p_{Z_i} ta_{ij} \left(te_{ij} b_i + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right)} - \frac{N_i \overline{r}_{ij}}{p_{Z_i} ta_{ij}} \right) \tag{17}$$

$\overline{r}_{ij}$ are the per-firm revenues divided by group-specific import tariffs. Henceforth, $N_i \overline{r}_{ij}$ represents the value of trade before group-specific import tariffs are paid. Generalized marginal cost does not fall proportionally in the amount of input bundles Z_i, as the number of varieties N_i does not increase proportionally with the amount of input bundles Z_i. N_i is calculated

[2]Derivations in Appendix A

by combining factor market equilibrium and the free entry condition. Since transport services are sourced employing separate input bundles, they have to be subtracted in calculating the demand for input bundles from a specific country and sector. So an increase in transport costs leads to less labor demand for given zero-profit-revenues. As a resut higher transport costs raise the number of varieties for a given number of input bundles.[3]

Representative output x_i can be transformed into q_{ij} accounting for the iceberg trade costs τ_{ij}.There is no demand externality in the Ethier-Krugman economy, so we have:

$$t_{ij} = \tau_{ij} \tag{18}$$

$$e_j^s = 1 \tag{19}$$

So, the Ethier/Krugman economy is characterized by equations (1)-(9) with c_i, t_{ij} and e_j as defined in equations (15)-(19).

2.4 Melitz Economy

In the Melitz economy preferences are like in Ethier/Krugman characterized by love for variety over varieties produced by different firms from different countries as in equation (13)-(14). Goods are produced by firms with heterogeneous productivity. To start producing, firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The distribution of initial productivities is Pareto with a shape parameter θ and a size parameter κ_i:

$$G_i(\varphi) = 1 - \frac{\kappa_i^\theta}{\varphi^\theta} \tag{20}$$

A higher θ reduces the dispersion of the productivity distribution and a higher κ_i raises all initial productivity draws proportionally. We impose $\theta > \sigma - 1$ to guarantee that expected revenues are finite.

The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants drawing a productivity parameter, implying that the productivity distribution of producing firms is constant.

[3]An increase in transport costs raises input bundle demand also through the demand for transport services, but in the transport sector we assume perfect competition so there is no number of firms externality.

Firms produce with an increasing returns to scale technology with marginal cost equal to $\frac{1}{\varphi}$. We assume that productivity φ operates both on the costs of production and on the transport sector. This means that more productive firms also need less transport services, an assumption also made for iceberg trade costs τ_{ij}. If productivity would only operate on the cost of production in a setting where the costs for transport services operate as per unit trade costs, the model would become intractable in a multicountry, multisector setting. Firms pay fixed costs f_{ij} for each market in which they sell. The fixed costs are paid partly in input bundles of the source country and partly in bundles of the destination country according to a Cobb Douglas specification with a fraction μ paid in source country input bundles. Upon paying the fixed entry costs for a destination market, firms can sell goods to all three groups of agents.

Since preferences are characterized by love for variety and production occurs with increasing returns to scale, an increase in the number of input bundles leads to a more than proportional increase in utility. To account for this externality, representative output is like in the Ethier/Krugman economy defined as variety scaled output.

Since productivity is heterogeneous, variety scaled output is also affected by input costs. Following Head and Mayer (2013) changes in costs lead to an adjustment in output along three margins, an intensive margin, an extensive margin and a compositional margin. Lower costs lead to more sales of firms already in the market, the intensive margin. This is a price effect and hence does not affect variety scaled output. Lower costs also raises the mass of firms that can produce profitably, the extensive margin. This leads to a rise in variety scaled output. And finally, lower costs reduces the average productivity of firms in the market, as more firms can survive, the compositional margin. This margin also affects variety scaled output. Accounting for the latter two margins, generalized marginal costs c_i can be written as:

$$c_i = \gamma_m \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta e n_i} \right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu \frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{21}$$

The expression for $\widetilde{Z}_i$ is identical to the expression in the Ethier-Krugman model and is given in equation (17). γ_m is a function of σ and θ and an additional conversion parameter ψ for later use set equal to 1:

$$\gamma_m = \psi \left(\frac{\sigma}{\sigma - 1} \right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta - \sigma + 1} \tag{22}$$

x_i can be transformed into q_{ij} accounting for generalized iceberg trade costs, which are a function of iceberg trade costs τ_{ij}, fixed trade costs f_{ij}, import tariffs c_{ij} and the cif price $te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}$. Iceberg and fixed trade costs affect the transformation in the same way through the extensive and compositional margin as the price of input bundles p_{Z_i} affect generalized marginal costs.[4] We get the following expression for generalized iceberg trade costs:

$$t_{ij} = \left(\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}+\frac{\theta-\sigma+1}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \right) \tau_{ij} \tag{23}$$

The four terms between brackets represent the effects of the cif-price, tariffs, and iceberg and fixed trade costs through the extensive and compositional margin on converting fob variety scaled output into cif variety scaled output. Iceberg trade costs also have a direct effect through the intensive margin, represented by the last term outside of the brackets.

Finally, the demand externality does play a role under firm heterogeneity, again driven by the extensive and compositional margin. The following expression can be derived for the demand externality e_j:

$$e_j^s = \left(\frac{\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{24}$$

E_j^{ag} is expenditure by ag in country j. Both larger price indexes P_j^{ag}, larger market sizes E_j^{ag} and lower group-specific tariffs ta_j^{ag} for the different groups of agents ag raise the extensive margin relative to the compositional margin and thus reduce the price index P_j^{ag} and raise utility q_j^{ag}. A lower price of input bundles p_{Z_j} in the destination country also raises utility, as it raises welfare through the extensive margin relative to the compositional margin.

The Melitz economy is characterized by equations (1)-(9) with the expressions c_i, t_{ij} and e_j given in equations (21)-(24).

2.5 Nesting

From the expressions in the previous 3 subsections it follows directly that Krugman/Ethier is a special case of Melitz up to a constant and Armington is a special case of both.

[4] Profits are calculated dividing revenues inclusive of tariffs by tariffs, $\pi = \frac{r}{1+ta} - cq - f$. Costinot and Rodriguez-Clare call this demand shifting. The alternative would be cost shifting with profits calculated as $\pi = r - c(1+ta)q - f$. This makes it impossible to find an expression for the mass of firms as a function of market size, a problem also occuring in the Ethier/Krugman model.

Melitz can be converted into an Ethier/Krugman model by setting θ equal to $\sigma - 1$, the size parameter of the productivity distribution κ_i equal to the inverse of marginal cost $\frac{1}{b_i}$, sunk entry costs times the death probability δen_i divided by the size parameter of the productivity distribution κ_i, $\delta en_i/\kappa_i$ equal to the fixed cost a_i and the conversion parameter ψ in equation (22) as follows:

$$\psi = \left(\frac{\sigma}{\sigma - 1}\right)^{\theta-\sigma+2} \sigma^{\frac{\theta}{\sigma-1}} (\theta - \sigma + 1) \tag{25}$$

$\theta = \sigma - 1$ implies that the demand externality e_j^s is 1. It can be easily verified that the expressions for c_i and t_{ij} in equations (21)-(23) become equal to the price of the representative good in the Ethier/Krugman economy in equations (15)-(18). Ethier/Krugman can be converted into Armington by setting the marginal cost parameter c_i equal to 1 and thus dropping the variety scaling.

The intuition for why $\theta = \sigma - 1$ implies that Melitz leads to Krugman/Ethier is the following. As pointed out above a change in trade costs generates a change in trade flows along three margins, an intensive margin of already exporting firms, an extensive margin representing an increase in the mass of varieties and a compositional margin representing the change in average productivity of firms exporting. If trade costs fall, trade rises with an elasticity of $\sigma - 1$ along the intensive margin and with an elasticity θ along the extensive margin. It falls along the compositional margin with an elasticity $\sigma - 1$. So, if $\theta = \sigma - 1$, the extensive and compositional margin cancel out and only the intensive margin remains. Therefore, the model with heterogeneous firms works out identically as a model with homogeneous firms.

The conversion factor ψ in moving from Melitz to Ethier/Krugman is necessary. Without this conversion factor utility would become infinite in Melitz with $\theta = \sigma - 1$. The reason is that $\theta = \sigma - 1$ would imply that average productivity would become infinite. Still, when θ approaches $\sigma - 1$ the effect of changes in trade costs will be identical to the effect in an Ethier/Krugman economy. So, we can see the Ethier/Krugman model as a limiting case of the Melitz model.

3 Margin Decomposition of Trade in Melitz Model

Total trade flows as measured in cif-terms, inclusive of bilateral import tariffs, but exclusive of group-specific importer tariffs, can be written as:

$$V_{ij} = N_{ij}\widetilde{\overline{r}}_{ij} = N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int_{\varphi_{ij}^{*}}^{\infty}\overline{r}_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi \tag{26}$$

Log differentiating equation (26) on the RHS and LHS wrt to the endogenous variables gives:

$$\begin{aligned} d\ln V_{ij} = {} & d\ln N_{ij} + N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int_{\varphi_{ij}^{*}}^{\infty} d\ln\overline{r}_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{r_{ij}\left(\widetilde{\varphi}\right)}g\left(\varphi\right)d\varphi \\ & + \frac{\partial\ln\left(1-G\left(\varphi_{ij}^{*}\right)\right)}{\partial\ln\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\left(\frac{\overline{r}_{ij}\left(\varphi_{ij}^{*}\right)}{\overline{r}_{ij}\left(\widetilde{\varphi}\right)}-1\right) \end{aligned} \tag{27}$$

The first term represents the extensive margin, EM, the second term the intensive margin, IM, and the third term the compositional margin, CM. To elaborate on these expressions, we first log differentiate the expression for φ_{ij}^{*} in equation (B.7):

$$\begin{aligned} \widehat{\varphi_{ij}^{*}} = {} & \frac{\mu}{\sigma-1}\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} + \widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}} + \frac{1}{\sigma-1}\widehat{f_{ij}} \\ & - \frac{1}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}}p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{ag}}\right) \end{aligned} \tag{28}$$

We can elaborate on the extensive margin, employing the expression for N_{ij} and NE_i in equations (B.17)-(B.18) and the expression for $\widehat{\varphi_{ij}^{*}}$ in equation (28):

$$EM = d\ln N_{ij} = -\theta\widehat{\varphi_{ij}^{*}} + \widehat{NE_i} \tag{29}$$

We can elaborate on the intensive margin, IM, employing the expression for $r_{ij}^{ag}(\varphi)$ and $p_{ij}^{ag}(\varphi)$ in equations (B.3)-(B.4) and summing over the three income groups:

$$
\begin{aligned}
IM =& \frac{\theta-\sigma+1}{\theta} \\
&* \int_{\varphi_{ij}^*}^{\infty} d\ln\left(\left(\frac{\sigma}{\sigma-1}\frac{ta_{ij}\tau_{ij}\left(te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi}\right)^{1-\sigma}\sum_{ag=\{s,p,f\}}\left(ta_j^{ag}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}\right)\frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)}d\varphi \\
&= (1-\sigma)\left(\widehat{\tau_{ij}}+\widehat{ta_{ij}}+\left(\widehat{te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right)\right)+\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left((\sigma-1)\widehat{P_j^{ag}}+\widehat{E_j^{ag}}-\sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}
\tag{30}
$$

Finally, we can express the compositional margin, CM, as follows, using the distribution function of the Pareto distribution in equation (20) and the expression for $r_{ij}(\varphi)$ in equation (B.3):

$$CM = -\theta\widehat{\varphi_{ij}^*}\left(\frac{\theta-\sigma+1}{\theta}-1\right) = (\sigma-1)\widehat{\varphi_{ij}^*} \tag{31}$$

Adding up the three margins, we can express the overall margin thus as follows:

$$
\begin{aligned}
d\ln V_{ij} =& TM = EM + IM + CM \\
&= -\frac{\theta-\sigma-1}{\sigma-1}\mu\widehat{p_{Z_i}}-(1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}}+\widehat{NE_i}-\left(\theta+\frac{\theta-\sigma-1}{\sigma-1}\right)\widehat{ta_{ij}}-\theta\widehat{\tau_{ij}} \\
&-\theta\left(\widehat{te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right)-\frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}}+\frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left((\sigma-1)\widehat{P_j^{ag}}+\widehat{E_j^{ag}}-\sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}
\tag{32}
$$

4 Parameter Estimation

In the Armington and Ethier-Krugman model we only need estimates of the substitution elasticity, whereas the firm heterogeneity model requires estimates of both the substitution elasticity σ and the shape parameter θ of the productivity distribution. In the Eaton and Kortum model we need estimates of the dispersion parameter of the productivity distribution ρ. We write down the gravity equation of our general model to reveal which parameters can be identified by estimating a gravity equation. The value of sales from country i to country j in cif-terms, v_{ij}, follows from multiplying the volume of trade in equation (5) by the price, p_{ij}. Since p_{ij} is the price inclusive of bilateral tariffs ta_{ij}, we have to divide $p_{ij}q_{ij}$ by ta_{ij} to get the value of trade in cif-terms:

$$v_{ij} = \frac{p_{ij}q_{ij}}{ta_{ij}} = \frac{p_{ij}^{1-\sigma}}{ta_{ij}} \left(p_j^m\right)^{-\sigma} q_j^m = ta_{ij}^{-\sigma} \left(t_{ij}c_i te_{ij} b_i p_{Z_i} \left(1 + itm_{ij}\right)\right)^{1-\sigma} \left(p_j^m\right)^{\sigma} q_j^m \tag{33}$$

itm_{ij} is the international transport margin defined as the value of payments to international transport services $vits_{ij}$ divided by the fob-value of trade, v_{ij}^{fob}, $itm_{ij} = \frac{p_{ij}^{tr} tr_{ij}}{te_{ij} b_i p_{Z_i} q_{ij}^{fob}}$.[5] Since we have observable values for tariffs ta_{ij}, we employ estimates of the tariff elasticity $\varepsilon_{v,ta}$ to identify the parameters in the different models.[6] Equation (33) shows that σ is equal to the tariff elasticity $\varepsilon_{v,ta}$ in the Armington and Ethier-Krugman model, where t_{ij} is equal to 1. In the Melitz model instead t_{ij} is a function of bilateral tariffs ta_{ij} implying that the tariff elasticity is not equal to σ. Substituting the expression for t_{ij} in equation (23) into the general gravity equation (33) gives:

$$v_{ij} = ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)} \left(te_{ij} b_i p_{Z_i}\right)^{-\theta} \left(1 + itm_{ij}\right)^{-\theta} \tau_{ij}^{-\theta} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}} c_i \left(p_j^m\right)^{\sigma} q_j^m \tag{34}$$

The tariff elasticity is determined by both σ and θ, so additional information is required to identify both parameters. The reason why the tariff elasticity is not identical to the trade elasticity θ is twofold. First, we estimate the gravity equation employing cif-values and therefore have to divide by the power of the tariff ta_{ij} implying a tariff elasticity $\theta + 1$. Second, in the Melitz model tariffs affect trade flows also through the cutoff productivity. Higher tariffs reduce trade flows because less firms can enter the market profitably (the extensive margin relative to compositional margin effect), responsible for the second part ($\frac{\theta-\sigma+1}{\sigma-1}$) of the elasticity. As discussed in Appendix B this additional effect occurs with tariffs based on the landed price (revenue shifting). Since iceberg trade costs τ_{ij} and export taxes te_{ij} are based on the cost-price (cost-shifting), the additional effect through the cutoff productivity is absent in the elasticities of these variables.

We discuss three possibilities to identify both parameters in the Melitz model in combination with the tariff elasticity $\theta + 1 + \frac{\theta-\sigma+1}{\sigma-1}$. First, we can try to find observable trade costs that

[5] We have used $q_{ij}^{fob} = a_{ij}^{tr} tr_{ij}$, because of the Leontief specification of transport services and fob-quantities.

[6] Some papers in the recent quantitative trade models literature concentrate estimation of the trade elasticity $\varepsilon_{v,\tau}$, the elasticity of trade values with respect to iceberg trade costs. In some models the trade elasticity is equal to the tariff elasticity. Since we do not have values for iceberg trade costs and since the trade elasticity deviates from the tariff elasticity in the Melitz model, we do not focus on identification of the parameters employing the trade elasticity. Still, we observe that the trade elasticity is equal to $\sigma - 1$, $\sigma - 1$, θ and ρ in respectively the Armington, Ethier-Krugman, Melitz and Eaton-Kortum model.

are proportional with iceberg trade costs τ_{ij} or fixed trade costs f_{ij}. Although fixed trade cost measures are available such as the World Bank cost of doing business data, we do not have information to determine whether these measures are exactly or more or less than proportional with fixed trade costs. Therefore, this is a not a viable option. Second, we can use information on the international transport margin to identify θ. The coefficient on one plus the international transport margin in the Melitz gravity equation (34) enables us to identify θ and with the tariff elasticity we can then obtain σ. We can use data on the international transport margin from the GTAP dataset. We disregard this option, because of poor data quality. Third, we can use the fact that a productivity distribution with shape parameter θ implies a firm size distribution with a shape parameter equal to $\theta/(\sigma-1)$. So $\theta/(\sigma-1)$ can be estimated from log-firm-size-log-rank regressions (Axtell (2001)).

As equation (34) shows, iceberg and fixed trade costs enter together in multiplicative form in the expression for trade flows and for import shares. This implies that we can use the conventional approach for Armington CGE-models and calibrate the combination of iceberg and fixed trade costs such that the trade shares in the baseline simulation are equal to the trade shares in the data. Therefore, we do not need information on the value of fixed trade costs separately. Balistreri (2012) estimate the source- and destination-specific components of fixed trade costs structurally from the model, but add a bilateral residual term to obtain a perfect fit between actual and fitted trade flows. We do not follow this route, since it is unclear to what extent source- and destination-specific components of fixed trade costs obtained in this way really represent fixed trade costs instead of iceberg trade costs, given that iceberg and fixed trade costs enter as a combined term in the theoretical gravity equation. So possible simulations on the effects of reductions in source- and destination-specific components of fixed trade costs do not properly inform us about the effects of reductions in fixed trade costs. Moreover, we think it is more interesting to include observable variables in the gravity equation and subsequently also in the CGE model to evaluate the effect of changing observable variables instead of unobservable source- and destination-specific components of fixed trade costs.

5 Evaluating the Effect of Trade Cost Measures

We can include continuous measures of bilateral trade costs like distance and NTBs denoted by bc_{ij}, and dummy measures of bilateral trade costs like contiguity, common language, common

AVE	Armington	Ethier-Krugman	Melitz	Eaton-Kortum
ave_{bc}	$\frac{a_{bc}}{\varepsilon_{v,t}-1}$	$\frac{a_{bc}}{\varepsilon_{v,t}-1}$	$\frac{a_{bc}}{\varepsilon_{v,t}-\frac{1}{\xi}}$	$\frac{a_{bc}}{\varepsilon_{v,t}-1}$
ave_{bd}	$\frac{\exp a_{bd}-1}{\varepsilon_{v,t}-1}$	$\frac{\exp a_{bd}-1}{\varepsilon_{v,t}-1}$	$\frac{\exp a_{bd}-1}{\varepsilon_{v,t}-\frac{1}{\xi}}$	$\frac{\exp a_{bd}-1}{\varepsilon_{v,t}-1}$

Table 1: Ad valorem equivalents of the four models

religion and membership of an FTA denoted by the vector bd_{ij}. In the Armington and Ethier-Krugman model we can easily calculate the ad valorem equivalent (AVE) of these measures. The AVE of a measure calculates the equivalent ad valorem trade cost reduction of a 1% change of the measure or in case of dummy variable of a change of the measure from 0 to 1. We estimate the following equation in logs:

$$\ln v_{ij} = \frac{p_{ij}q_{ij}}{ta_{ij}} = \frac{p_{ij}^{1-\sigma}}{ta_{ij}} \left(p_j^m\right)^{-\sigma} q_j^m = -\sigma \ln ta_{ij} - (\sigma-1)\left(1+itm_{ij}\right) + a_{bc}\ln bc_{ij} + a_{bd}bd_{ij} + \nu_i + \eta_j + \varepsilon_{ij}$$

The AVE of bc and bd can be calculated by dividing the estimated coefficient on bc by one minus the estimated tariff elasticity. Formally, we calculate the elasticity of trade flows with respect to an observable trade cost measure divided by the elasticity of trade flows with respect to (ad-valorem) iceberg trade costs, the trade elasticity $\varepsilon_{v,\tau}$:

$$ave_{bc} = \frac{\frac{\partial \ln v_{ij}}{\partial \ln bc_{ij}}}{\frac{\partial \ln v_{ij}}{\partial \ln \tau_{ij}}} = \frac{a_{bc}}{\varepsilon_{v,\tau}} \tag{35}$$

For the dummy trade cost measure we divide the semi-elasticity of trade flows with respect to a dummy observable trade cost measure with the trade elasticity:

$$ave_{bd} = \frac{\frac{\partial \ln v_{ij}}{\partial bd_{ij}}}{\frac{\partial \ln v_{ij}}{\partial \ln \tau_{ij}}} = \frac{\frac{\frac{dv_{ij}}{dbd_{ij}}}{v_{ij}}}{\varepsilon_{v,\tau}} = \frac{\frac{v_{ij}|bd_{ij}=1 - v_{ij}|bd_{ij}=0}{v_{ij}|bd_{ij}=0}}{\varepsilon_{v,\tau}} = \frac{\exp a_{bd}-1}{\varepsilon_{v,\tau}} \tag{36}$$

We can identify the trade elasticity $\varepsilon_{v,\tau}$ using the tariff elasticity $\varepsilon_{v,t}$. The expressions for the two types of AVEs are displayed in Table 1.

So to calculate the AVE in the Melitz model also an estimate of the degree of granularity ξ is required. There is a second difference between the AVE in the conventional Armington model and the Melitz model. In the Armington model the AVE of a trade cost measure is equal to the elasticity of iceberg trade costs with respect to the trade cost measure, for example for distance the elasticity of iceberg trade costs with respect to distance. In the Melitz model instead a trade cost measure can affect trade flows both through the iceberg trade costs and through fixed trade costs. Henceforth, there is for example an elasticity of both iceberg and fixed trade costs with respect to distance. In running policy experiments it does not matter whether a reduction in a trade cost measure is operationalized through calculation of the AVE and a reduction in iceberg trade costs, or whether it is operationalized through calculation of the elasticity of fixed trade costs with respect to the trade cost measure and implementation of the corresponding reduction in the trade cost measure. To see the last point, we write the gravity equation of the Melitz model with iceberg and fixed trade costs as a function of observable trade cost measures obs_{ij}^{τ} and obs_{ij}^{f}:

$$v_{ij} = ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)} \left(te_{ij} b_i p_{Z_i}\right)^{-\theta} \left(1+itm_{ij}\right)^{-\theta} \tau_{ij}^{-\theta} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}} c_i \left(p_j^m\right)^{\sigma} q_j^m$$

$$\ln v_{ij} = -\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right) \ln ta_{ij} - \theta\left(1+itm_{ij}\right) + a_{\tau} \ln obs_{ij}^{\tau} + a_f obs_{ij}^{f} + \nu_i + \eta_j + \varepsilon_{ij} \quad (37)$$

Comparing equation (34) and (37), the elasticity of iceberg and fixed trade costs with respect to the observable trade cost measures are respectively given by:

$$ave_{melitz}^{\tau} = \frac{a_{\tau}}{\varepsilon_{v,t} - \frac{1}{\xi}} \quad (38)$$

$$ave_{melitz}^{f} = \frac{a_f}{\frac{1}{\xi} - 1} \quad (39)$$

Implementation of changes in obs_{ij}^{τ} and obs_{ij}^{f} in the model shows that changes in the two variables work out exactly the same. In the expression for generalized iceberg trade costs t_{ij}, the term in τ_{ij} and f_{ij} is given by $\left(\tau_{ij}^{\theta} f_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}}\right)^{\frac{1}{\sigma-1}}$. Therefore, given that $\theta = \varepsilon_{v,\tau} - \frac{1}{\xi}$ and $\frac{\theta-\sigma+1}{\sigma-1} = \frac{1}{\xi} - 1$, the elasticities of t_{ij} with respect to obs_{ij}^{τ} and obs_{ij}^{f} are identical and given by $\frac{a_{\tau}}{\sigma-1}$ and $\frac{a_f}{\sigma-1}$. So for the overall effects it does not matter conceptually whether a change in an observable trade cost measure is operationalized as a reduction in iceberg or in fixed trade costs. Only for the margin decomposition it does matter how the change is operationalized. In general though it will be hard to determine conceptually whether observable trade cost measures affect trade flows through iceberg or through fixed trade costs. For example, it is likely that distance or the presence of a free trade agreement (FTA) affects trade flows through both. It is less costly to start exporting in a country at a shorter distance or in an FTA-partner. It is hard to find measures that only affect one of the two trade costs.

Equation (39) indicates that ave_{melitz}^{f} goes to infinity when the firm size distribution is granular. The reason is that fixed exporting costs should not have any effect under granularity under trade flows thus implying that the effect of fixed trade costs should be zero.

6 Simulation Results

We present two sets of results. First, we present results of multi-country multi-sector simulations with the adapted GTAP-GEMPACK model. Second, we replicate and extend the symmetric two-country single-sector simulations in Melitz and Redding (2013) to be able to compare our multi-sector simulation results with the results in Melitz and Redding (2013).

6.1 Multi-Sector Multi-Country Results

ADD RESULTS

6.2 Symmetric Two-Country Single-Sector Model

Melitz and Redding (2013) compare the welfare effects of trade and trade liberalization under firm heterogeneity and firm homogeneity in a symmetric two-country single-sector single-production factor setting. In the simulations Melitz and Redding (2013) compare the firm heterogeneity model with a homogeneous firms model where only a proportion of firms exports and firms pay fixed export costs. Moreover, they set the common structural parameter in the two models, the substitution elasticity, equal and calibrate to the export share of firms and the share of exporting firms. With this setup they find that the welfare gains (losses) from lower (higher) trade costs are unambiguously larger (smaller) under firm heterogeneity than under firm homogeneity. These results seem at odds with our multi-sector simulations where welfare effects are larger under firm homogeneity than firm heterogeneity in some scenarios. But we parameterize the model differently. In our simulations we set the structural parameters of the model such that empirically observable parameters such as the tariff or trade elasticity are identical in the two models. This calibration implies a different substitution elasticity in the two models. In table 2 we display different choices for the two structural parameters, the substitution elasticity σ and the Pareto shape parameter θ, and the two empirical parameters, the tariff elasticity $\varepsilon_{v,ta}$ and the granularity of the firm size distribution, ξ. In the first four columns we calibrate the parameters starting from the empirically observable parameters $\varepsilon_{v,ta}$ and ξ with θ and σ following from these values. In column 5 instead we follow the approach in

Param.	Heterogeneous	Homogeneous	Plain EK	Granular	Melitz-Redding
$\varepsilon_{v,ta}$	4	4	4	4	$\theta + \frac{\theta}{\sigma-1} = 5\frac{2}{3}$
ξ	$\frac{3}{4\frac{1}{4}}$	$\frac{3}{4\frac{1}{4}}$	$-$	1	$\frac{\sigma-1}{\theta} = \frac{3}{4\frac{1}{4}}$
θ	$\varepsilon_{v,ta} - \frac{1}{\xi} = 2\frac{7}{12}$	$\frac{\varepsilon_{v,ta}-1}{\xi} = 4\frac{1}{4}$	$-$	$\varepsilon_{v,ta} - \frac{1}{\xi} = 3$	$4\frac{1}{4}$
σ	$\varepsilon_{v,ta}\xi = 2\frac{14}{17}$	$\varepsilon_{v,ta} = 4$	$\varepsilon_{v,ta} = 4$	$\varepsilon_{v,ta}\xi = 4$	4

Table 2: Parameterization of the five models starting from tariff elasticity in homogeneous firms model Melitz and Redding (2014)

Melitz and Redding with the structural parameters set at certain values with implied values for the empirical counterparts. Melitz and Redding set the structural parameters θ and σ, implying values for the empirical parameters $\varepsilon_{v,ta}$ and ξ.

In the first four columns we set the tariff elasticity at 4, such that the tariff elasticity is the same as the tariff elasticity in the homogeneous firms model in Melitz and Redding. We set the granularity parameter ξ at $\frac{3}{4\frac{1}{4}}$, the value implied by the parameters of Melitz and Redding. With this choice we take the homogeneous firms model in Melitz and Redding as starting point (column 2). We need a value for ξ as well in this model, since only a fixed fraction of firms enters in their baseline and this fraction is determined by the size of fixed costs and the parameter θ. In column 1 we derive the structural parameters θ and σ in the heterogeneous firms model from the empirical counterparts $\varepsilon_{v,ta}$ and ξ. Column 3 displays the plain Ethier-Krugman homogeneous firms model, so without destination specific fixed costs. Column 4 finally follows our approach with structural parameters following from the empirical counterparts in the case of $\xi = 1$, corresponding to granularity.

The fixed exporting costs f_{ij} and iceberg trade costs τ_{ij} are set such that the overall import share is 0.89 and the share of exporting firms is 0.18. The other parameters do not affect the results and are thus set at 1, $f_{ii} = \kappa_i = L_i = en_i = \delta = 1$. We solve the model employing the equilibrium equations in Melitz and Redding (2013). The details are presented in the webappendix, where we show as well that the Melitz-Redding equilibrium equations lead to the same solution as the general setup model with the appropriate choices for e_j, t_{ij} and c_i.[7] Figure 1 displays the real wage for the five parameter settings as a function of percentage increases and decreases in iceberg trade costs τ_{ij}. The figure conveys three important messages. First, as in Melitz and Redding (2013) the welfare gains from trade liberalization are larger in the heterogeneous firms model than in the homogeneous firms model when the structural parameter,

[7] We also show equivalence between the Melitz model equilibrium equations and the general setup model in the presence of intermediate linkages.

the substitution elasticity is set identical in the two models. Second, when the empirically observable parameter, the tariff elasticity, is identical in the two models, this conclusion is reversed and the homogeneous firms model generates larger welfare gains. This shows that the way the model is calibrated is crucial for the relative welfare effects in the hetereogeneous and homogeneous firms models. Third, the welfare effects in the homogeneous firms models and in the model with a granular firm size distribution are identical.

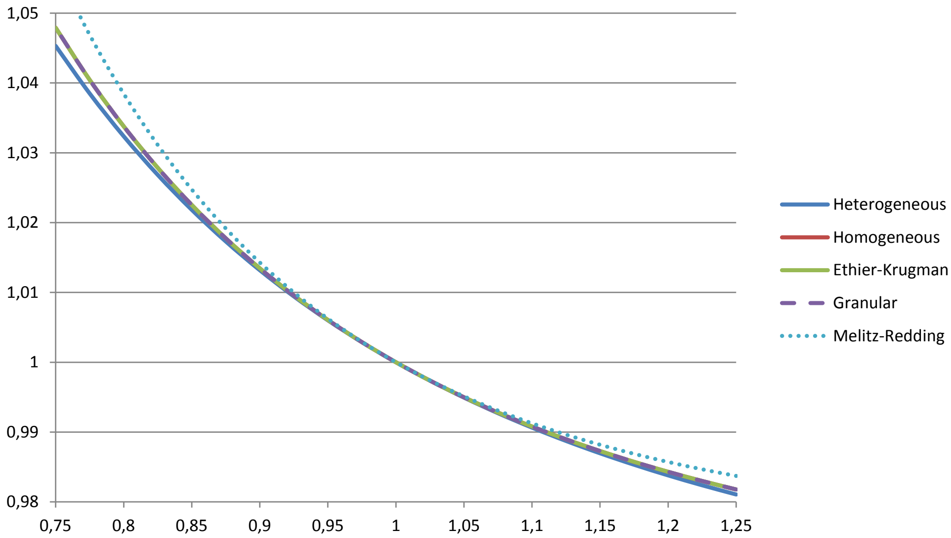

Figure 1: The real wage as a function of percentage change in iceberg trade costs for different ways of calibrating the modeling parameters θ and σ and calibration to the tariff elasticity, the overall import share and the share of exporting firms in symmetric two-country single-sector model

As a first robustness check, we calibrate like in Melitz and Redding (2013) to the export share of firms (0.14) and the share of exporting firms (0.18). The figure reported in the webappendix shows that the first two conclusions of the baseline calibration to the overall import share also hold with calibration to the export share of firms: the welfare gains from trade liberalization are larger in the heterogeneous firms model than in the homogeneous firms model when the substitution elasticity is identical in the two models, but is smaller in the heterogeneous firms model when the estimated tariff elasticity is identical. Like in the baseline calibration the plain Ethier-Krugman model generates the same welfare effects as the firm heterogeneity model under granularity. In deviation from the baseline calibration the welfare gains and losses from respectively lower and higher trade costs are larger in the plain Ethier-Krugman and granular firm heterogeneity model than in the firm heterogeneity model for an identical substitution elasticity (Melitz-Redding calibration). This finding is in line with the findings of the multi-sector simulations.

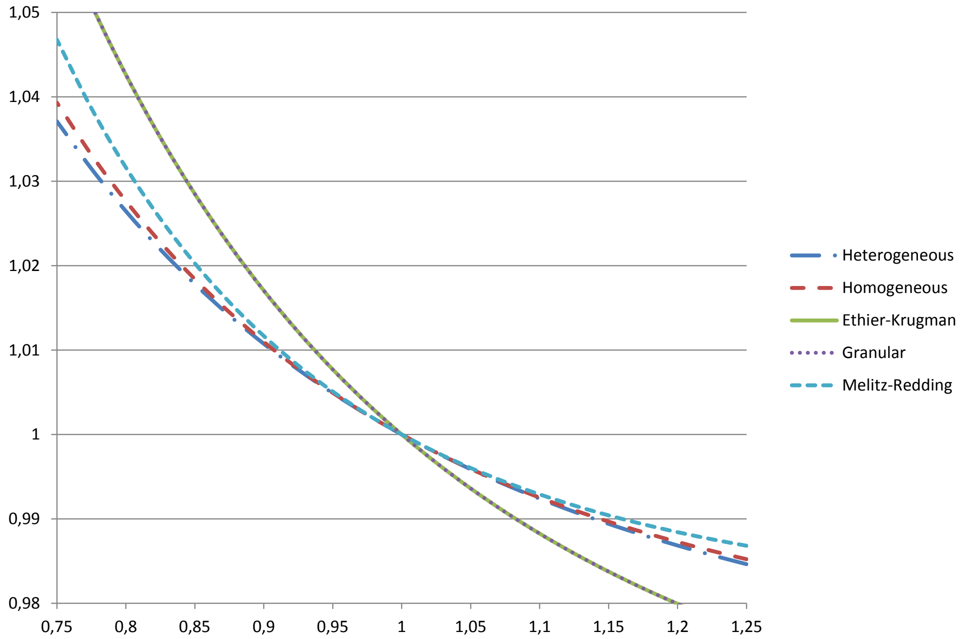

Figure 2: The real wage as a function of percentage change in iceberg trade costs for different ways of calibrating the modeling parameters θ and σ and calibration to the tariff elasticity, the export share of firms and the share of exporting firms in symmetric two-country single-sector model

Param.	**Heterogeneous**	**Homogeneous**	**Plain EK**	**Granular**	**Melitz-Redding**
$\varepsilon_{v,\tau}$	3	3	3	3	$\theta = 4\frac{1}{4}$
ξ	$\frac{3}{4\frac{1}{4}}$	$\frac{3}{4\frac{1}{4}}$	–	1	$\frac{\sigma-1}{\theta} = \frac{3}{4\frac{1}{4}}$
θ	$\varepsilon_{v,\tau} = 3$	$\varepsilon_{v,\tau}/\xi = 4\frac{1}{4}$	–	$\varepsilon_{v,\tau} = 3$	$4\frac{1}{4}$
σ	$\varepsilon_{v,\tau}\xi + 1 = 3\frac{2}{17}$	$\varepsilon_{v,\tau} + 1 = 4$	$\varepsilon_{v,\tau} + 1 = 4$	$\varepsilon_{v,\tau}\xi + 1 = 4$	4

Table 3: Parameterization of the five models starting from trade elasticity in homogeneous firms model Melitz and Redding (2014)

As a second robustness check we do not calibrate to the tariff elasticity $\varepsilon_{v,ta}$, but to the trade elasticity $\varepsilon_{v,\tau}$, the elasticity of trade values with respect to iceberg trade costs. We return to calibration to the overal import share and the share of export firms as in the baseline. Table 3 displays the implied values for θ and σ starting from a trade elasticity of 3 and degree of granularity of $3/4\frac{1}{4}$, as in the homogeneous firms model in Melitz and Redding. We have $\theta = \varepsilon_{v,\tau}$ and $\sigma = \xi\varepsilon_{v,\tau} + 1$ in the firm heterogeneity model and $\sigma = \varepsilon_{v,\tau} + 1$ in the homogeneous firms model. We also need to determine a θ for the homogeneous firms model and use $\theta = (\sigma - 1)\,\xi = \varepsilon_{v,\tau}\xi$. Finally, in the Melitz-Redding firm heterogeneity model we start with values for θ and σ implying values for the empirical parameters $\varepsilon_{v,\tau}$ and ξ.

Figure 3 shows that calibration to the substitution elasticity as in Melitz and Redding would still generate a larger welfare effect under firm heterogeneity. But with calibration to the trade elasticity, the welfare effects of trade liberalization would be exactly identical in the different models. This result is in line with the argument in Arkolakis, et al. (2012) that the welfare gains from trade are identical in the different models with an identical trade elasticity. Their result though is on the welfare gains from trade as operationalized by a larger import share instead of trade liberalization as operationalized by lower iceberg trade costs.

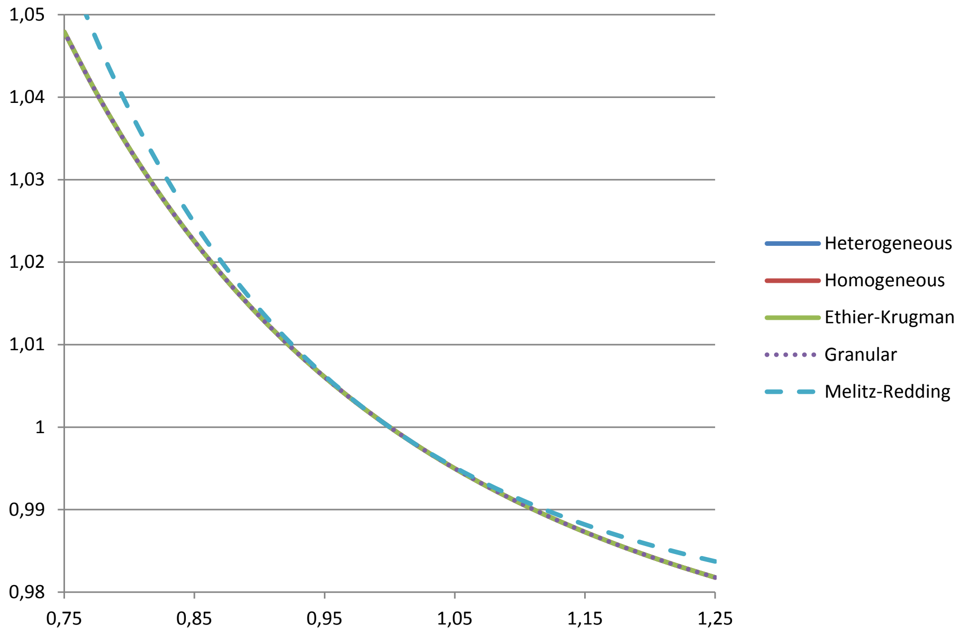

Figure 3: The real wage as a function of percentage change in iceberg trade costs for different ways of calibrating the modeling parameters θ and σ and calibration to the trade elasticity, the overall import share and the share of exporting firms in symmetric two-country single-sector model

Next we evaluate the effect of variations in the empirical parameters ξ and $\varepsilon_{v,ta}$ and in the structural parameters θ and σ on the welfare gains from trade in the Melitz firm heterogeneity model, calibrating trade costs such that the actual import share is equal to the import share in the baseline. With these simulations we can interpret several of the findings in the recent trade literature. Figure 4 shows that a fall in ξ reduces the welfare gains from trade. So moving towards granularity raises the welfare gains from trade liberalization. From figure 5 we can draw the conclusion that an increase in the tariff elasticity raises the welfare gains from trade liberalization. This result shows that the findings on the welfare gains from small trade cost reductions on global welfare in Fan et al. (2013) do not seem to be very interesting. The graph shows that variations in the trade elasticity only have a negligible impact on the welfare effects of lower iceberg trade costs for changes in these trade costs of about 5%. For changes of 10% and larger the welfare effects start to vary considerably.

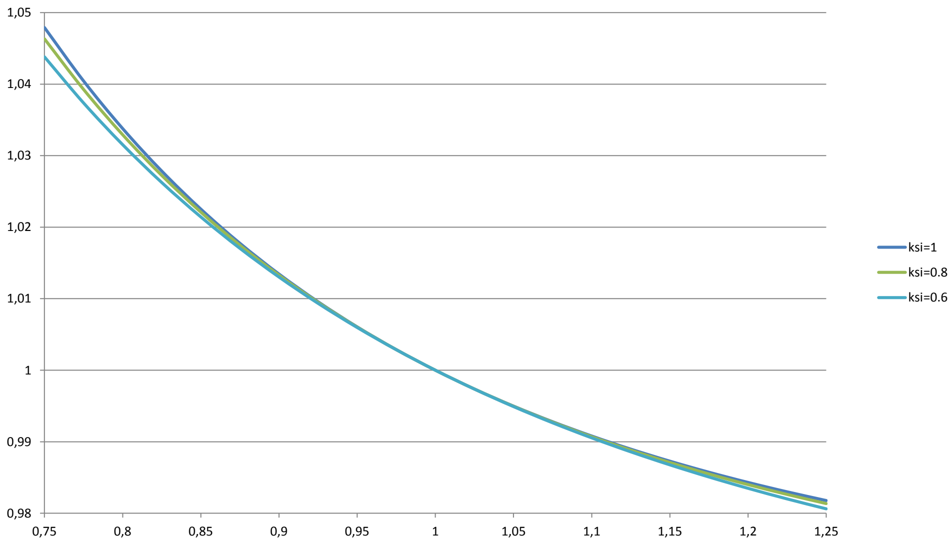

Figure 4: The real wage as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model

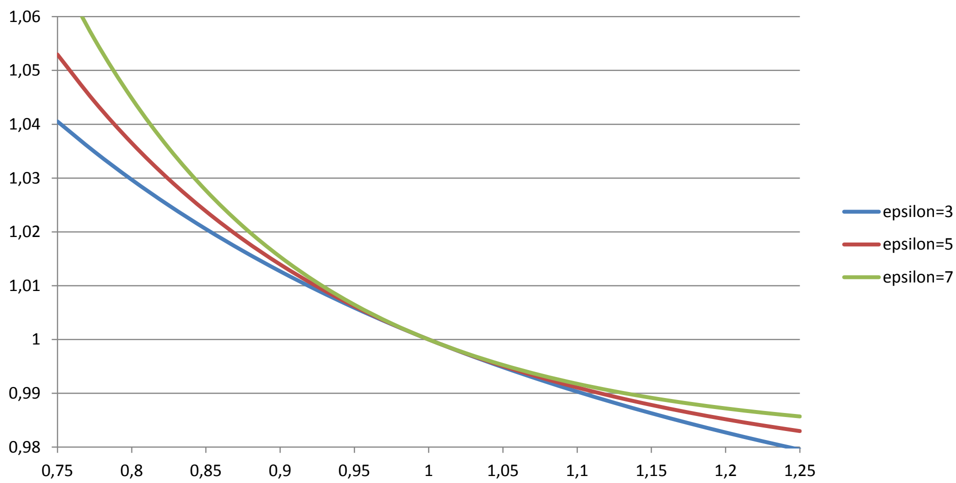

Figure 5: The real wage as a function of percentage change in iceberg trade costs for different values of $\varepsilon_{v,ta}$ and $\xi = 0.8$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model

Figure 6 shows that an increase in θ generates larger welfare gains from trade liberalization. This result aligns with our finding above that the Melitz and Redding firm heterogeneity calibration leads to larger gains than the Ethier-Krugman calibration, which is the limiting case of granularity with the same substitution elasticity as under Melitz and Redding and only a lower shape parameter θ. The result seems however at odds with Proposition 4 in Melitz and Redding (2013), where it is argued that a lower shape parameter leads to larger welfare gains from trade. As shown in Figure 7 we find that this result holds if one starts from identical iceberg trade costs for different values of θ, instead of starting from identical import shares. The latter approach seems to be more reasonable, since iceberg trade costs are unobservable and should thus be calibrated to the observable import share. The experiment in Figure 7 is similar to the experiment conducted in di Giovanni and Levchenko (2012). These authors show that a move towards granularity has a strong positive effect on the welfare gains from lower iceberg trade costs. They keep σ constant and model a move towards granularity by a reduction in the shape parameter θ close to $\sigma - 1$. Based on our baseline results in Figure 4 we draw a similar conclusion: a move towards granularity raises the welfare gains from lower iceberg trade costs. But our calibration differs in two crucial ways. First, we model a variation in granularity, keeping the tariff elasticity $\varepsilon_{v,ta}$ constant, whereas di Giovanni and Levchenko (2012) vary both the tariff elasticity $\varepsilon_{v,ta}$ and the granularity parameter ξ by varying θ with constant σ. Second, we calibrate unobserved trade costs to observable import shares, whereas di Giovanni and Levchenko (2012) keep unobserved trade costs constant when varying θ. They obtain their trade costs from a gravity regression of trade flows on several explanatory variables. Their calibration with constant trade costs across the different scenarios for θ implies that the elasticity of for example trade costs with respect to distance would vary as θ as varied, since the fitted coefficient of trade values with respect to distance stays constant across the different scenarios. Comparing Figure 4 employing our proposed calibration with Figure 7 employing the calibration used in di Giovanni and Levchenko (2012) shows that the effects of moving towards granularity are much more modest in our calibration with the implicit variation in the degree of granularity ξ in Figure 7 identical to the variation in Figure 4, from 1 to 0.6, with θ falling from 5 to 3. So it seems that di Giovanni and Levchenko (2012) overstate the effects of granularity on the welfare gains from reductions in iceberg trade cost reductions.

As we only use the experiments with constant iceberg trade costs across the different scenarios to replicate findings in the literature, we delegate a discussion of the effect of variation in the other parameters on the welfare effects from trade liberalization under this calibration to a webappendix.

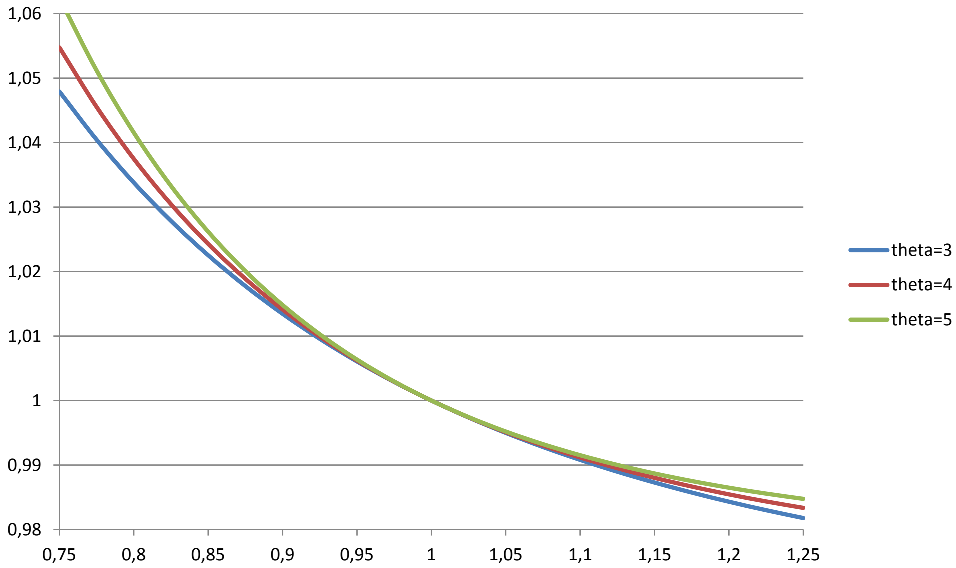

Figure 6: The real wage as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model

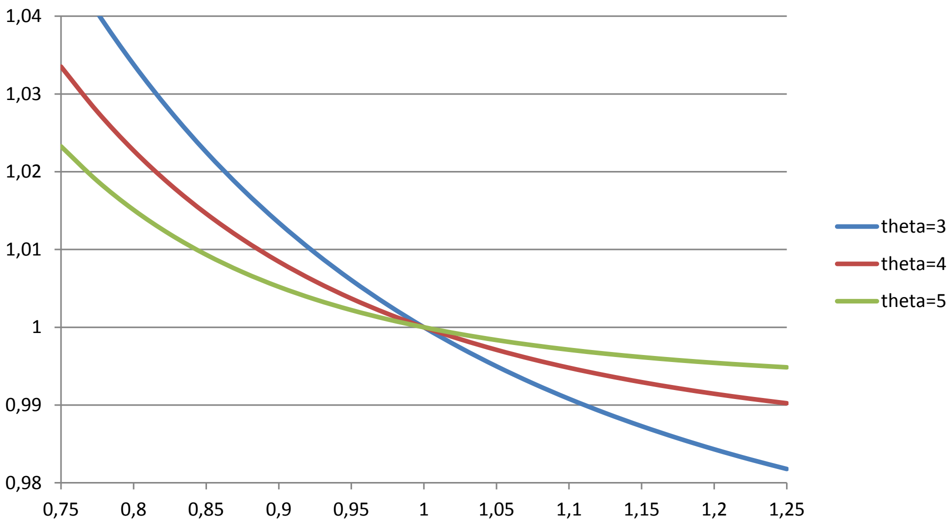

Figure 7: The real wage as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model

Figure 8 shows that variation in the substitution elasticity σ does not have any effect on the welfare effects of changes in iceberg trade costs. Evaluating the underlying ξ and $\varepsilon_{v,ta}$, we see that an increase in σ for given θ, raises $\varepsilon_{v,ta}$, which makes the welfare effects larger and decreases ξ, which reduces the welfare effects. Figure 8 indicates that the two opposite forces

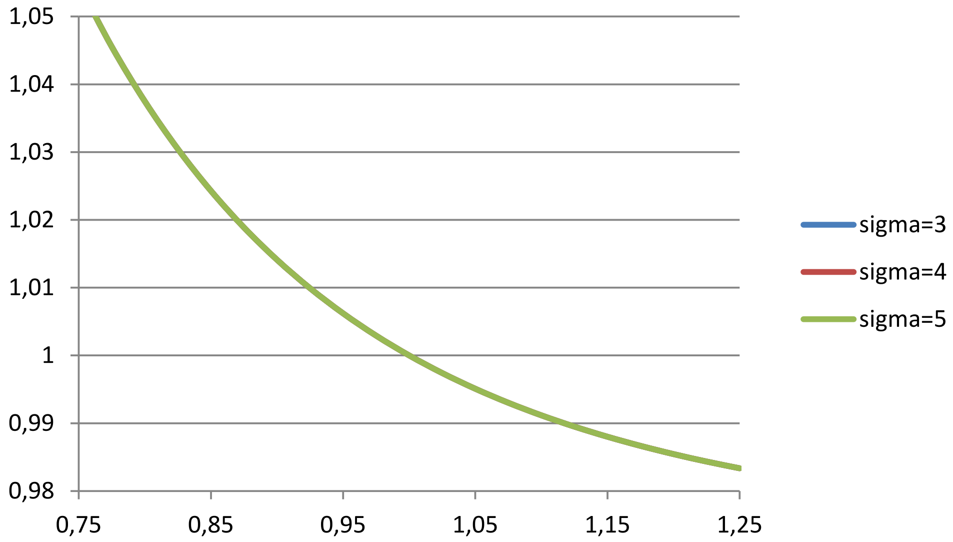

Figure 8: The real wage as a function of percentage change in iceberg trade costs for different values of σ and $\theta = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model

exactly cancel out.

We can summarize the symmetric two-country simulations as follows. First, our multi-country multi-sector simulations are not at odds with the findings in Melitz and Redding (2013). Second, calibration to the empirically observable parameters generates results opposite to calibration to the structural model parameters: calibration to the empirically observable tariff elasticity generates smaller welfare gains from trade liberalization under firm heterogeneity than under firm homogeneity also in a symmetric two-country setting. Third, in the comparison between the firm homogeneity and heterogeneity model it matters whether iceberg and fixed trade costs are calibrated to the overall import share or the import share of firms.

As a next step we include intermediate linkages in the symmetric single-sector two-country model. We model intermediate linkages with input bundles being a Cobb-Douglas aggregate of labor and intermediate input bundles with intermediate input bundles identical to final goods bundles. We evaluate how the welfare effects of trade cost changes vary with the empirical parameters ξ and $\varepsilon_{v,ta}$ and the structural parameters θ and σ. We calibrate iceberg trade costs such that import shares in the baseline are equal to 0.89, normalizing both domestic and exporting fixed costs to 1. Figures 9 and 10 show that the same pattern emerges for variations in $\varepsilon_{v,ta}$ and θ as in the models without intermediate linkages. Figures 11 and 12 instead show that the patterns change for variations in σ and ξ. Figure 11 shows that trade cost changes generate larger welfare effects when σ is smaller, whereas without intermediate linkages the welfare effects

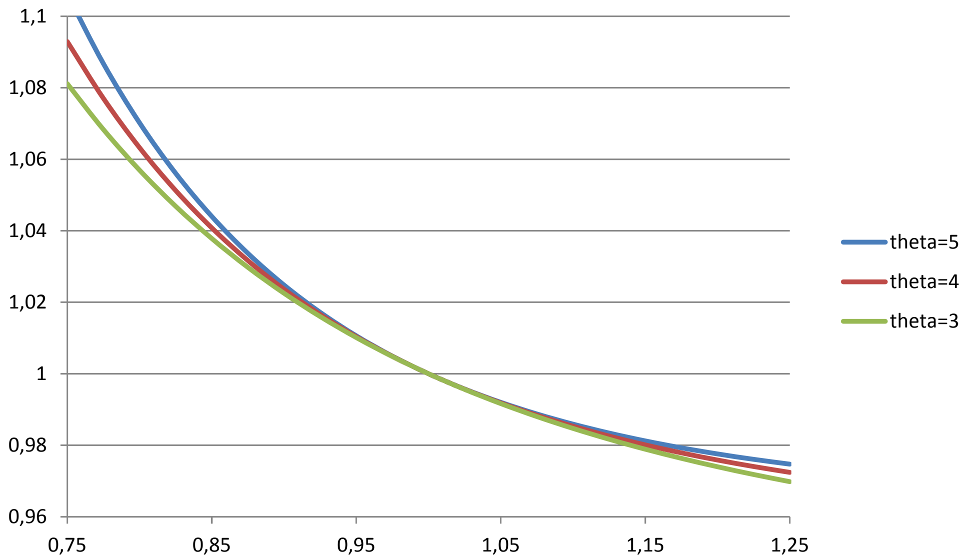

Figure 9: The real wage as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model with intermediate linkages

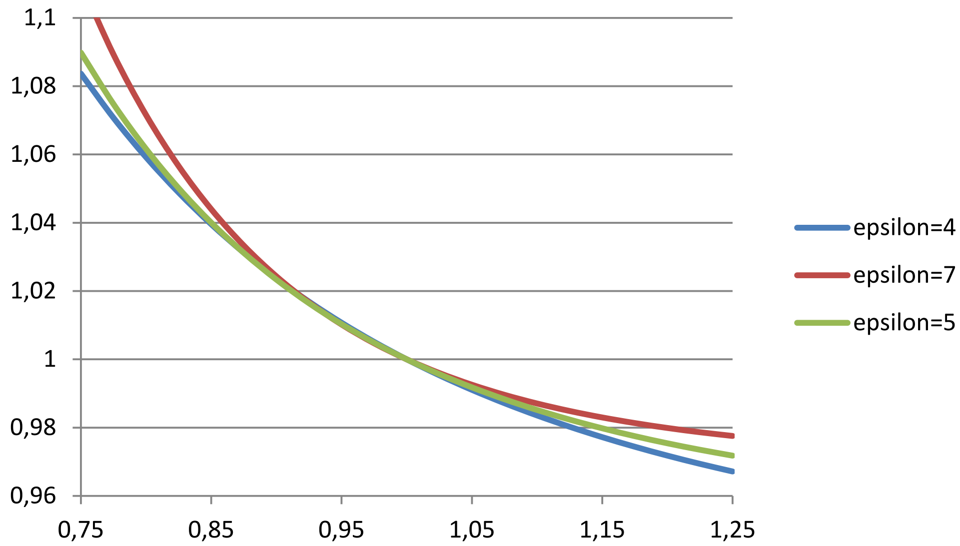

Figure 10: The real wage as a function of percentage change in iceberg trade costs for different values of $\varepsilon_{v,ta}$ and $\xi = 0.8$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model with intermediate linkages

did not vary with σ. From Figure 12 we see that the effect of ξ on the welfare effects of variations in trade costs changes. Without intermediate linkages a larger ξ corresponding with a move to granularity leads to larger welfare gains from trade liberalization and smaller welfare gains from larger trade costs. In the model with intermediate linkages instead the welfare gains from trade liberalization are larger when moving away from granularity, whereas the losses from higher trade costs are still smaller with more granularity. In Figure 13 we display the welfare effects of lower trade costs as a function of ξ for various values of β, the share of gross output spent on labor. The figure makes clear that the new pattern emerges for labor shares β of 0.8 and smaller. Comparing these results of the single-sector model with intermediate linkages with the multi-sector model results in the previous subsection shows that for both variations in ξ and σ the patterns are opposite. A larger ξ and a larger σ lead in the multisector model (respetively Figures 14 and 17) to larger welfare effects, whereas they lead in the single-sector model with intermediate linkages (respectively Figures 12 and 11) to smaller effects.

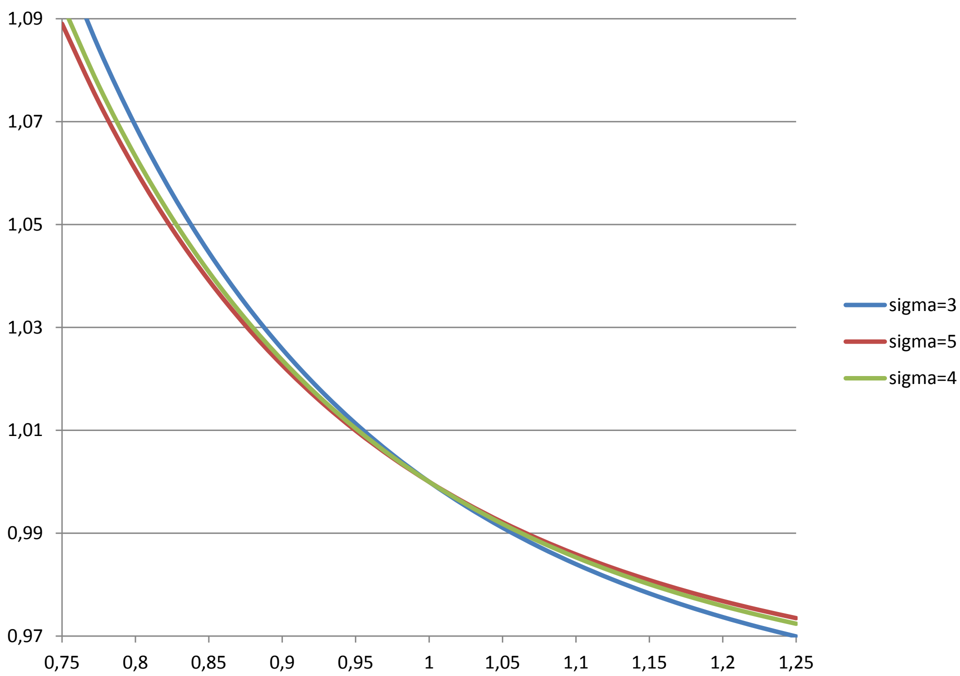

Figure 11: The real wage as a function of percentage change in iceberg trade costs for different values of σ and $\theta = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model with intermediate linkages

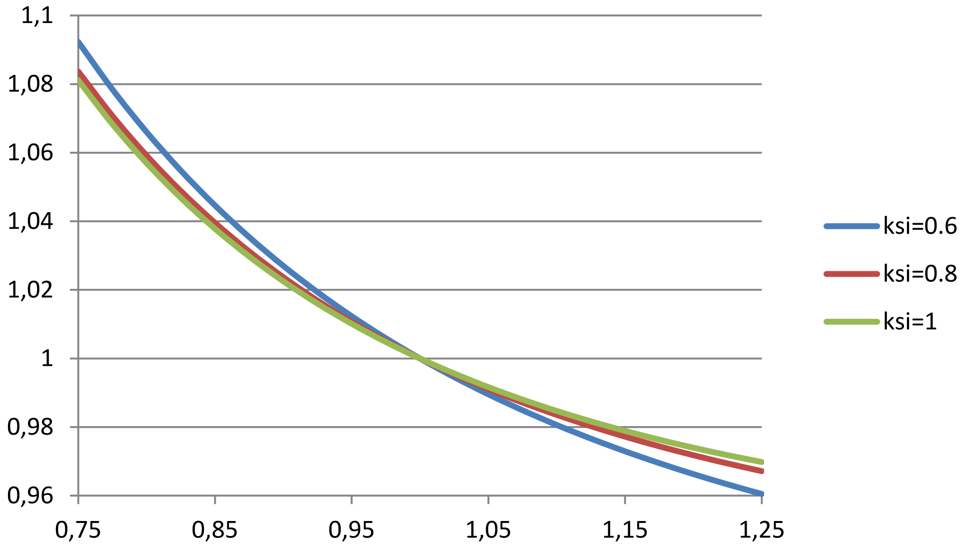

Figure 12: The real wage as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$ and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model with intermediate linkages

7 Concluding Remarks

We have shown that both the Ethier-Krugman monopolistic competition model and the Melitz firm heterogeneity model can be defined as a generalized Armington model. From this representation of the two models it also follows immediately that the Melitz model generates the same equilibrium outcome as the Ethier-Krugman model when the firm size distribution becomes granular. Using the Armington representation of the Ethier-Krugman and Melitz models makes it possible to incorporate these models in a multi-sector multi-country setting without generating computational problems. The Armington representation does not require solving for additional pairwise variables like the cutoff productivity and the mass of firms, keeping the dimensionality of the model limited. Parameters are based on estimates of the gravity model following directly from the theoretical model and on measures of the granularity of the firm size distribution taken from the literature. The model is calibrated such that import shares in the baseline are equal to actual import shares in the data.

We have also undertaken simulations with a symmetric two-country model to clarify the seemingly contradiction between our results on the ambiguity of the welfare effects across the different models and the results in Melitz and Redding (2013) that welfare effects are always larger under firm heterogeneity. These differences stem from the fact that we set the parameters in the firm heterogeneity and firm homogeneity model such that the implied empirically observable trade elasticity is identical in the two models, whereas Melitz and Redding (2013) set the

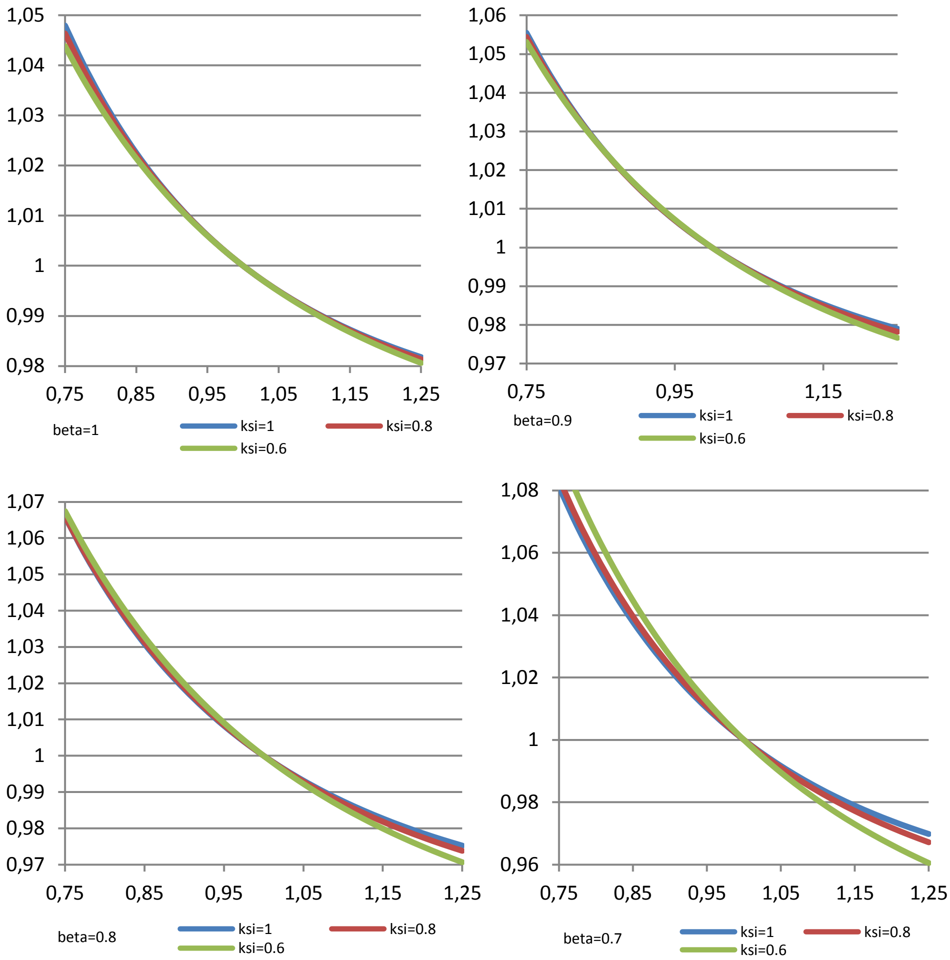

Figure 13: The real wage as a function of percentage change in iceberg trade costs for different values of ξ, $\varepsilon_{v,ta} = 4$, different values of β and calibration to the overall import share and the share of exporting firms in symmetric two-country single-sector model with intermediate linkages

common structural parameter in the two models, the substitution elasticity, equal, implying a different trade and tariff elasticity.

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Appendix A Ethier/Krugman Economy

The goal of this section is to derive the expressions for c_i and t_{ij} in the main text in equations (15)-(18). Before we go into the Ethier-Krugman model, we first rewrite the expressions for demand and the price index in the general model. The general setup-expressions for $q_{ij}e_j^s$ and P_j^{ag} implied by equations (3)-(9) are given by:

$$q_{ij}e_j^s = \left(\frac{p_{ij}}{e_j^s}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}} \tag{A.1}$$

$$P_j^{ag} = \left(\sum_{i=1}^{J} \left(\frac{p_{ij}ta_j^{s',ag}}{e_j^{s\prime}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{A.2}$$

With p_{ij} defined as follows:

$$p_{ij} = ta_{ij}t_{ij}c_i \left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{A.3}$$

With $s' = d$ if $i = j$ and $s' = m$ if $i \neq j$ and $ta_{ii} = t_{ii} = te_{ii} = 1$ and $\frac{p_{ii}^{tr}}{a_{ii}^{tr}} = 0$.

To show equivalence between the general model-representation and the normal representation of different models, we have to show that the expressions for demand in equation (A.1) and for the price index in equation (A.2) with the appropriate choices for c_i, t_{ij} and e_j^s in the general model-representation are identical to the demand and price index expressions in the normal representation of the different models.

In the Ethier-Krugman model agents of group $ag = \{s, p, f\}$ with g government, p private sector and f firms in country j have CES preferences over physical quantities $o(\omega)$ of varieties ω from different countries. The quantity and price index are defined in equations (13)-(14). Demand for a variety ω shipped from i to j and sold to group ag is equal to:

$$o_{ij}(\omega) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag}p_{ij}(\omega)\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{A.4}$$

Varieties are produced by identical firms with an increasing returns to scale technology with fixed cost a_i and marginal cost b_i, implying that each firm produces a unique variety. As firms are identical, ω can be dropped in the remainder.

Firms face iceberg trade costs τ_{ij}, bilateral export taxes te_{ij}, bilateral import tariffs ta_{ij}, and group specific import tariffs ta_j^{ag}. Moreover, there is a transport sector with firms having to spend a fixed quantity share of sales on transport services. Technically, the cif-quantity traded o_{ij}^{cif} is a Leontief function of the quantity in fob-terms o_{ij}^{fob} and transport services tr_{ij}:

$$o_{ij}^{cif} = \min\left(o_{ij}^{fob}, a_{ij}^{tr} tr_{ij}\right) \tag{A.5}$$

Profits are therefore given by:

$$\begin{aligned}
\pi_{ij} &= ta_j^{ag} p_{ij}^o o_{ij} - \left(ta_j^{ag} - 1\right) p_{ij}^o o_{ij} - \frac{ta_{ij} - 1}{ta_{ij}} p_{ij} o_{ij} - \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) o_{ij} \\
&= \frac{p_{ij} o_{ij}}{ta_{ij}} - \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) o_{ij} \qquad \text{(A.6)}
\end{aligned}$$

This expression for profit implies the following markup pricing rule:

$$p_{ij}^o = \frac{\sigma}{\sigma - 1} ta_{ij} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{A.7}$$

p_{ij}^o is the cif price of physical output o_{ij} before the group-specific import tariff ta_j^{ag} is applied. Firms do not face destination specific fixed costs and can enter all markets upon paying the fixed costs a_i. Profits from sales to all markets are thus equal to:

$$\pi_i = \sum_{j=1}^{J} \frac{p_{ij}^o o_{ij}}{\sigma ta_{ij}} - a_i p_{Z_i} \tag{A.8}$$

As a next step, N_i is defined as the mass of varieties produced in country i. N_i is identical for all destinations by absence of destination specific fixed costs. It follows from the following labor market equilibrium:

$$\left(\sum_{j=1}^{J} \tau_{ij} o_{ij} + a_i\right) N_i = Z_i \tag{A.9}$$

To rewrite this expression, we first rewrite the expression for $\tau_{ij} o_{ij}$ using the markup equation (A.7):

$$\tau_{ij} o_{ij} = \frac{\sigma - 1}{\sigma} \frac{p_{ij}^o o_{ij}}{p_{Z_i} ta_{ij}} + \frac{\sigma - 1}{\sigma} \frac{p_{ij}^o o_{ij}}{p_{Z_i} ta_{ij}} \left(\frac{1}{te_{ij} b_i + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} - 1\right) \tag{A.10}$$

Using equations (A.8) and (A.10), we can solve for N_i from equation (A.9) as follows:

$$N_i = \frac{\widetilde{Z}_i}{\sigma a_i} = \frac{Z_i - \frac{\sigma-1}{\sigma}\left(\sum_{j=1}^{J} \frac{N_i \bar{r}_{ij}}{p_{Z_i} ta_{ij}\left(te_{ij} b_i + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right)} - \frac{N_i \bar{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}{\sigma a_i} \tag{A.11}$$

With $\widetilde{Z}_i$ as defined in equation (17).

The price index in (14) can be written as equation (A.2) with $e_j^s = 1$ and p_{ij} defined as:

$$p_{ij} ta_j^{s,ag} = \left(\int_{\omega \in \Omega_{ij}^{ag}} p^{ag,o}(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}} \tag{A.12}$$

Therefore, we only need to elaborate on $p_{ij} ta_j^{s,ag}$ to show equivalence of the price index. Given that all firms are identical and all varieties N_i are exported to all destinations, equation (A.12) can be rewritten as:

$$p_{ij} ta_j^{s,ag} = N_i^{\frac{1}{1-\sigma}} ta_j^{s,ag} p_{ij}^o = \left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}} ta_j^{s,ag} p_{ij}^o \tag{A.13}$$

Substituting equation (A.7) for p_{ij}^o leads to:

$$p_{ij} = ta_j^{s,ag} ta_{ij} \tau_{ij} \left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \tag{A.14}$$

Equation (A.14) shows that the externality is applied after expenditures on the transport sector have been incurred. t_{ij} is thus equal to 1 and we can write generalized marginal costs c_i thus as follows with $\widetilde{Z}_i$ as defined in equation (17):

$$c_i = \frac{\sigma}{\sigma - 1} \left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}}$$

Appendix B Melitz Economy

Appendix B.1 Demand and Production

Like in the Ethier/Krugman economy the goal of this section is to derive the expressions for generalized marginal costs c_i, generalized iceberg trade costs t_{ij} and the demand externality e_j in the Melitz economy in equations (21)-(24) and to derive the demand externality.

Agents of group ag in country j have the same CES preferences over varieties ω from different countries as in the Ethier/Krugman economy. The quantity and price index are thus given by equations (13)-(14) and demand for physical quantities $o_{ij}(\omega)$ of a variety ω by equation (A.4). In contrast to the Ethier/Krugman economy goods are produced by firms with heterogeneous productivity. Firms can sell both in domestic and foreign markets and have to pay fixed costs f_{ij} to sell in each market. The fixed costs are paid in wages of both countries with according to a Cobb Douglas specification a fraction μ paid in domestic input bundles. The fixed costs are destination-specific, but not agent-specific. So a firm pays the fixed costs ij only once for sales to all three groups of agents. Exporting firms also face iceberg trade costs τ_{ij}, bilateral tariffs ta_{ij}, agent-specific tariffs ta_j^g, export taxes te_{ij}. Moreover, there is a transport sector with firms having to spend a fixed quantity share of sales on transport services as in the Ethier-Krugman model with the cif-quantity traded o_{ij}^{cif} defined as in equation (A.5). Profits are therefore given by:

$$\begin{aligned}\pi_{ij} &= ta_j^{ag} p_{ij}^o o_{ij} - \left(ta_j^{ag} - 1\right) p_{ij}^o o_{ij} - \frac{ta_{ij}-1}{ta_{ij}} p_{ij}^o o_{ij} - \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi} \\ &= \frac{p_{ij}^o o_{ij}}{ta_{ij}} - \tau_{ij} p_{Z_i}\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi} \end{aligned} \tag{B.1}$$

We assume that productivity φ operates both on the costs of production and on the transport sector. b_i is interpreted as a source-country marginal cost shifter.[8] Each firm produces a unique variety, so we can identify demand for variety ω by the productivity φ of the firm producing this variety. Demand $o_{ij}(\varphi)$ and revenues $r_{ij}(\varphi)$ of a firm with productivity φ producing in i and selling in j are equal to:

$$o_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{B.2}$$

$$r_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{B.3}$$

Maximizing profits implies the following markup pricing rule:

$$p_{ij}^o(\varphi) = \frac{\sigma}{\sigma-1} \frac{ta_{ij}\tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi} \tag{B.4}$$

Substituting equation (B.4) back into equation (B.1) shows that profits for sales to destination

[8]In line with the GTAP model we define p_{ij}^o as the price before group specific import tariffs ta_j^{ag} are paid.

market j are equal to:

$$\pi_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij} \sigma} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{B.5}$$

So we add up the revenues for sales to the three groups of agents to calculate profit. In the profit expression in equation (B.1) we have assumed that the bilateral tariffs ta_{ij} and the group-specific importer-specific tariffs ta_j^{ag} are paid both based on the marked-up price over marginal cost, respectively on the landed cif-price and on the landed cif-price inclusive of bilateral tariffs. Iceberg trade costs τ_{ij} and export taxes te_{ij} instead are paid based on the cost level, respectively the cif cost level (so inclusive of transport costs) and fob cost level. Both types of trade costs (based on marked-up landed prices and based on cost levels) affect the optimal markup price in equation (B.4) identically, but they affect the expression for profit as a function of revenues in equation (B.5) differently. Revenues are divided by import tariffs based on landed prices to calculate profit. Import tariffs are therefore revenue-shifting, whereas iceberg trade costs and export subsidies are cost-shifting. The distinction is relevant for the gravity equation in the Melitz model, since the revenue shifting tariffs affect the cutoff productivity and therefore display a different elasticity.

Appendix B.2 Entry and Exit

Entry and exit are like in Melitz (2003), i.e. firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants NE_i drawing a productivity parameter, implying that the productivity distribution of producing firms is constant. Denoting φ_{ij}^* as the cutoff productivity, only firms with a productivity $\varphi \geq \varphi_{ij}^*$ from country i sell in market j.

Appendix B.3 Free Entry and Zero Cutoff Profit Conditions

Equilibrium is defined with a zero cutoff profit condition (ZCP) and a free entry condition (FE). According to the zero cutoff profit condition firms from country i with cutoff productivity φ_{ij}^*

can just make zero profit from sales in country j:

$$\sum_{ag=\{p,g,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{B.6}$$

Since the fixed costs are destination-specific and not group-specific there is only one ZCP for each source-destination pair and thus also only one cutoff productivity level φ_{ij}^*. Using equations (B.3)-(B.5) the ZCP can be written as follows:

$$\varphi_{ij}^* = \frac{\frac{\sigma}{\sigma-1} ta_{ij}\tau_{ij}\left(te_{ij}b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{1}{1-\sigma}} \tag{B.7}$$

The free entry condition (FE) equalizes the expected profits before entry with the sunk entry costs:

$$\sum_{ag=\{p,g,f\}} \sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^*\right)\right) \pi_{ij}^{ag}\left(\widetilde{\varphi}_{ij}\right) = \delta en_i p_{Z_i} \tag{B.8}$$

$\widetilde{\varphi}_{ij}$ is a measure of average productivity and defined as:

$$\widetilde{\varphi}_{ij} = \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1 - G_i\left(\varphi_{ij}^*\right)} d\varphi\right)^{\frac{1}{\sigma-1}} \tag{B.9}$$

Using $\frac{r_{ij}^{ag}(\varphi_1)}{r_{ij}^{ag}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ and the ZCP in equation (B.6), the FE in equation (B.8) can be written as:

$$\sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^*\right)\right) p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} f_{ij} \left(\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^*}\right)^{\sigma-1} - 1\right) = \delta en_i p_{Z_i} \tag{B.10}$$

The distribution of initial productivities $G_i(\varphi)$ is Pareto:

$$G_i(\varphi) = 1 - \frac{\kappa_i^{\theta}}{\varphi^{\theta}} \tag{B.11}$$

with θ the shape parameter and κ_i the size parameter. We impose $\theta > \sigma - 1$ to guarantee that expected revenues are finite. With a Pareto distribution $\widetilde{\varphi}_{ij}$ is proportional to φ_{ij}^*:

$$\widetilde{\varphi}_{ij} = \left(\frac{\theta}{\theta - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \varphi_{ij}^* \tag{B.12}$$

Substituting equations (B.11)-(B.12) into the fe, equation (B.10), gives:

$$\sum_{j=1}^{J}\left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} f_{ij} \frac{\sigma-1}{\theta-\sigma+1} = \delta en_i p_{Z_i} \tag{B.13}$$

Appendix B.4 Equivalence of The Price Index

To show equivalence of the price index in the general representation version of the Melitz model and the normal version, we write the price index in (14) as equation (A.2) with the representative price $\frac{p_{ij} ta_j^{s,ag}}{e_j^s}$ defined as:

$$\frac{p_{ij} ta_j^{s,ag}}{e_j^s} = \left(\int\limits_{\omega\in\Omega_{ij}^{ag}} p^o(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} \tag{B.14}$$

$\frac{p_{ij} ta_j^{s,ag}}{e_j^s}$ is the representative price including the demand externality. The representative price in equation (B.14) can be redefined as an integral over productivities of the producing firms as follows:

$$\frac{p_{ij} ta_j^{s,ag}}{e_j^s} = \left(\int_{\varphi_{ij}^*}^{\infty} N_{ij} p_{ij}^{ag,o}(\varphi)^{1-\sigma} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^*\right)} d\varphi\right)^{\frac{1}{1-\sigma}} \tag{B.15}$$

Using equations (B.4) and (B.9) the representative price in equation (B.15) can be rewritten as a function of average productivities:

$$\frac{p_{ij} ta_j^{s,ag}}{e_j^s} = \frac{\sigma}{\sigma-1}\left(N_{ij}\left(ta_{ij} ta_j^{s,ag} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma} \widetilde{\varphi}_{ij}^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \tag{B.16}$$

The mass of varieties sold from country i to country j, N_{ij} is related to the mass of entrants NE_i and the cutoff productivity φ_{ij}^* by the following steady state condition:

$$N_{ij} = \frac{\left(1-G_i\left(\varphi_{ij}^*\right)\right) NE_i}{\delta} = \left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta} \frac{NE_i}{\delta} \tag{B.17}$$

The steady state of entry and exit implies that NE_i can be written as a function of the number of input bundles Z_i:

$$NE_i = \frac{\sigma-1}{\theta\sigma}\frac{\widetilde{Z}_i}{en_i} = \frac{\sigma-1}{\theta\sigma}\frac{Z_i - \sum_{j=1}^{J} N_{ij}\frac{\sigma-1}{\sigma}\left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{b_i p_{Z_i} a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{b_i p_{Z_i} a_{ij}^{tr}}}\right)\frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_j^{g} ta_{ij}}}{en_i} \tag{B.18}$$

Since $N_{ij}\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)$ is equal to the value of trade (inclusive of bilateral import tariffs ta_{ij}, but inclusive of group- and importer-specific tariffs ta_j^{ag}) and thus equal to $N_i\overline{r}_{ij}$ in the Ethier-Krugman model, we can use the same definition for $\widetilde{Z}_i$ in both models. Using equations (B.12), (B.17) and (B.18), the representative price in equation (B.16) can be written as:

$$\frac{p_{ij}ta_j^{s,ag}}{e_j^s} = \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\frac{\left(ta_{ij}ta_j^{s,ag}\tau_{ij}\left(b_i te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^{*}\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \tag{B.19}$$

The final step is to substitute the ZCP solved for φ_{ij}^{*} in equation (B.7) into equation (B.19) generating the following expression:

$$\frac{p_{ij}ta_j^{s,ag}}{e_j^s} = \left(\frac{\gamma_m\kappa_i^{\theta}\widetilde{Z}_i\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}\left(ta_{ij}^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}\left(ta_j^{s,ag}\right)^{1-\sigma}}{\delta en_i}\right)^{\frac{1}{1-\sigma}}$$

$$* \left(\frac{\sum_{ag=\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.20}$$

γ_m is defined in equation (22) in the main text. From equation (B.20) we can easily determine the source-specific component, c_i, the bilateral component, $ta_{ij}t_{ij}$, and the destination specific component, e_j^s, in equation (A.3), the general setup-expression for the price in the Melitz model. The source specific component in equation (B.20) is equal to:

$$c_i = \left(\frac{\gamma_m\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.21}$$

The pairwise component in equation (B.20) is given by:

$$t_{ij}ta_{ij}\left(b_i te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)=\left(b_i te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}}\left(ta_{ij}\tau_{ij}\right)^{\frac{\theta}{\sigma-1}}\left(ta_{ij}f_{ij}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.22}$$

Rearranging leads to the expression for t_{ij} in the main text, equation (23):

$$t_{ij}=\left(\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}}ta_{ij}^{\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}}f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)\tau_{ij} \tag{B.23}$$

Finally, the destination specific terms in equation (B.20) represent the demand externality, giving:

$$e_j^s=\left(\frac{\sum_{ag=\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.24}$$

So we have shown that the general setup-expression for the price index in equation (A.2) employing expressions for c_i in equation (21), t_{ij} in equation (23) and e_j^s in equation (24) follows from a Melitz structure and is thus equivalent to a Melitz structure.

Appendix B.5 Equivalence of Quantity Index

To prove equivalence between the general setup and the Melitz setup, we also show that the general setup-expression for demand in equation (A.1) is equivalent to the expression for demand following from the Melitz structure. Substituting the expressions for t_{ij}, c_i and e_j^s into the expression for $q_{ij}e_j^s$ in equation (A.1) leads to:

$$\begin{aligned} q_{ij}e_j^s=&\left(\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}}\tau_{ij}^{\frac{\theta}{\sigma-1}}ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}\left(\frac{\gamma_m\kappa_i^\theta\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}}\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}\\ &*\left(\sum_{ag\in\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}\end{aligned} \tag{B.25}$$

Next we show that the expression for quantity $q_{ij}e_j^s$ inclusive of the demand-side externality starting from the Melitz-setup is identical to the expression in equation (B.25). We can write the quantity starting from the Melitz-setup as follows:

$$q_{ij}e_j^s = \left(\int\limits_{\omega\in\Omega_{ij}} o\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{B.26}$$

Redefining quantity in equation (B.26) as an integral over the productivity of producing firms gives:

$$q_{ij}e_j^s = \left(N_{ij} \int\limits_{\varphi_{ij}^*}^{\infty} o_{ij}\left(\varphi\right)^{\frac{\sigma-1}{\sigma}} \frac{g\left(\varphi\right)}{1-G\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{\sigma}{\sigma-1}} \tag{B.27}$$

Substituting the expression for $q_{ij}\left(\varphi\right)$ in equation (B.2), representative quantity in equation (B.27) can be written as a function of average productivity:

$$q_{ij}e_j^s = N_{ij}^{\frac{\sigma}{\sigma-1}} o_{ij}\left(\widetilde{\varphi}_{ij}\right) \tag{B.28}$$

The next step is to use $\frac{o_{ij}(\varphi_1)}{o_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$ and equation (B.12) to write $o_{ij}\left(\widetilde{\varphi}_{ij}\right)$ as a function of cutoff quantity $o_{ij}\left(\varphi_{ij}^*\right)$:

$$q_{ij}e_j^s = N_{ij}^{\frac{\sigma}{\sigma-1}} o_{ij}\left(\varphi_{ij}^*\right) \left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{\sigma}{\sigma-1}} \tag{B.29}$$

The ZCP in equation (B.6) can be employed to express cutoff quantity $o_{ij}^{ag}\left(\varphi_{ij}^*\right)$ as follows:

$$o_{ij}\left(\varphi_{ij}^*\right) = \left(\sigma-1\right) \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij}\left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)} \varphi_{ij}^* \tag{B.30}$$

Substituting equation (B.30) and also the expressions for N_{ij} and NE_i in equations (B.17)-(B.18) into equation (B.29) leads to:

$$q_{ij}e_j^s = \frac{\left(\frac{\sigma-1}{\sigma(\theta-\sigma+1)}\right)^{\frac{\sigma}{\sigma-1}} \left(\sigma-1\right) \left(\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij}\left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}}{\left(\varphi_{ij}^*\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}} \tag{B.31}$$

Finally, the ZCP solved for φ_{ij}^* in equation (B.7) can be substituted into equation (B.31) and after several rearrangings, we get the same expression as the general setup-expression in equation (B.25).

Appendix C Symmetric Two-Country Single-Sector Model

Appendix C.1 Equilibrium Equations

We follow Melitz and Redding (2013) and define equilibrium of the Melitz model in a single-sector setting with two identical countries and a single factor of production L as a solution for φ_d and φ_x as follows:

$$f_d J(\varphi_d) + f_x J(\varphi_x) = \delta e n \tag{C.1}$$

$$\varphi_x = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d \tag{C.2}$$

With two symmetric countries we do not need country subscripts and use instead subscripts for domestic (d) and exporting (x) values. f_d and f_x are domestic and exporting fixed costs, φ_d and φ_x the domestic and exporting cutoff productivities. τ represents iceberg trade costs with domestic iceberg trade costs normalized at 1. $J(\varphi)$ is defined as follows:

$$J(\varphi) = (1 - G(\varphi)) \left(\left(\frac{\widetilde{\varphi}(\varphi)}{\varphi}\right)^{\sigma-1} - 1\right) \tag{C.3}$$

With a Pareto distribution as in equation (B.11) this gives:

$$J(\varphi) = \frac{\kappa^\theta}{\varphi^\theta}\left(\frac{\theta}{\theta-\sigma+1} - 1\right) = \frac{\kappa^\theta}{\varphi^\theta}\frac{\sigma-1}{\theta-\sigma+1} \tag{C.4}$$

Hence, equation (C.1) can be written as follows:

$$\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{f_d \varphi_{\min}^\theta}{\left(\varphi_d^T\right)^\theta} + \frac{f_x \varphi_{\min}^\theta}{\left(\varphi_x^T\right)^\theta}\right) = en \tag{C.5}$$

Next, we derive an expression for welfare, equal to the real wage w/P, based on the cutoff productivity φ_d. With the wage w normalized at 1, welfare is equal to the inverse of P:

$$\frac{w}{P} = \frac{\sigma-1}{\sigma}\left(\frac{L(1+\chi)}{\sigma F^T}\left(\widetilde{\varphi}_t^T\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{C.6}$$

With $\widetilde{\varphi}_t$, F, and χ respectively average productivity, a composite of fixed costs and the probability of exporting and defined as:

$$(\widetilde{\varphi}_t)^{\sigma-1} = \frac{1}{1+\chi}\left[(\widetilde{\varphi}_d)^{\sigma-1} + \chi\left(\tau^{-1}\widetilde{\varphi}_x\right)^{\sigma-1}\right] \tag{C.7}$$

$$F = \frac{en}{1-G(\varphi_d)} + f_d + \chi f_x \tag{C.8}$$

$$\chi = \frac{1-G(\varphi_x)}{1-G(\varphi_d)} = \left(\frac{\varphi_x}{\varphi_d}\right)^{-\theta} \tag{C.9}$$

Using equation (B.12), welfare can thus be written as follows:

$$\frac{w}{P} = \frac{\sigma-1}{\sigma}\left(\frac{L\frac{\theta}{\theta-\sigma+1}\left[\left(\varphi_d^T\right)^{\sigma-1} + \chi\left(\tau^{-1}\varphi_x^T\right)^{\sigma-1}\right]}{\sigma F^T}\right)^{\frac{1}{\sigma-1}} \tag{C.10}$$

In the homogeneous firms model defined by Melitz and Redding for comparison with the heterogeneous firms model, we use the same expressions for welfare, except for the fact that with changes in trade costs τ, the cutoff productivities serve as average productivities and do not change.

To allow for the case of granularity where $\theta = \sigma - 1$, we drop the constant $\frac{\sigma-1}{\theta-\sigma+1}$ in equation (C.5) determining φ_d and φ_x and we drop the constant $\frac{\theta}{\theta-\sigma+1}$ in the expression for welfare in equation (C.10). These changes do not affect the results on the effect of changes in trade costs on welfare, since both changes only shift the level of welfare proportionally for all levels of trade costs.

Since we do not impose an identical substitution elasticity in the different models, we do not have an identical level of welfare at the starting level of trade costs. This does not affect our results though and to account for different levels of welfare in the starting position we report the welfare effects as a function of percentage changes in trade costs. Since the models are structurally different, it is logical that welfare levels in the starting position are different. We could have generated an identical level of welfare in all different models at the starting level of trade costs by adjusting parameters that do not affect the results like the Pareto shift parameter κ.

Appendix C.2 Calibration

Calibration of the parameters θ and σ is discussed in the main text. To calibrate to the overall import share λ and the share of exporting firms χ, we define the import share λ as:

$$\lambda = \frac{1}{1 + \tau^{1-\sigma}\left(\frac{\varphi_x}{\varphi_d}\right)^{-(\theta-\sigma+1)}} \tag{C.11}$$

The share of exporting firms is equal to the probability of exporting χ defined in equation (C.9). We solve for f_x/f_d and τ as a function of λ and ef, the fraction of exporting firms. Using equation (C.2) for the ratio of productivities we get for ef and λ the following two equations:

$$ef = \tau^{-\theta}\left(\frac{f_x}{f_d}\right)^{-\frac{\theta}{\sigma-1}} \tag{C.12}$$

$$\lambda = \frac{1}{1 + \tau^{-\theta}\left(\frac{f_x}{f_d}\right)^{-\frac{\theta-\sigma+1}{\sigma-1}}} \tag{C.13}$$

Solving for $\tau^{-\theta}$ from equation (C.12) gives:

$$\tau^{-\theta} = ef\left(\frac{f_x}{f_d}\right)^{\frac{\theta}{\sigma-1}} \tag{C.14}$$

Substituting (C.14) into equation (C.13) and solving for f_x/f_d gives:

$$f_x/f_d = \frac{1-\lambda}{\lambda ef} \tag{C.15}$$

Substituting back into equation (C.14) and solving for τ gives:

$$\tau = \exp - \frac{\ln ef\left(\frac{1-\lambda}{\lambda ef}\right)^{\frac{\theta}{\sigma-1}}}{\theta} \tag{C.16}$$

We can calibrate to the export share of firms es and the share of exporting firms ef, using the expression for ef in equation (C.12) and the following expression for es:

$$es = \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \tag{C.17}$$

Solving equation (C.17) for τ gives:

$$\tau = \exp\left(-\frac{\ln\frac{es}{1-es}}{\sigma - 1}\right) \tag{C.18}$$

f_x/f_d then follows from equation (C.12):

$$\frac{f_x}{f_d} = \exp -\xi \ln ef\tau^{\theta} \tag{C.19}$$

We cannot calibrate both to the fraction of export sales and the import share if we want to allow for the granularity case. The fraction of export sales would fix a value for τ, so the calibrated import share would be determined by values of f_x/f_d, creating a problem in the case of granularity, since the coefficient on fixed costs in the expression for the import share λ is $\theta - \sigma + 1$ and would thus become zero in case of granularity.

Appendix C.3 Equivalence with General Setup Model

We check the correctness of the Melitz general setup model by comparing the model outcomes with the outcomes of the Melitz model employing the full set of equations. We do this in two settings. First, we take the symmetric two-country model used above and second, we look at a two-country model with intermediate linkages. We also explore the second setting since with intermediate linkages the demand externality e_j plays a role in determining the price of input bundles p_{Z_i} and the wage w_i, enabling us to check the correctness of the way we deal with the demand side.

We eliminate several institutional details from GTAP in our two-counry model with intermediate linkages. So we examine a model without a transport sector, without import tariffs and export subsidies, without a separate nest between domestic and imported goods and with only one group of agents, private households. Imposing the general equilibrium condition that gross output $p_{Z_i}Z_i$ is equal to the value of exports to all destination countries j and using the fact that the absence of tariffs and trade imbalances implies that demand E_j is equal to $p_{Z_j}Z_j$, gives us the following general equilibrium condition:

$$p_{Z_i}Z_i = \sum_{j=1}^{J}\left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} p_{Z_j}Z_j \tag{C.20}$$

In a setting without intermediate linkages, the price and quantity of input bundles are respectively equal to wages w_i and quantity of labor L_i. The expressions for the price index and the underlying generalized price are:

$$P_j = \left(\sum_{i=1}^{J} p_{ij}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{C.21}$$

$$p_{ij} = \frac{ta_{ij}t_{ij}c_i p_{Z_i}}{e_j} \tag{C.22}$$

We have the following expressions for c_i, t_{ij} and e_j. Because of the absence of per unit transport services, we have included the term in p_{Z_i} in t_{ij} in c_i, which is possible :

$$c_i = \gamma_m \left(\frac{\kappa_i^{\theta} Z_i}{\delta e n_i}\right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\frac{\theta-\sigma+1}{\sigma-1}+\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{C.23}$$

$$t_{ij} = \tau_{ij}^{\frac{\theta}{\sigma-1}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{C.24}$$

$$e_j = \left(\frac{P_j^{\sigma-1} p_{Z_j} Z_j}{p_{Z_j}^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{C.25}$$

Without intermediate linkages we solve equations (C.20)-(C.25) for w_i, P_i, p_{ij}, c_i and e_j. With intermediate linkages we add the following two additional equilibrium equations implying that P_j has to be solved simultaneously with the other variables:

$$p_{Z_i} = w_i^{\beta_i} P_i^{1-\beta_i} \tag{C.26}$$

$$p_{Z_i} Z_i = \frac{w_i L_i}{\beta_i} \tag{C.27}$$

We assume that input bundles are a Cobb-Douglas aggregate over labor and intermediates with intermediates identical to final goods.[9] So with intermediates we solve equations (C.20)-(C.27) for p_{Z_i}, Z_i, w_i, P_i, p_{ij}, c_i and e_j.

We compare the Melitz general setup model in the symmetric two-country case with the equilibrium in the previous subsection. So we solve equations (C.2) and (C.5) for φ_d and φ_x and determine the real wage from equation (C.10). In the model with intermediates we compare the Melitz general setup model with the following set of equilibrium equations: the

[9] With a Leontief specification, we would have the following expressions:

$$p_{Z_i} = w_i\beta + (1-\beta)\, P_i$$

$$Z_i = \frac{L_i}{\beta}$$

expression for the price index following from equation (B.19); the expression for the number of varieties following from equations (B.17) and (B.18); a demand equation; an expression for cutoff revenues following from equation (B.3); a markup pricing expression in equation (B.4); and a zero cutoff profit condition in equation (B.6). The free entry condition is substituted in both the expression for the number of varieties and the demand equation. This gives the following set of equations:

$$(P_i)^{1-\sigma} = \sum_{j=1}^{J} N_{ji} \frac{\theta}{\theta - \sigma + 1} p_{ji} \left(\varphi_{ji}^*\right)^{1-\sigma} \tag{C.28}$$

$$N_{ij} = \left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta_i} \frac{\sigma - 1}{\sigma\theta_i} \frac{Z_i}{\delta en_i} \tag{C.29}$$

$$p_{Z_i} Z_i = \sum_{j=1}^{J} N_{ij} \frac{\theta}{\theta - \sigma + 1} r_{ij} \left(\varphi_{ij}^*\right) \tag{C.30}$$

$$r_{ij} \left(\varphi_{ij}^*\right) = p_{ij} \left(\varphi_{ij}^*\right)^{1-\sigma} (P_i)^{\sigma-1} E_j \tag{C.31}$$

$$p_{ij} \left(\varphi_{ij}^*\right) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} p_{Z_i}}{\varphi_{ij}^*} \tag{C.32}$$

$$r_{ij} \left(\varphi_{ij}^*\right) = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{\mu} \tag{C.33}$$

We solve equations (C.28)-(C.33) together with equations (C.26)-(C.27) for P_i, $p_{ij}\left(\varphi_{ij}^*\right)$, N_{ij}, φ_{ij}^*, Z_i, p_{Z_i}, $r_{ij}\left(\varphi_{ij}^*\right)$, w_i.

We show that relative welfare changes as a function of relative changes in iceberg trade costs are identical employing the general setup set of equations and employing respectively the Melitz-Redding set of equations (for the symmetric two-country model) and the full set of equations (for the model with intermediate linkages). We do this for the three different parameter values also used for the Melitz model in the main text in Subsection 6.2.

Appendix C.4 Calibration Model with Intermediate Linkages

We use the symmetric two-country single-sector model with intermediate linkages also to run robustness checks on the influence of variation in parameters on the welfare effects of trade costs changes. Therefore, we discuss here calibration of trade costs in this model. The share of goods imported into country j from country i, s_{ij} is given by:

$$s_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{1-\sigma}$$

In the model with all equilibrium equations we write the import share as follows:

$$\begin{aligned} s_{ij} &= \left(\frac{p_{ij}\left(\widetilde{\varphi}_{ij}\right)}{P_j}\right)^{1-\sigma} \\ &= \left(\frac{\left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{1}{1-\sigma}} p_{ij}\left(\varphi_{ij}^{*}\right)}{P_j}\right)^{1-\sigma} \\ &= \frac{\theta}{\theta-\sigma+1}\left(\frac{p_{ij}\left(\varphi_{ij}^{*}\right)}{P_j}\right)^{1-\sigma} \end{aligned}$$

To determine the baseline level of iceberg trade costs τ_{ij}, we endogenize τ_{ij}, imposing the above equations. We normalize domestic iceberg trade costs τ_{ii} at 1. We set f_{ij} at 1.

Instead of setting f_{ij} at 1, we can also calibrate to the share of exporting firms as follows to determine the size of f_{ij}, again normalizing f_{ii} at 1:

$$\chi = \frac{N_{ij}}{N_{ii}} \tag{C.34}$$

We find an expression for t_{ij} from the baseline simulation. Using equation (C.24), we can then write τ_{ij} as a function of t_{ij} and f_{ij}:[10]

$$\tau_{ij} = t_{ij}^{\frac{\sigma-1}{\theta}} f_{ij}^{-\frac{(\theta-\sigma+1)}{\theta(\sigma-1)}} \tag{C.35}$$

Appendix D Implementation in GTAP GEMPACK

We implement the Melitz structure with demand and supply side externalities and generalized iceberg trade costs in the GTAP model programmed in GEMPACK. We outline for each of the three topics first the blocks added to the GEMPACK code and then how the existing code is adjusted. Then we discuss parameterization in GEMPACK to continue this section with a discussion of how to move between the different models employing closure swaps. We finish this section with a discussion of the margin decomposition in GEMPACK. In the implementation we assume that all fixed exporting costs are paid in the source country, i.e. $\mu = 1$.

[10] We normalize f_{ij} at 1.

Appendix D.1 Supply-Side Externality

The supply-side externality in the Ethier-Krugman and Melitz model can be gathered by log differentiating respectively equations (15) and (21):

$$\widehat{c_i} = -\frac{1}{\sigma-1}\widehat{N_i} \tag{D.1}$$

$$\widehat{c_i} = -\frac{1}{\rho}\widehat{T_i\left(Z_i\right)} = -\frac{1}{\rho}\widehat{Z_i} \tag{D.2}$$

$$\widehat{c_i} = -\frac{1}{\sigma-1}\widehat{NE_i} + \frac{\theta-\sigma+1}{(\sigma-1)^2}\widehat{p_{Z_i}} \tag{D.3}$$

In GEMPACK we model respectively the Ethier-Krugman, Eaton-Kortum and Melitz supply-side externality as follows:

$$oscaleek(i,r) = ekscale(i,r) - [1/(\sigma-1)] * nne(i,r) \tag{D.4}$$

$$oscaleeako(i,r) = eakoscale(i,r) - \frac{1}{\sigma} * qo(i,r) \tag{D.5}$$

$$\begin{aligned} oscalem(i,r) &= mscale(i,r) - [1/(\sigma-1)] * nne(i,r) \\ &+ \frac{\theta-\sigma+1}{(\sigma-1)^2} * [ps(i,r) - pfactwld] \end{aligned} \tag{D.6}$$

In equation (D.5) we have used that in the Eaton-Kortum model $\sigma = \rho$.

We deflate the price change term $ps\left(i,r\right)$ in the calculation of the Melitz-externality in equation (D.6) by the numeraire $pfactwld$, such that a change in all prices does not change the size of the externality and is neutral. To move between the different supply-side externalities we add the following additional equation:

$$oscaleekm(i,r) = ekscale(i,r) + eakoscale\left(i,r\right) + emscale(i,r) - sext(i,r) \tag{D.7}$$

We use the same variable for the relative change in the number of firms in the Ethier-Krugman model and in the number of entrants in the Melitz model, $nne\left(i,r\right)$, since the two are identical. This becomes clear by log differentiating equation (A.11) or equivalently equation (B.18). In GEMPACK notation we get:

$$N_i = \frac{\widetilde{Z}_i}{\sigma a_i} = \frac{Z_i - \frac{\sigma-1}{\sigma}\left(\sum_{j=1}^{J}\frac{N_i\overline{r}_{ij}}{p_{Z_i}ta_{ij}\left(te_{ij}b_i + \frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)} - \frac{N_i\overline{r}_{ij}}{p_{Z_i}ta_{ij}}\right)}{\sigma a_i}$$

$$
\begin{aligned}
nneh(i,r) = {} & \frac{VOM\,(i,r)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)} qo\,(i,r) \\
& - \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma}VXMD\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)} (pcif\,(i,r,s) + qxs\,(i,r,s) \\
& - \frac{VXWD\,(i,r,s)}{VIWS\,(i,r,s)} \left(ps\,(i,r) + ao\,(i,r) - tx\,(i,r) - tx\,(i,r,s)\right) \\
& - \frac{VIWS\,(i,r,s) - VXWD\,(i,r,s)}{VIWS\,(i,r,s)} ptrans\,(i,r,s)) \\
& + \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma}VIWS\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)} \\
& * \left(pcif\,(i,r,s) + qxs\,(i,r,s) - \left(ps\,(i,r) + ao\,(i,r)\right)\right) - nne\,(i,r) \qquad \text{(D.8)}
\end{aligned}
$$

So the expression for the number of varieties contains additional terms, reflecting the size of transport services and export subsidies to all destination partners. Moreover, we have to take into account that the variety scaling term has to be applied to the cif-price, so inclusive of transport costs, for the international price and quantity. Therefore, we have to write the iceberg trade costs technology shifter $ams\,(i,r,s)$ as a function of the supply-side externality. We cannot include the supply-side externality before the transport sector is added, since we would have to multiply all terms by $1/FOBSHR\,(i,r,s)$ which would be destination specific. Since the domestically sold goods do not feature transport costs, but do benefit from variety scaling, the variety scaling term also affects domestic prices and quantities, i.e. *ppd*, *pgd* and pfd and *qpd*, *qgd* and qfd.

Appendix D.2 Demand-Side Externality

To model the demand-side externality, we add a block to the model calculating the demand-side externality and we adjust the price and quantity expressions for domestic and imported goods for the three groups of agents, private households, governments and firms.

First, we discuss the additional block for the demand-side externality. Log differentiating the theoretical expression for the externality in equation (24) gives:

$$e_j^s = \left(\frac{\sum\limits_{ag=\{s,p,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}$$

$$\widehat{e_j^s} = \sum_{ag=\{g,p,f\}} \frac{\left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{\sum\limits_{ag'=\{s,p,f\}} \left(\frac{P_j^{ag'}}{ta_j^{s,ag'}} \right)^{\sigma-1} \frac{E_j^{ag'}}{ta_j^{s,ag'}}} \left(\frac{\theta-\sigma+1}{\sigma-1} \left(\widehat{P_j^{ag}} - \widehat{ta_j^{s,ag}} \right) + \frac{\theta-\sigma+1}{(\sigma-1)^2} \left(\widehat{E_j^{ag}} - \widehat{ta_j^{s,ag}} \right) \right) \tag{D.9}$$

Multiplying the numerator and denominator of the coefficient by $\left(\frac{p_j^s}{e_j^s}\right)^{1-\sigma}$, we can rewrite equation (D.9) as follows:

$$\widehat{e_j^s} = \sum_{ag=\{g,p,f\}} \frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}} \left(\frac{\theta-\sigma+1}{\sigma-1} \left(\widehat{P_j^{ag}} - \widehat{ta_j^{s,ag}} \right) + \frac{\theta-\sigma+1}{(\sigma-1)^2} \left(\widehat{E_j^{ag}} - \widehat{ta_j^{s,ag}} \right) \right) \tag{D.10}$$

To find the equivalent expression in GTAP notation, we observe that $p_j^s q_j^{s,ag}$ represents the expenditures of group $ag = f, p, g$ on source $s = d, m, V, S, AG, M$. So, equation (D.10) can be written in GEMPACK notation as follows with $s = m, d$:

$$\begin{aligned} dscale1s\,(i,r) = {} & \frac{\theta-\sigma+1}{\sigma-1} \left(priceDs\,(i,r) - pfactwld\right) + \frac{\theta-\sigma+1}{(\sigma-1)^2} \left(valueDs\,(i,r) - pfactwld\right) \\ & - \frac{\sigma\left(\theta-\sigma+1\right)}{(\sigma-1)^2} tariffDs\,(i,r) \end{aligned} \tag{D.11}$$

With $priceDs\,(i,r)$ the price index term of the externality in sector i in country r for source $s = d, m$, $valueDs\,(i,r)$ the value term and $tariffDs\,(i,r)$ the tariff term and defined for $s = m$ as (the expressions for $s = d$ are similar):

$$\begin{aligned} priceDm(i,r) = {} & SHRIPM * [pp(i,r)] + SHRIGM * [pg(i,r)] \\ & + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * [pf(i,j,r)]) \end{aligned} \tag{D.12}$$

And:

$$
\begin{aligned}
valueDm(i,r) = {} & SHRIPM * [pp(i,r) + qp\,(i,r)] \\
& + SHRIGM * [pg(i,r) + qg\,(i,r)] \\
& + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * [pf(i,j,r) + qf\,(i,j,r)]) \quad \text{(D.13)}
\end{aligned}
$$

And:

$$
\begin{aligned}
tariffDm(i,r) = {} & SHRIPM * tpm(i,r) + SHRIGM * tgm(i,r) \\
& + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * tfm(i,j,r)) \quad \text{(D.14)}
\end{aligned}
$$

pp, pg, and pf are the relative price changes for private households, government and firms and qp, qg, and qf the quantity equivalents. $SHRIPM\,(i,r)$ is defined as:

$$
SHRIPM\,(i,r) = \frac{VIPM\,(i,r)}{VIM\,(i,r)} \quad \text{(D.15)}
$$

With $VIM\,(i,r)$ the sum of import demand at market prices:

$$
VIM(i,r) = VIPM(i,r) + VIGM(i,r) + sum(j, PROD\_COMM, VIFM(i,j,r)) \quad \text{(D.16)}
$$

$SHRIGM\,(i,r)$ and $SHRIFM\,(i,j,r)$ are defined similarly. As for the supply-side externality, we deflate the price and value changes (based on price changes) in the calculation of the externality by the numeraire, such that a change in all prices does not change the externality.

To determine how the expressions for domestic and importer demand and price for the three groups of agents in the GTAP model change, we define the domestic and importer price, inclusive of the externality and the agent-specific tax, $\widetilde{p}_j^{s,ag}$, as follows:

$$
\widetilde{p}_j^{s,ag} = \frac{ta_j^{s,ag} p_j^{s,ag}}{e_j^s} \quad \text{(D.17)}
$$

Log differentiating both equation (D.17) and the rewritten expression for demand in equation

(3) gives:

$$\widehat{q_j^{s,ag}} = \sigma \left(\widehat{P_j^{ag,e}} - \widehat{\widetilde{p}_j^{s,ag}} \right) + \widehat{q_j^{ag,e}} - \widehat{e_j^s} \tag{D.18}$$

$$\widehat{\widetilde{p}_j^{s,ag}} = \widehat{ta_j^{s,ag}} + \widehat{p_j^{s,ag}} - \widehat{e_j^s} \tag{D.19}$$

The equivalent expressions in GTAP for domestic government goods is given by:

$$qgd(i,s) = ESUBD(i) * [pg(i,s) - pgd(i,s)] + qg(i,s) - Dextd(i,s) \tag{D.20}$$

$$pgd(i,s) = tgd(i,s) + pm(i,s) - Dextd(i,s) \tag{D.21}$$

$$pgm(i,s) = tgm(i,s) + pim(i,s) - Dextm(i,s) \tag{D.22}$$

with qgd and qg the domestic and total government demand; pgd, pgm and pg, the domestic, imported and overall price of government consumption; tgd and tgm the tax on domestic and imported government consumption; pm and pim the domestic and import price of goods; and $Dextd$ the domestic demand externality. So we model the demand externality as a technology shifter to domestic and imported demand.

Appendix D.3 Generalized Iceberg Trade Costs

The generalized iceberg trade costs are equal to the normal iceberg trade costs in the Armington, Ethier-Krugman and Eaton-Kortum model. Only in the Melitz model the two are distinct and generalized iceberg trade costs are defined in equation (23). Log differentiating this equation gives:

$$\begin{aligned} \widehat{t_{ij}} &= \frac{\theta - \sigma + 1}{\sigma - 1} \widehat{te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}} + \left(1 + \frac{\theta - \sigma + 1}{\sigma - 1} \right) \widehat{\tau_{ij}} \\ &+ \frac{\sigma \left(\theta - \sigma + 1 \right)}{\left(\sigma - 1 \right)^2} \widehat{ta_{ij}} + \frac{\theta - \sigma + 1}{\left(\sigma - 1 \right)^2} \widehat{f_{ij}} \end{aligned} \tag{D.23}$$

In the GTAP model (with all variables expressed in relative change terms) bilateral ad-valorem tariffs $\widehat{ta_{ij}}$ consist of import tariffs tm and tms and the iceberg trade costs $\widehat{\tau_{ij}}$ consist of an iceberg-trade-costs-like technology shifter ams. Tariffs are paid based on the marked-up prices, whereas iceberg trade costs and the transport margin operate on the physical quantities and are thus based on costs. As a result, the coefficient on tariffs in generalized trade costs is different.

Since both the generalized iceberg trade costs t_{ij} and the generalized marginal costs c_i are applied on the cif-price, we endogenize the iceberg-trade-cost-like technology shifter $ams\,(i,r,s)$ as a function of the supply-side externality $sext\,(i,r)$ and generalized iceberg trade costs. In GEMPACK notation we get in the Ethier-Krugman/Eaton-Kortum and Melitz model respectively:

$$genitcekh(i,r,s) = -sext(i,r) + itc(i,r,s) - genitcek(i,r,s) \tag{D.24}$$

$$\begin{aligned} genitcmh(i,r,s) = &-sext(i,r) + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2}(tm(i,s)+tms(i,r,s)) + \left(1+\frac{\theta-\sigma+1}{\sigma-1}\right) itc(i,r,s) \\ &+ \frac{\theta-\sigma+1}{(\sigma-1)^2} fex\,(i,r,s) + \frac{\theta-\sigma+1}{\sigma-1} pcif\,(i,r,s) - genitcm\,(i,r,s) \end{aligned} \tag{D.25}$$

We shift between the Ethier-Krugman/Eaton-Kortum and Melitz model with the following equation:

$$genitcekm(i,r,s) = genitcek(i,r,s) + genitcm(i,r,s) + ams(i,r,s) \tag{D.26}$$

We add the variable *itc* to the model, which represents normal iceberg trade cost in the Ethier-Krugman and Melitz specification of the model. Since $ams\,(i,r,s)$ is a technology-shifter and a positive shock to *ams* represents a reduction in iceberg trade costs in the standard model, we add *ams* in the above equation instead of subtracting it. The existing code of the model does not have to be adjusted to account for Melitz-generalized trade costs and only requires a closure swap. Since $sext\,(i,r)$ can be either Ethier-Krugman, Eaton-Kortum or Melitz depending on the swap chosen in equation (D.7) and since the generalized trade cost is given by iceberg trade costs τ_{ij} (*itc* in GTAP relative changes) in both Ethier-Krugman and Eaton-Kortum, we can use one equation, equation (D.24), for both models.

Appendix D.4 Parameterization

We need values for the parameters σ in the Armington, Ethier-Krugman and Melitz model, θ in the Melitz model and ρ and η in the Eaton-Kortum model. From the empirics we have estimates for the tariff elasticity $\varepsilon_{v,ta}$ and the degree of granularity ξ. By varying the parameters *etil* and *gran*, based on the estimated $\varepsilon_{v,ta}$ and ξ, we switch between the parameterizations of the different models.

Starting with the Melitz model, we have:

$$\varepsilon_{v,ta} = \theta + 1 + \frac{\theta - \sigma + 1}{\sigma - 1} \tag{D.27}$$

$$\xi = \frac{\sigma - 1}{\theta} \tag{D.28}$$

We can thus express θ and σ as a function of the estimated $\widetilde{e}$ and g as follow:

$$\sigma = \xi * \varepsilon_{v,ta} \tag{D.29}$$

$$\theta = \varepsilon_{v,\tau} - \frac{1}{\xi} \tag{D.30}$$

Granularity ξ approaching 1 means that the model is approaching so-called "full granularity" with $\theta = \sigma - 1$.

Melitz and Redding (2013) compare the Melitz and Ethier-Krugman model imposing an identical value for the substitution elasticity σ instead of identical value for the tariff and trade elasticity. To mimic their experiment we impose $\sigma = \varepsilon_{v,ta}$ and $\theta = \frac{\varepsilon_{v,ta}-1}{\xi_{MR}}$ with ξ_{MR} the degree of granularity implied by the experiments in Melitz and Redding (2013). Since these authors set σ at 4 and θ at 4.25, we have $\xi_{MR} = \frac{\sigma-1}{\theta} = \frac{3}{4.25} \approx 0.71$. To run this experiment, we will have to use a separate tabfile, since σ and θ are different functions of $\varepsilon_{v,ta}$ and ξ than in the other calibrations.

In the Armington and Ethier-Krugman model we only need a value for σ, which is equal to $\varepsilon_{v,ta}$. In the Eaton and Kortum model we need a value for the dispersion parameter ρ, which is equal to the tariff elasticity minus one, $\varepsilon_{v,ta} - 1$. In the implementation in GTAP we do not replace the substitution elasticity $\sigma = esubd$ in the code by $\rho = rho$, but keep working with $esubd$ and recognize that we get the Eaton-Kortum equations if we impose $esubd = rho = \varepsilon_{v,ta} - 1$ and adjust the parameter values accordingly.[11] To work with $esubd$ set equal to $\varepsilon_{v,ta} - 1$, we introduce the parameter $etil$ in the parameter file based on the estimated tariff elasticity and set it at $\varepsilon_{v,ta} - 1$ in the Eaton-Kortum model.

We thus introduce the parameters $gran$ as a measure for granularity ξ and $etil$ as a measure

[11] In the quantity equations for qpd, qpm, qgd, qgm, qfd, qfm, and qxs, σ is equal to ρ, so we impose $\sigma = \rho$ in the quantity equations. In the price equations σ is equal $\rho + 1$, but in relative changes the parameter ρ does not play a role, so we do not have to allow for the different value of σ in the pricing equations.

Parameters	Armington	Ethier-Krugman	Melitz	Eaton-Kortum
$etil$	$\varepsilon_{v,ta}$	$\varepsilon_{v,ta}$	$\varepsilon_{v,ta}$	$\varepsilon_{v,ta} - 1$
$gran$	1	1	ξ	1
$esubd$	$\varepsilon_{v,ta}$	$\varepsilon_{v,ta}$	$\xi * \varepsilon_{v,ta}$	$\varepsilon_{v,ta} - 1$
$theta$	$-$	$-$	$\varepsilon_{v,ta} - \frac{1}{\xi}$	$-$

Table 4: Parameterization of the four models

for the tariff elasticity $\varepsilon_{v,ta}$ and employ the following equations in all four models:

$$esubd = gran * etil \tag{D.31}$$

$$theta = etil - \frac{1}{gran} \tag{D.32}$$

$esubd$ is the substitution elasticity σ in the original GTAP model and $theta$ is the dispersion parameter θ in the added Melitz-block of the model. By varying the values for $gran$ and $etil$, we can then move between the different models. First, in the Ethier-Krugman and Armington model the substitution elasticity $esubd$ is equal to the tariff elasticity $\varepsilon_{v,ta}$, thus requiring $gran = 1$ and $etil = \varepsilon_{v,ta}$. Second, in the Melitz model we have the expressions (D.29)-(D.30) for $esubd = \sigma$ and $theta = \theta$, thus requiring $etil = \varepsilon_{v,\tau}$ and $gran = \xi$. Third, by setting $gran$ at 1 and $etil$ at $\widetilde{e} - 1$, we get the Eaton-Kortum parameterization with $esubd = rho = \varepsilon_{v,ta} - 1$. The parameterization is summarized in Table 4. The table shows the values required for the parameters $etil$ and $gran$ read from the parameter file and the implied values for $esubd$ and $theta$ based on the use of different parameter files in the command file.

Appendix D.5 Moving between Different Models with Closure Swaps

We move between the different models using closure swaps and employing different parameter files with different parameter values. First we discuss closure swaps. The baseline model with the additional blocks and without closure swaps implies the Armington model. We move from Armington to Ethier-Krugman by turning on the Ethier-Krugman supply-side externality and by endogenizing iceberg trade costs. We move from Armington to Melitz by turning on the Melitz supply-side and demand-side externalities and by endogenizing iceberg trade costs. We move from Armington to Eaton-Kortum by turning on the Eaton-Kortum supply-side externality and by endogenizing iceberg trade costs.

By swapping $oscaleekm$ with $sext$ in equation (D.7) and $nneh$ with nne in equation (D.8) for the Ethier-Krugman and Melitz model and $tekh$ with tek in the Eaton-Kortum model we turn on the supply-side externality. By swapping *oscaleek* with *ekscale*, *oscalem* with *mscale* or *eakoscale* with *eakoscale* in respectively equations (D.4)-(D.6) we turn respectively the Ethier-Krugman, Melitz and Eaton-Kortum supply-side externality on.

To turn on the Melitz demand-side externality, we swap *dscaled* with *Dextd* (*dscalem* with *Dextm*) in the following equation:

$$dscale2d(i,r) = dscale1d(i,r) - Dextd(i,r) \tag{D.33}$$

Finally, to model generalized trade costs in Ethier-Krugman, Eaton-Kortum or Melitz, $ams\,(i,r,s)$ is swapped with $genitcekm\,(i,r,s)$ in equation (D.26). By swapping *genitcekh* with *genitcek* or *genitcmh* with *genitcm* in respectively equations (D.24)-(D.25) we choose for respectively Ethier-Krugman/Eaton-Kortum or Melitz generalized iceberg trade costs.

To move between the different models, we also have to use different parameter values. We do this by employing different parameter files in the command file, with the parameter files differing in their values of *etil* and *gran* according to Table 4. The table makes clear that the values for *etil* and *gran* are identical for Armington and Ethier-Krugman. Hence, we use the same parameter file for these two models, whereas Melitz and Eaton-Kortum have their own parameter files.

Appendix D.6 Margin Decomposition

To calculate the three margins in GEMPACK, we rewrite equations (28)-(32) in GEMPACK notation as follows:

$$\begin{aligned} psistarh\,(i,r,s) = {} & \frac{1}{\sigma-1}[ps(i,r) + ao\,(i,r) - pfactwld] + \left(1 + \frac{1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\ & + pcif\,(i,r,s) + itc\,(i,r,s) + \frac{1}{\sigma-1} fex\,(i,r,s) \\ & - priceDs(i,s) - \frac{1}{\sigma-1} valueDs\,(i,s) + \frac{\sigma}{\sigma-1} tariffDs\,(i,s) - psistar\,(i,r,s) \end{aligned}$$

The extensive margin is given by:

$$extm\,(i,r,s) = -\theta psistar(i,r,s) + nne(i,r)$$

And the intensive margin is defined by:

$$\begin{aligned} intm\,(i,r,s) = &-(\sigma-1)\,(itc\,(i,r,s)+tm\,(i,s)+tms\,(i,r,s)+pcif\,(i,r,s)) \\ &+(\sigma-1)\,priceDs(i,s)+valueDs\,(i,s)-\sigma tariffDs\,(i,s) \end{aligned}$$

The compositional margin can be expressed as:

$$compm\,(i,r,s) = (\sigma-1)\,psistar\,(i,r,s)$$

And finally the overall effect can be written as:

$$\begin{aligned} d\ln V_{ij} &= TM = EM + IM + CM \\ &= -\frac{\theta-\sigma-1}{\sigma-1}\,(ps\,(i,r)+ao\,(i,r)-pfactwld)+nne(i,r) \\ &-\left(\theta+\frac{\theta-\sigma-1}{\sigma-1}\right)(tm\,(i,s)+tms\,(i,r,s)) \\ &-\theta\,(itc\,(i,r,s)+pcif\,(i,r,s))-\frac{\theta-\sigma-1}{\sigma-1}fex\,(i,r,s) \\ &+\theta priceDs(i,s)+\frac{\theta}{\sigma-1}valueDs\,(i,s)-\frac{\sigma\theta}{\sigma-1}tariffDs\,(i,s) \end{aligned} \tag{D.34}$$

With $priceDs$, $valueDs$ and $tariffDs$ defined as in equations (D.12)-(D.14), except for the fact that values are expressed employing agents prices instead of market prices.

Supplementary Appendices of Derivations

Additional Multi-Country Multi-Sector Simulation Results

We present additional results on the effect of variations in the empirical parameters ξ and $\varepsilon_{v,ta}$ and in the structural parameters θ and σ on the welfare gains from trade in the Melitz model, working with the model with a Melitz-structure in all sectors and considering variations in trade costs in all sectors. While varying ξ we keep $\varepsilon_{v,ta}$ constant and vice versa and while varying θ we keep σ constant and vice versa. The results displayed in Figures 14-17 deliver a clear message. Moving away from granularity, be it through a reduction in ξ, an increase in θ for given σ or a decrease in σ for given θ, leads to smaller welfare effects: smaller welfare gains from reductions in iceberg trade costs and smaller welfare losses from increases in iceberg trade costs.

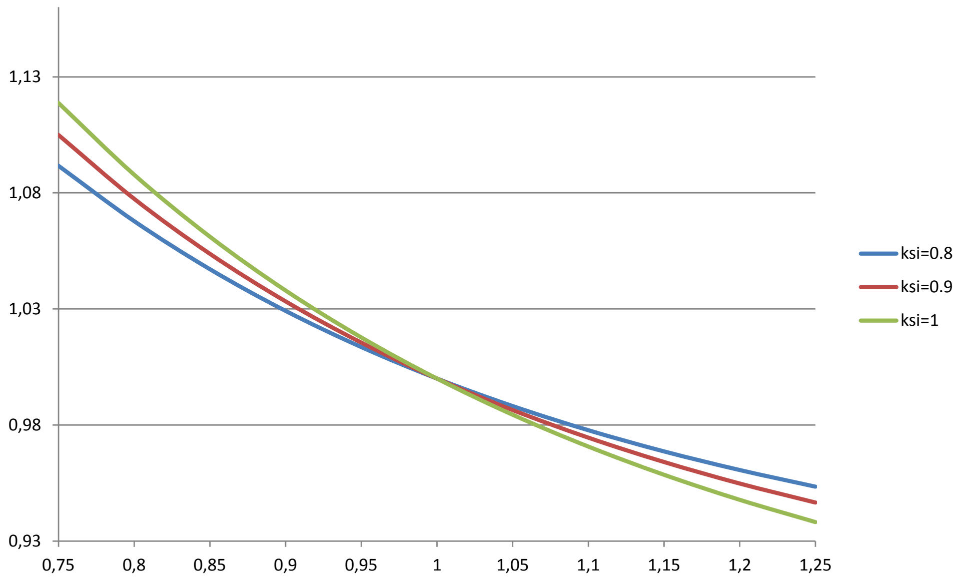

Figure 14: Average weighted utility as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$

To better compare the results generated with GAMS in a two-country setting with the simulations employing GTAP-GEMPACK, we also ran simulations with GTAP-GEMPACK in a single-sector setting by aggregating the data. Figures 18 and 19 display the results of variations in ξ and in θ, giving the same picture as the multi-sector model. A movement away from granularity with more action on the extensive versus decompositional margin, either through a decrease in ξ for given $\varepsilon_{v,ta}$ or an increase in θ for given σ, leads to smaller welfare effects, both smaller gains from trade liberalization and smaller losses from increases in trade costs. The only difference with the multi-sector simulations is that for larger reductions in trade

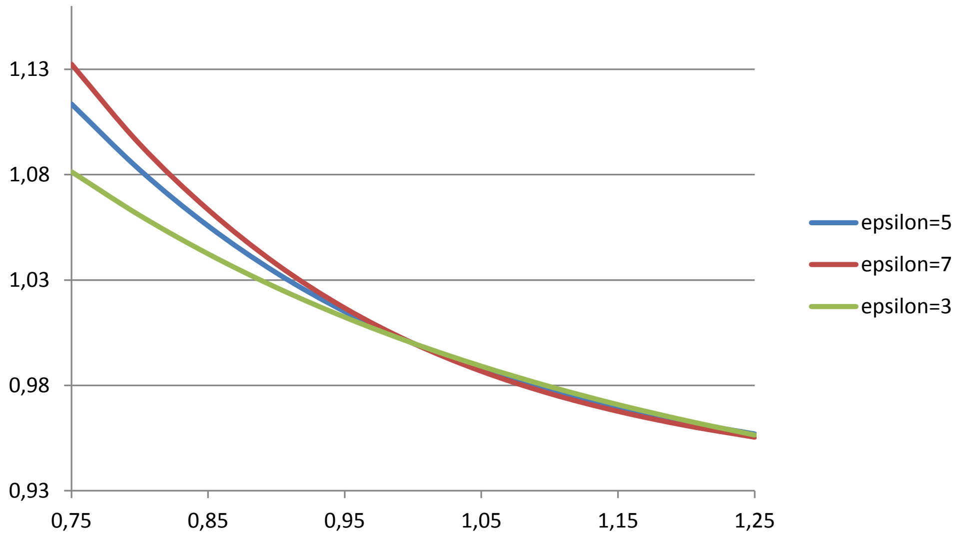

Figure 15: Average weighted utility as a function of percentage change in iceberg trade costs for different values of $\varepsilon_{v,ta}$ and $\xi = 0.8$

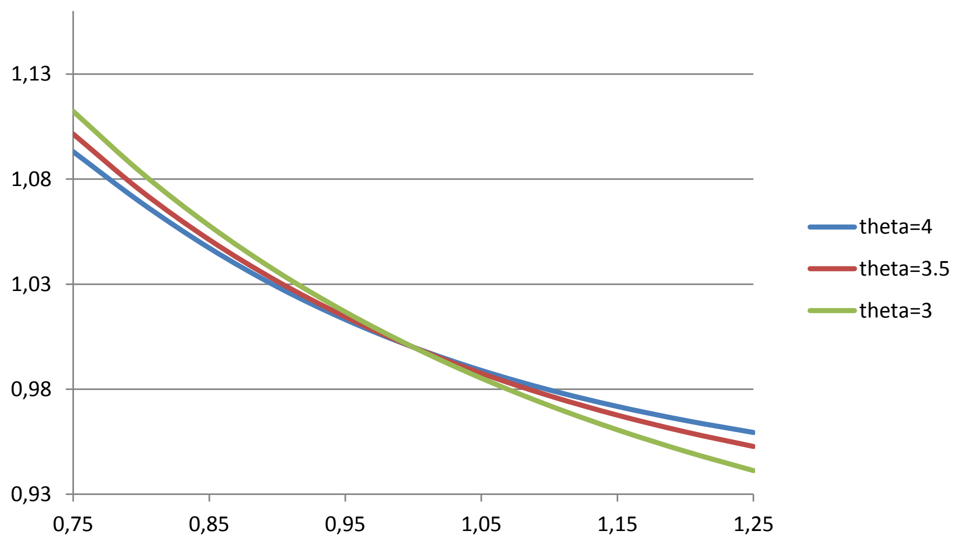

Figure 16: Average weighted utility as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$

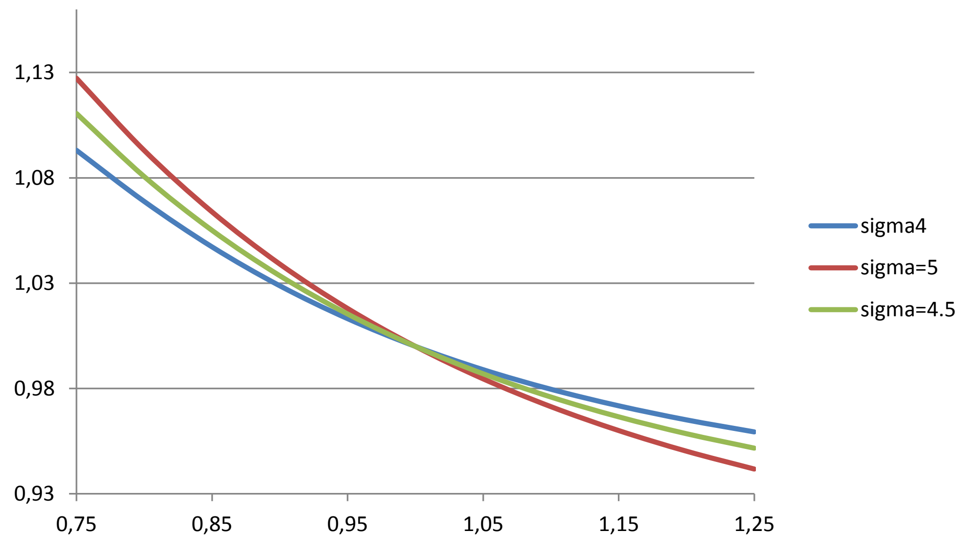

Figure 17: Average weighted utility wage as a function of percentage change in iceberg trade costs for different values of σ and $\theta = 4$

costs, the welfare gains become closer when moving away from granularity become closer again to the gains under granularity. So utility with $\xi = 0.8$ approaches utility $\xi = 0.9$ and utility with $\theta = 4$ approaches utility with $\theta = 3.5$.

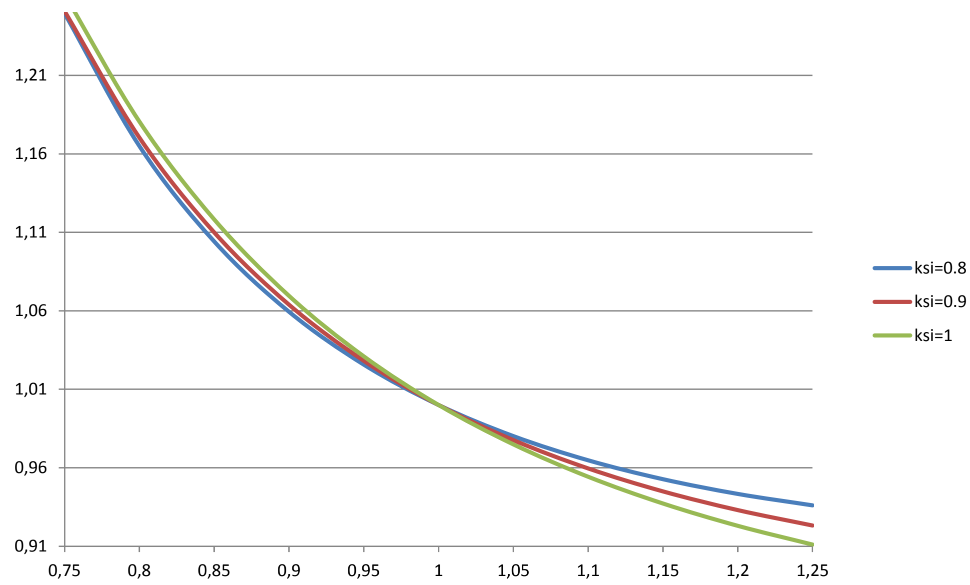

Figure 18: Average weighted utility as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$ in a single-sector model

Additional Symmetric Two-Country Single-Sector Simulation Results

In Figures 21-22 we display the effect of variations in respectively $\varepsilon_{v,ta}$, ξ, and σ on the welfare gains from lower iceberg trade costs, calibrating to the same iceberg trade costs for the different parameter values in each figure. Whereas calibration to import shares always

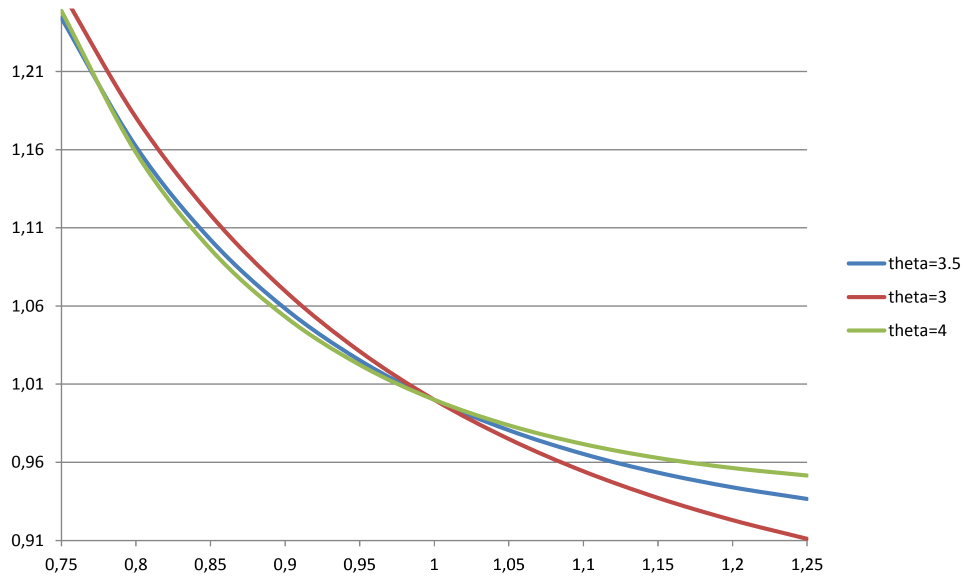

Figure 19: Average weighted utility as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$ in a single-sector model

generate both larger welfare gains and smaller welfare losses when parameters are changed in a certain direction, calibration to iceberg trade costs leads to larger welfare gains and losses when parameters are changed in a certain direction. Moreover for $\varepsilon_{v,ta}$ and ξ the effect of the parameters on the welfare gains from lower iceberg trade costs is reverted. With calibration to import shares a larger tariff elasticity leads to larger gains from lower iceberg trade costs, whereas with calibration to iceberg trade costs a larger tariff elasticity leads to smaller gains. Similarly, we find that with calibration to import shares a larger ξ leads to larger gains from lower iceberg trade costs, whereas with calibration to iceberg trade costs a larger ξ leads to smaller gains.

In Figures 23 and 24 we vary the trade elasticity instead of the tariff elasticity for both calibrations, i.e. with iceberg trade costs calibrated such that the import share is identical in all baselines and with iceberg trade costs identical in all baselines. We find the same results as with variation in the tariff elasticity. A larger tariff elasticity raises welfare when trade costs are calibrated to the import share and the welfare effect becomes smaller with a larger trade elasticity when trade costs are equal in the different baselines.

In Figures 25-28 we display the results of simulations of the model with intermediate linkages with trade costs calibrated as in di Giovanni and Levchenko (2012), so with identical baseline trade costs for different parameter values. We find identical results as in the model without intermediate linkages. Welfare effects are stronger for a smaller tariff elasticity $\varepsilon_{v,ta}$, a smaller

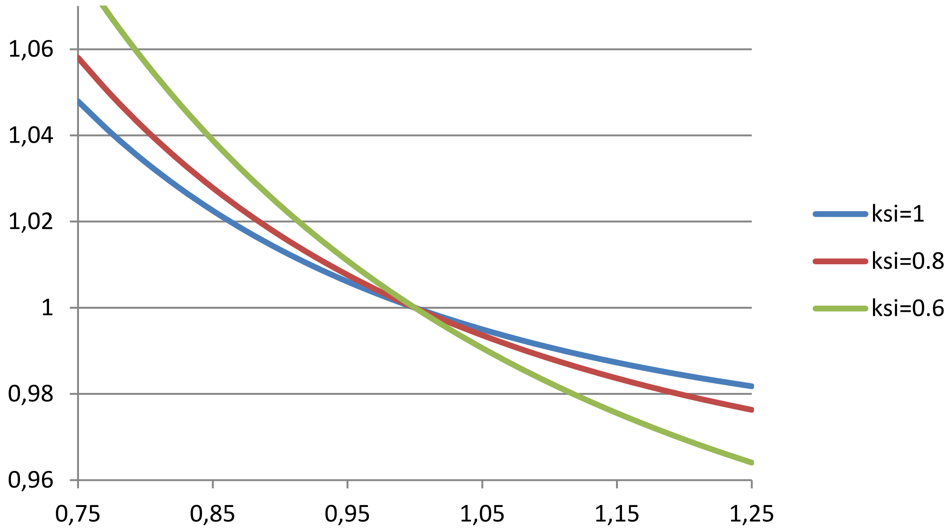

Figure 20: The real wage as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

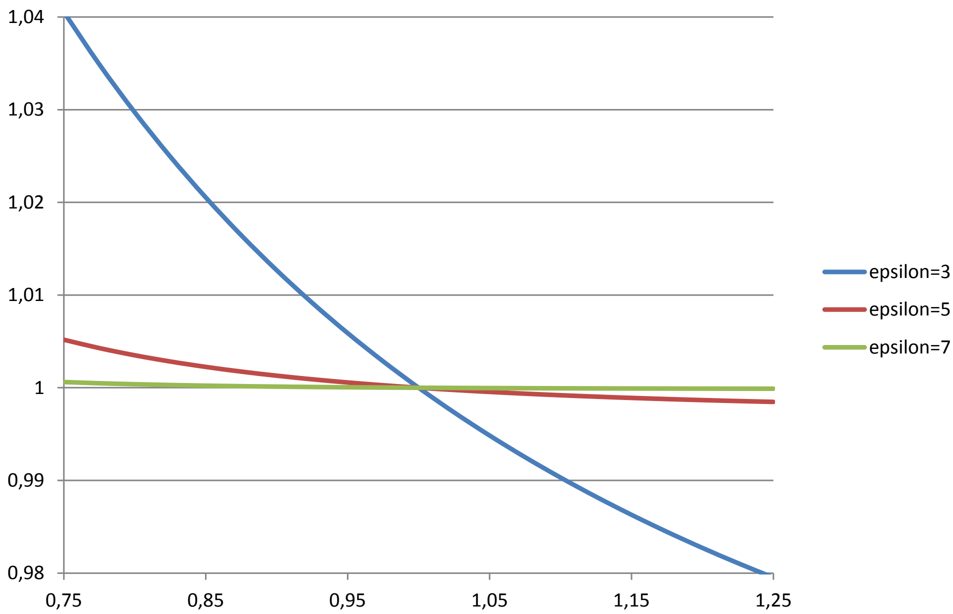

Figure 21: The real wage as a function of percentage change in iceberg trade costs for different values of $\varepsilon_{v,ta}$ and $\xi = 0.8$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

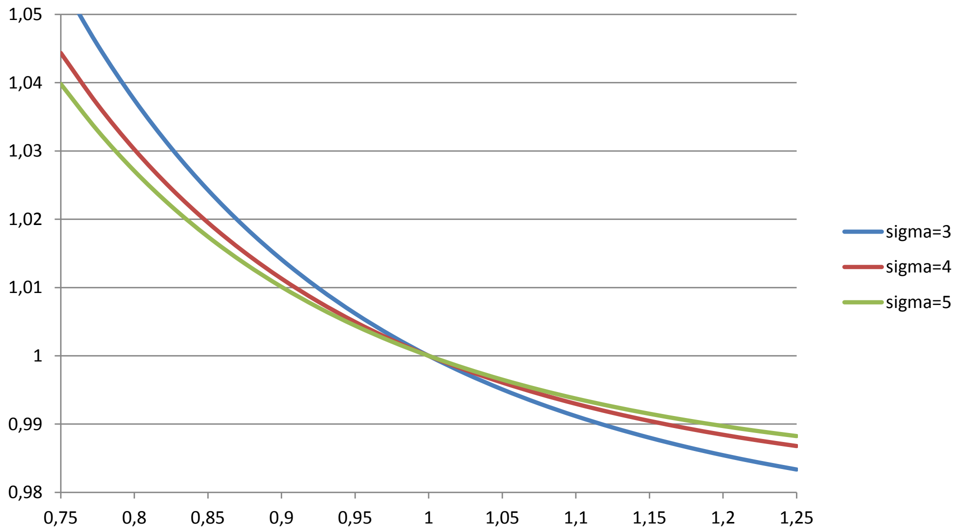

Figure 22: The real wage as a function of percentage change in iceberg trade costs for different values of σ and $\theta = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

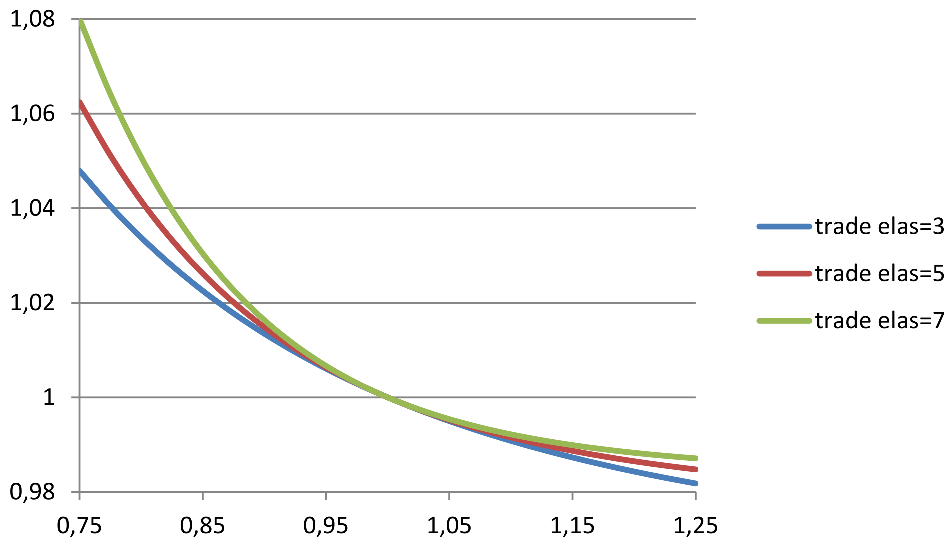

Figure 23: The real wage as a function of percentage change in iceberg trade costs for different values of the trade elasticity and calibration to the overall import share and the share of exporting firms

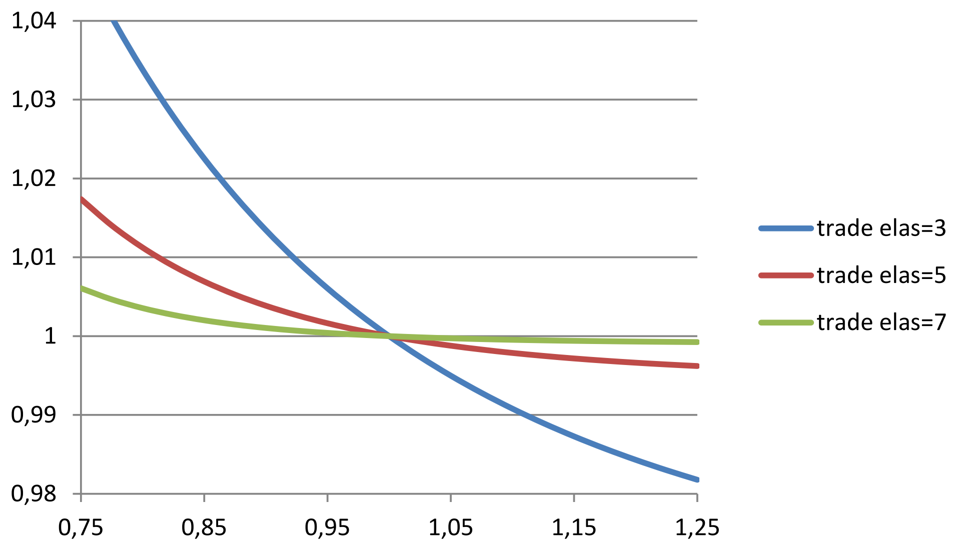

Figure 24: The real wage as a function of percentage change in iceberg trade costs for different values of the trade elasticity, with identical iceberg trade costs for different parameter values in the baseline

degree of granularity ξ, a smaller σ and a smaller θ. The last result is in line with the finding in di Giovanni and Levchenko (2012), who show that welfare effects of trade liberalization are larger when moving towards granularity, as modelled by a lower θ in a setting with identical baseline trade costs for different parameter values.

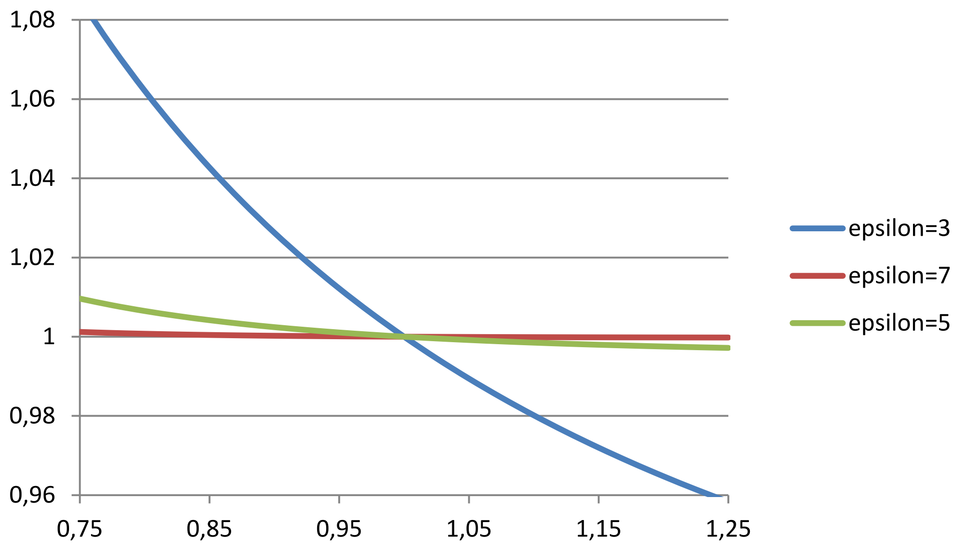

Figure 25: The real wage as a function of percentage change in iceberg trade costs for different values of $\varepsilon_{v,ta}$ and $\xi = 0.8$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

Equation (25)

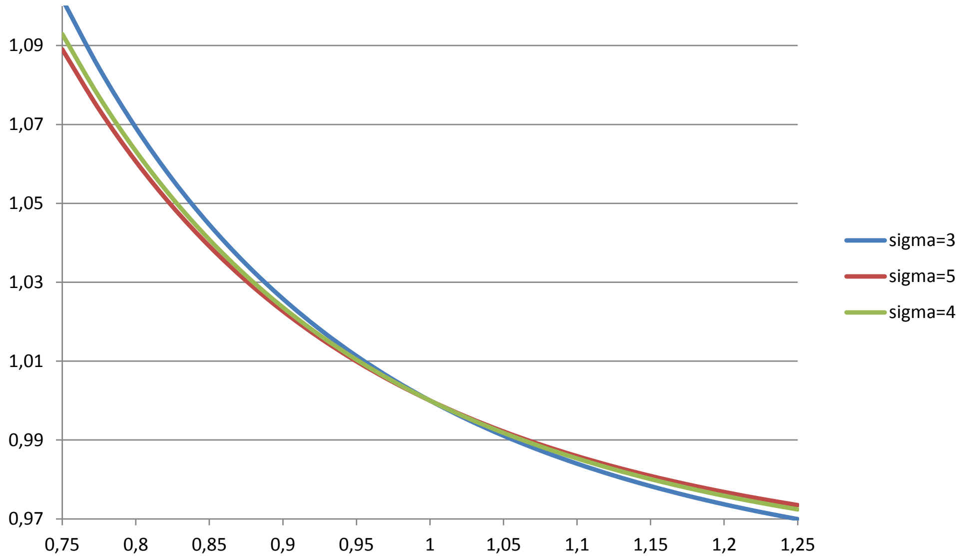

Figure 26: The real wage as a function of percentage change in iceberg trade costs for different values of σ and $\theta = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

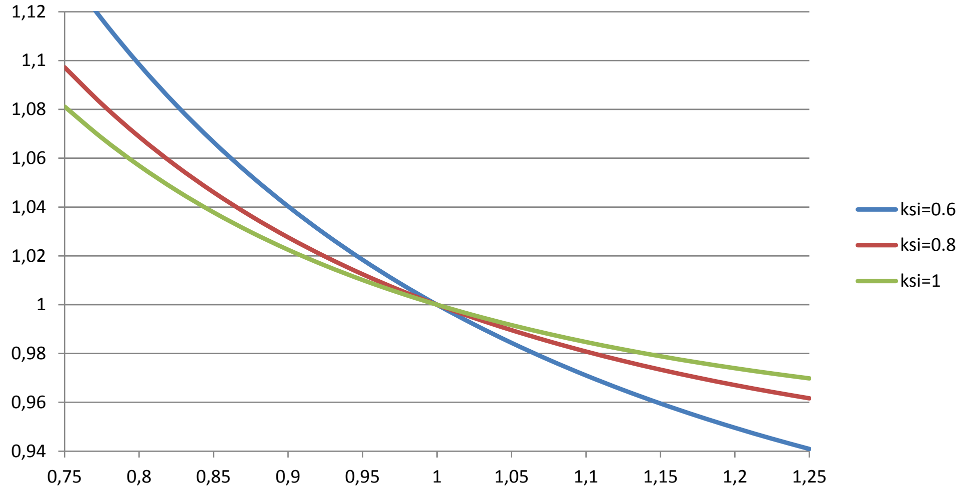

Figure 27: The real wage as a function of percentage change in iceberg trade costs for different values of ξ and $\varepsilon_{v,ta} = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

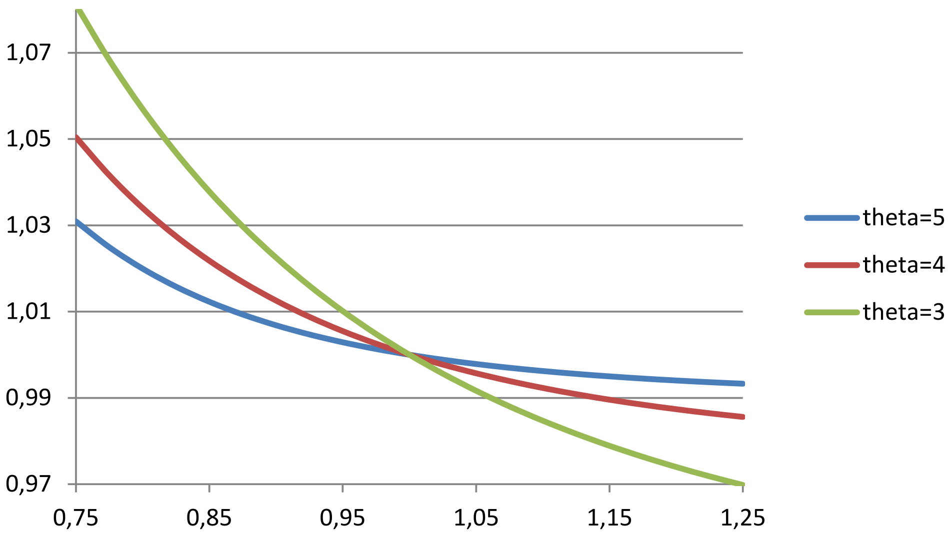

Figure 28: The real wage as a function of percentage change in iceberg trade costs for different values of θ and $\sigma = 4$, with identical iceberg trade costs for different parameter values in the baseline in symmetric two-country single-sector model with intermediate linkages

To convert Melitz into Ethier/Krugman the following should hold:

$$\gamma_m^{\frac{1}{\sigma-1}} = \gamma_{ek}$$

Substituting the expressions for γ_{ek} and γ_m in equation (22) leads to the following expression for ψ:

$$\left(\psi\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)}\frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}\right)^{\frac{1}{\sigma-1}} = \frac{\sigma-1}{\sigma}\sigma^{\frac{1}{1-\sigma}}$$

$$\psi\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)}\frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{1}{\sigma}$$

$$\psi = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\theta+1}\frac{\theta-\sigma+1}{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}+1}}$$

$$\psi = \left(\frac{\sigma}{\sigma-1}\right)^{\theta-\sigma+2}\frac{\theta-\sigma+1}{\sigma^{-\frac{\theta}{\sigma-1}}}$$

$$= \left(\frac{\sigma}{\sigma-1}\right)^{\theta-\sigma+2}\sigma^{\frac{\theta}{\sigma-1}}(\theta-\sigma+1)$$

Equation (27)

Differentiating equation (26) on the RHS and LHS wrt to the endogenous variables gives:

$$dV_{ij} = dN_{ij}\widetilde{r}_{ij} + N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty} dr_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi - N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)g\left(\varphi_{ij}^{*}\right)d\varphi_{ij}^{*}$$

$$+ N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty} r_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi\frac{g\left(\varphi_{ij}^{*}\right)}{1-G\left(\varphi_{ij}^{*}\right)}d\varphi_{ij}^{*}$$

$$= dN_{ij}\widetilde{r}_{ij} + N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty} dr_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi - N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)g\left(\varphi_{ij}^{*}\right)d\varphi_{ij}^{*}$$

$$+ V_{ij}\frac{g\left(\varphi_{ij}^{*}\right)}{1-G\left(\varphi_{ij}^{*}\right)}d\varphi_{ij}^{*}$$

Writing in logs and using $g\left(\varphi_{ij}^{*}\right)=-\frac{\partial\left(1-G\left(\varphi_{ij}^{*}\right)\right)}{\partial\varphi_{ij}^{*}}$:

$$\begin{aligned}
d\ln V_{ij} &= d\ln N_{ij}\frac{N_{ij}}{V_{ij}}\widetilde{r}_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)r_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi\frac{1}{V_{ij}}\\
&- N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)g\left(\varphi_{ij}^{*}\right)d\ln\varphi_{ij}^{*}\varphi_{ij}^{*}\frac{1}{V_{ij}}\\
&- V_{ij}\frac{\frac{\partial\left(1-G\left(\varphi_{ij}^{*}\right)\right)}{\partial\varphi_{ij}^{*}}}{1-G\left(\varphi_{ij}^{*}\right)}d\ln\varphi_{ij}^{*}\varphi_{ij}^{*}\frac{1}{V_{ij}}\\
&= d\ln N_{ij}+\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi\\
&+ N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)\frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{1-G\left(\varphi_{ij}^{*}\right)}{\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\frac{\varphi_{ij}^{*}}{V_{ij}}\\
&- \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{1-G\left(\varphi_{ij}^{*}\right)}{\varphi_{ij}^{*}}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}d\ln\varphi_{ij}^{*}\varphi_{ij}^{*}\\
&= d\ln N_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi\\
&+ \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{\partial\ln\varphi_{ij}^{*}}{\partial\ln\tau_{ij}}\frac{r_{ij}\left(\varphi_{ij}^{*}\right)}{\overline{r}_{ij}}-\frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\\
&= d\ln N_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\alpha\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi\\
&+ \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\left(\frac{r_{ij}\left(\varphi_{ij}^{*}\right)}{\overline{r}_{ij}}-1\right)
\end{aligned}$$

Equation (32)

Adding up the three margins in equations (29)-(31), we get:

$$
\begin{aligned}
d\ln V_{ij} &= TM = EM + IM + CM \\
&= -\frac{\mu\theta}{\sigma-1}\widehat{p_{Z_i}} - \frac{\theta}{\sigma-1}\left(1-\mu\right)\widehat{p_{Z_j}} - \theta\left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} \\
&- \theta\widehat{\tau_{ij}} - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) - \frac{\theta}{\sigma-1}\widehat{f_{ij}} + \widehat{NE_i} \\
&+ \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right) \\
&- \left(\sigma-1\right)\left(\widehat{\tau_{ij}} + \widehat{ta_{ij}} + \left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right)\right) \\
&+ \sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right) \\
&+ \mu\widehat{p_{Z_i}} + \left(1-\mu\right)\widehat{p_{Z_j}} + \sigma\widehat{ta_{ij}} + \left(\sigma-1\right)\widehat{\tau_{ij}} + \left(\sigma-1\right)\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) + \widehat{f_{ij}} \\
&- \sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}
$$

Elaborating and merging terms, we get:

$$\begin{aligned}
TM = & -\frac{\theta\mu}{\sigma-1}\widehat{p_{Z_i}} + \mu\widehat{p_{Z_i}} + \widehat{NE_i} \\
& - \frac{\theta}{\sigma-1}(1-\mu)\widehat{p_{Z_j}} + (1-\mu)\widehat{p_{Z_j}} \\
& - \theta\left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} - (\sigma-1)\widehat{ta_{ij}} + \sigma\widehat{ta_{ij}} \\
& - \theta\widehat{\tau_{ij}} - (\sigma-1)\widehat{\tau_{ij}} + (\sigma-1)\widehat{\tau_{ij}} \\
& - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) - (\sigma-1)\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) + (\sigma-1)\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) \\
& - \frac{\theta}{\sigma-1}\widehat{f_{ij}} + \widehat{f_{ij}} \\
& + \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left((\sigma-1)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right) \\
= & -\mu\frac{\theta-\sigma+1}{\sigma-1}\widehat{p_{Z_i}} - (1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}} \\
& - \left(\theta\left(1+\frac{1}{\sigma-1}\right)-1\right)\widehat{ta_{ij}} - \theta\widehat{\tau_{ij}} - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) - \frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}} \\
& + \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left((\sigma-1)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}$$

So we have:

$$\begin{aligned}
TM = & -\mu\frac{\theta-\sigma+1}{\sigma-1}\widehat{p_{Z_i}} - (1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}} - \left(\theta + \frac{\theta-\sigma-1}{\sigma-1}\right)\widehat{ta_{ij}} - \theta\widehat{\tau_{ij}} \\
& - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) - \frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}} + \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left((\sigma-1)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}$$

Equation (28)

Log differentiating the expression for φ^*_{ij} in equation (B.7) gives:

$$\begin{aligned}
\widehat{\varphi^*_{ij}} &= \left(1+\frac{\mu}{\sigma-1}\right)\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} \\
&\quad + \frac{1}{1-\sigma}\sum_{ag=\{s,p,f\}}\widehat{\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}} \\
&= \left(1+\frac{\mu}{\sigma-1}\right)\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} \\
&\quad - \frac{1}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{\frac{E_j^{ag}}{ta_j^{ag}}}{\sum_{ag'=\{s,p,f\}}\frac{E_j^{ag'}}{ta_j^{ag'}}}\left((\sigma-1)\left(\widehat{P_j^{ag}}-\widehat{ta_j^{ag}}-\widehat{\tau_j^{ag}}\right)+\widehat{E_j^{ag}}-\widehat{ta_j^{ag}}\right)
\end{aligned}$$

Equation (34)

Substituting equation (23) into equation (33) gives:

$$\begin{aligned}
v_{ij} &= ta_{ij}^{-\sigma}\left(\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}}ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}+\frac{\theta-\sigma+1}{(\sigma-1)^2}}f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\tau_{ij}c_i\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\sigma}\left(\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}}\tau_{ij}^{\frac{\theta}{\sigma-1}}ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}\frac{\sigma}{\sigma-1}}f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}c_i\right)^{1-\sigma}\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(1+\frac{\theta-\sigma+1}{\sigma-1}\right)\sigma}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\frac{\theta\sigma}{\sigma-1}}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\frac{\theta\sigma+\theta(\sigma-1)-\theta(\sigma-1)}{\sigma-1}}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+\frac{\theta\sigma-\theta\sigma+\theta}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+\frac{\theta}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m
\end{aligned}$$

Equation (A.1)

Substituting equations (6)-(8) into equations (2)-(5) gives for q_{ij}:

$$\begin{aligned} q_{ij} &= \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} q_j^m = \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} q_j^{m,ag} \\ &= \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(e_j^m\right)^{\sigma-1} \left(\frac{ta_j^{m,ag} p_j^m}{P_j^{ag}}\right)^{-\sigma} q_j^{ag} \\ &= p_{ij}^{-\sigma} \left(e_j^m\right)^{\sigma-1} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{m,ag}}\right)^{\sigma} q_j^{ag} \end{aligned} \tag{S.1}$$

Substituting equation (9) and rearranging gives:

$$q_{ij}e_j^m = \left(\frac{ta_{ij}t_{ij}c_i\left(te_{ij}b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{e_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{m,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{m,ag}} \tag{S.2}$$

To derive the expression for $q_j^{d,ag}$ we substitute equation (7) into equations (2)-(3):

$$q_j^d e_j^d = \left(\frac{c_j b_j p_{Z_j}}{e_j^d}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{d,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{d,ag}} \tag{S.3}$$

Together equations (S.2)-(S.3) imply the general expression for $q_j^{s,ag}$ in equation (A.1).

Equation (D.8)

Log differentiating equation (A.11) gives:

$$N_i = \frac{\widetilde{Z}_i}{\sigma a_i} = \frac{Z_i - \frac{\sigma-1}{\sigma}\left(\sum_{j=1}^{J} \frac{N_i \bar{r}_{ij}}{p_{Z_i} ta_{ij}\left(b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right)} - \frac{N_i \bar{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}{\sigma a_i}$$

$$
\begin{aligned}
\widehat{N_i} = & \frac{Z_i}{Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{Z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)} \widehat{Z}_i \\
& - \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(b_i p_{z_i} te_{ij} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)}}{Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{z_i} ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)} (\frac{\widehat{N_{ij}\overline{r}_{ij}}}{ta_{ij}} \\
& - \frac{p_{z_i} te_{ij}}{b_i p_{z_i} te_{ij} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\left(\widehat{p_{Z_i}} + \widehat{te_{ij}}\right) - \frac{\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}{b_i p_{z_i} te_{ij} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\widehat{p^{tr}_{ij}}) \\
& + \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}}{Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{z_i} ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}\left(\frac{\widehat{N_{ij}\overline{r}_{ij}}}{ta_{ij}} - \widehat{p_{Z_i}}\right) \\
= & \frac{p_{Z_i} Z_i}{p_{Z_i} Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{Z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)} \widehat{Z}_i \\
& - \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(te_{ij} + \frac{p^{tr}_{ij}}{b_i p_{Z_i} a^{tr}_{ij}}\right)}}{p_{z_i} Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{Z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)} (\frac{\widehat{N_{ij}\overline{r}_{ij}}}{ta_{ij}} \\
& - \frac{p_{z_i} te_{ij}}{p_{z_i} te_{ij} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\left(\widehat{p_{Z_i}} + \widehat{te_{ij}}\right) - \frac{\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}{b_i p_{z_i} te_{ij} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\widehat{p^{tr}_{ij}}) \\
& + \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}}{p_{z_i} Z_i - \sum_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(b_i te_{ij} + \frac{p^{tr}_{ij}}{p_{Z_i} a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)}\left(\frac{\widehat{N_{ij}\overline{r}_{ij}}}{ta_{ij}} - \widehat{p_{Z_i}}\right)
\end{aligned}
$$

In GEMPACK notation we get:

$$oscale(i,r) = nne(i,r) - \frac{VOM\,(i,r)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)} qo\,(i,r)$$
$$+ \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VXMD\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)} (pcif\,(i,r,s) + qxs\,(i,r,s)$$
$$- \frac{VXWD\,(i,r,s)}{VIWS\,(i,r,s)} \left(ps\,(i,r) + ao\,(i,r) - tx\,(i,r) - tx\,(i,r,s)\right)$$
$$- \frac{VIWS\,(i,r,s) - VXWD\,(i,r,s)}{VIWS\,(i,r,s)} ptrans\,(i,r,s))$$
$$- \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VIWS\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum\limits_{t=1}^{J}\left(VXMD\,(i,r,t) - VIWS\,(i,r,t)\right)}$$
$$* \left(pcif\,(i,r,s) + qxs\,(i,r,s) - \left(ps\,(i,r) + ao\,(i,r)\right)\right)$$

Equation (A.7)

Taking the FOC wrt p_{ij}^{o} in equation (A.6) gives:

$$0 = (1-\sigma)\frac{\left(\sum\limits_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}\right)}{ta_j^{ag} ta_{ij} te_{ij}}$$
$$+ \sigma\tau_{ij}\left(b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\left(\sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}\right)^{-(\sigma+1)} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}\right)$$
$$0 = (1-\sigma)\frac{1}{ta_j^{ag} ta_{ij} te_{ij}} + \sigma\tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{1}{ta_j^{ag} p_{ij}}$$

$$p_{ij} = \frac{\sigma}{\sigma-1} ta_{ij} te_{ij} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)$$

Equation (A.8)

Substituting equation (A.7) back into equation (A.6) gives:

$$\pi_{ij} = \frac{ta_j^{ag} p_{ij}\left(\sum\limits_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}^{o}\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}\right)}{ta_j^{ag} ta_{ij}} - \frac{p_{ij}^{o}}{ta_{ij}}\frac{\sigma-1}{\sigma}\left(\sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}\right)$$
$$= \frac{p_{ij}^{o} o_{ij}}{\sigma ta_{ij}}$$

Equation (A.11)

Substituting equations (A.8) and (A.10) into equation (A.9) gives:

$$\left(\sum_{j=1}^{J}\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}+\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)+a_i\right)N_i=Z_i$$

$$N_i\sigma a_i+\sum_{j=1}^{J}N_i\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)=Z_i$$

$$N_i=\frac{Z_i-\frac{\sigma-1}{\sigma}\sum_{j=1}^{J}N_i\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)}{\sigma a_i} \tag{S.4}$$

$$=\frac{Z_i-\frac{\sigma-1}{\sigma}\sum_{j=1}^{J}N_i\frac{\overline{r}_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)}{\sigma a_i} \tag{S.5}$$

Equation (B.5)

With tariffs as revenues shifters, profits for sales from i to j can be written as:

$$\begin{aligned}
\pi_{ij} &= \sum_{ag=\{s,p,f\}}\left(\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{ta_j^{ag}ta_{ij}}-\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi}\right)-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\\
&= \sum_{ag=\{s,p,f\}}\left(\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{ta_j^{ag}ta_{ij}}-\frac{\sigma-1}{\sigma}\frac{ta_j^{ag}p_{ij}o_{ij}^{ag}}{ta_j^{ag}ta_{ij}}\right)-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\\
&= \sum_{ag=\{s,p,f\}}\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{\sigma ta_j^{ag}ta_{ij}}-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\\
&= \sum_{ag=\{s,p,f\}}\frac{\left(ta_j^{ag}p_{ij}^{o}\left(\varphi\right)\right)^{1-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}}{ta_j^{ag}ta_{ij}\sigma}-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}
\end{aligned}$$

Equation (B.7)

Using equations (B.3)-(B.5) the ZCP can be written as follows:

$$p_{ij}^{o}\left(\varphi\right)=\frac{\sigma}{\sigma-1}\frac{ta_{ij}\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi}$$

$$\sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{S.6}$$

$$\sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}} \left(\frac{\sigma}{\sigma-1} \frac{ta_j^{ag} ta_{ij} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi_{ij}^*} \right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}} \left(\frac{\sigma}{\sigma-1} ta_j^{ag} ta_{ij} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{\sigma ta_j^{ag} ta_{ij} f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\frac{\left(\frac{\sigma}{\sigma-1} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} ta_{ij}\right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\frac{\left(\frac{\sigma}{\sigma-1} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} \sum_{ag=\{s,p,f\}} \frac{\left(\frac{P_j^{ag,e}}{ta_j^{ag} ta_{ij} \tau_{ij}}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\begin{aligned}
\varphi_{ij}^* &= \frac{\frac{\sigma}{\sigma-1} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag} ta_{ij} \tau_{ij}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag} ta_{ij}} \right)^{\frac{1}{1-\sigma}} \\
&= \frac{\frac{\sigma}{\sigma-1} ta_{ij} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}} \right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Equation (B.10)

Writing expected profit $\pi_{ij}\left(\widetilde{\varphi}_{ij}\right)$ as a function of expected revenues $r_{ij}\left(\widetilde{\varphi}_{ij}\right)$ using equation (B.5) and expressing expected revenues $r_{ij}\left(\widetilde{\varphi}_{ij}\right)$ as a function of cutoff revenues $r_{ij}\left(\varphi_{ij}^*\right)$ using $\frac{r_{ij}(\varphi_1)}{r_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ gives:

$$\pi_{ij}\left(\widetilde{\varphi}_{ij}\right) = \sum_{ag=\{s,p,f\}} \frac{r_{ij}^{ag}\left(\varphi_{ij}^*\right)}{ta_j^{ag} ta_{ij} \sigma} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^*}\right)^{\sigma-1} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

Using the ZCP in equation (B.6) this can be rewritten as:

$$\begin{aligned}\pi_{ij}\left(\widetilde{\varphi}_{ij}\right) &= f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \\ &= f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \left(\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} - 1\right)\end{aligned} \tag{S.7}$$

Substituting equation (S.7) into the FE, equation (B.8) leads to equation (B.10).

Equation (B.12)

Using the Pareto distribution in equation (B.11) average productivity $\widetilde{\varphi}_{ij}$ can be written as:

$$\begin{aligned}\widetilde{\varphi}_{ij}^{\sigma-1} &= \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^{*}\right)} d\varphi = \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-1} \frac{\theta \frac{\kappa_i^{\theta}}{\varphi^{\theta+1}}}{\left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{\theta_i}} d\varphi \\ &= \int_{\varphi_{ij}^{*}}^{\infty} \theta \left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{-\theta} \varphi^{\sigma-1-\theta+1} \frac{\kappa_i^{\theta}}{\varphi^{\theta+1}} d\varphi = \int_{\varphi_{ij}^{*}}^{\infty} \theta \varphi_{ij}^{*\theta} \varphi^{\sigma-1-\theta-1} d\varphi \\ &= \theta \varphi_{ij}^{*\theta} \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-\theta-2} d\varphi = \theta \varphi_{ij}^{*\theta} \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-\theta-2} d\varphi \\ &= \frac{\theta}{\sigma-\theta-1} \varphi_{ij}^{*-\theta} \varphi^{\sigma-\theta-1} \Big|_{\varphi_{ij}^{*}}^{\infty} = -\frac{\theta}{\sigma-\theta-1} \varphi_{ij}^{*\theta} \varphi_{ij}^{*\sigma-\theta-1} \\ &= \frac{\theta}{\theta-\sigma+1} \varphi_{ij}^{*\sigma-1}\end{aligned}$$

Equation (B.16)

Substituting equations (B.4) and (B.9) into equation (B.15) gives:

$$\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} = \left(\int_{\varphi_{ij}^*}^{\infty} N_{ij} \left(\frac{\sigma}{\sigma - 1} \frac{ta_{ij} ta_j^{s,ag} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\varphi} \right)^{1-\sigma} \frac{g_i(\varphi)}{1 - G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma - 1} \left(N_{ij} \left(ta_{ij} ta_j^{s,ag} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{1-\sigma} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1 - G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma - 1} \left(N_{ij} \left(ta_{ij} ta_j^{s,ag} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{1-\sigma} \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1 - G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{\sigma-1}{\sigma-1}} \right)^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma - 1} \left(\sum_{i=1}^{J} N_{ij} \left(\frac{ta_{ij} ta_j^{s,ag} \tau_{ij} \left(b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\widetilde{\varphi}_{ij}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Equation (B.18)

Equation (B.18) can be derived from labor market equilibrium. First, we write the expression for $q_{ij}(\varphi)$ as a function of revenues, using the rewritten markup equation $\frac{\tau_{ij}}{\varphi} = \frac{\sigma-1}{\sigma} \frac{p_{ij}^o(\varphi)}{\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right) ta_{ij} p_{Z_i}}$. This gives:

$$\frac{\tau_{ij} o_{ij}(\varphi)}{\varphi} = \frac{\sigma - 1}{\sigma} \frac{p_{ij}^o(\varphi)}{\left(b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right) ta_{ij} p_{Z_i}} o_{ij} = \frac{\sigma - 1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{\left(b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right) p_{Z_i} ta_{ij}}$$

$$= \frac{\sigma - 1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} + \frac{\sigma - 1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \left(\frac{1 - te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \tag{S.8}$$

Input bundle demand consists of demand for labor bundles in production, fixed costs and sunk entry costs. This gives the following equilibrium condition:

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\tau_{ij} o_{ij}(\varphi)}{\varphi} \frac{g(\varphi)}{1 - G\left(\varphi_{ij}^*\right)} d\varphi + \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}}$$

Substituting equation (S.8) and elaborating the expression using $\frac{r_{ij}(\varphi_1)}{r_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ gives:

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi$$

$$+ \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{ta_{ij}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_j^g ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\theta}{\theta-\sigma+1} \frac{r_{ij}\left(\varphi_{ij}^*\right)}{p_{Z_i} ta_j^g ta_{ij}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{r_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_j^g ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

Substituting equation (B.12) for the ratio of productivities and the ZCP in equation (B.6) gives:

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\theta}{\theta-\sigma+1} \sigma f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{r_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$= NE_i en_i + \sum_{j=1}^{J} \frac{\theta(\sigma-1)}{\theta-\sigma+1} N_{ij} f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$= NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\theta(\sigma-1) + \mu(\theta-\sigma+1)}{\theta-\sigma+1} f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

Next, equation (B.17) is used to express N_{ij} as a function of NE_i and φ^*_{ij}:

$$Z_i = NE_i en_i + \frac{NE_i}{\delta}\sum_{j=1}^{J}\left(\frac{\kappa_i}{\varphi^*_{ij}}\right)^{\theta}\frac{\theta(\sigma-1)+\mu(\theta-\sigma+1)}{\theta_i-\sigma+1}f_{ij}\frac{p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}}$$
$$+\sum_{j=1}^{J}N_{ij}\frac{\sigma-1}{\sigma}\left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}\right)\frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}}+\sum_{k=1}^{J}N_{ki}f_{ki}\frac{(1-\mu)\,p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

The next step is to substitute the FE from equation (B.13):

$$\begin{aligned}
Z_i &= NE_i en_i + NE_i\sum_{j=1}^{J}\left(\frac{\kappa_i}{\varphi^*_{ij}}\right)^{\theta_i}\frac{p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}}f_{ij}\frac{\sigma-1}{\theta_i-\sigma+1}\frac{1}{\delta}\frac{\theta_i(\sigma-1)+\mu(\theta_i-\sigma+1)}{\sigma-1}\\
&+\sum_{j=1}^{J}N_{ij}\frac{\sigma-1}{\sigma}\left(\frac{1-b_i te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{b_i te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}\right)\frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}}+\sum_{k=1}^{J}N_{ki}f_{ki}\frac{(1-\mu)\,p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{p_{Z_i}}\\
&= NE_i en_i + NE_i en_i\frac{\theta(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1}+\sum_{k=1}^{J}N_{ki}f_{ki}\frac{(1-\mu)\,p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{p_{Z_i}}\\
&= NE_i en_i\frac{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1}+\sum_{k=1}^{J}N_{ki}f_{ki}\frac{(1-\mu)\,p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{p_{Z_i}}\\
&+\sum_{j=1}^{J}N_{ij}\frac{\sigma-1}{\sigma}\left(\frac{1-b_i te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{b_i te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}\right)\frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}}
\end{aligned}\tag{S.9}$$

To rewrite the second term on the RHS of equation (S.9) we substitute the relation between $\widetilde{\varphi}_{ki}^{\sigma-1}$ and $\varphi_{ki}^{*\sigma-1}$ from equation (B.12) into the expression for the price index implied by equation (B.16) :

$$\begin{aligned}
P_i^{1-\sigma} &= \sum_{k=1}^{J}N_{ki}\left(\frac{\sigma}{\sigma-1}ta_{ki}\tau_{ki}\left(te_{ki}b_k p_{Z_k}+\frac{p_{ki}^{tr}}{a_{ki}^{tr}}\right)\right)^{1-\sigma}\widetilde{\varphi}_{ki}^{\sigma-1}\\
&= \sum_{k=1}^{J}N_{ki}\left(\frac{\sigma}{\sigma-1}ta_{ki}\tau_{ki}\left(te_{ki}b_k p_{Z_k}+\frac{p_{ki}^{tr}}{a_{ki}^{tr}}\right)\right)^{1-\sigma}\varphi_{ki}^{*\sigma-1}\frac{\theta}{\theta-\sigma+1}
\end{aligned}$$

P_i is the group-uniform price index before the group-specific tariff is imposed. Substituting the rewritten ZCP from equation (B.7) gives:

$$P_i^{1-\sigma} = \sum_{k=1}^{J} (N_{ki} \left(\frac{\sigma}{\sigma-1} ta_{ki} \tau_{ki} \left(te_{ki} b_k p_{Z_k} + \frac{p_{ki}^{tr}}{a_{ki}^{tr}} \right) \right)^{1-\sigma} \left(\frac{\sigma}{\sigma-1} ta_{ki} \tau_{ki} \left(te_{ki} b_k p_{Z_k} + \frac{p_{ki}^{tr}}{a_{ki}^{tr}} \right) \right)^{\sigma-1}$$
$$* \frac{\sigma f_{ki} ta_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}} \left(\frac{P_i^{ag}}{ta_i^{ag}} \right)^{\sigma-1} \frac{E_i^{ag}}{ta_i^{ag}}}) \frac{\theta}{\theta-\sigma+1}$$
$$= \sum_{k=1}^{J} N_{ki} \frac{\sigma f_{ki} ta_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}} P_i^{\sigma-1} \frac{E_i^{ag}}{ta_i^{ag}}} \frac{\theta}{\theta-\sigma+1} = P_i^{1-\sigma} \sum_{k=1}^{J} N_{ki} \frac{\sigma f_{ki} ta_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}} \frac{E_i^{ag}}{ta_i^{ag}}} \frac{\theta}{\theta-\sigma+1}$$

This expression can be written as:

$$\sum_{k=1}^{J} N_{ki} f_{ki} w_k^{\mu} w_i^{1-\mu} + \sum_{k=1}^{J} N_{ki} f_{ki} \left(ta_{ki} - 1 \right) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu} = \sum_{ag=\{s,p,f\}} \frac{E_i^{ag}}{ta_i^{ag}} \frac{\theta-\sigma+1}{\theta\sigma} \tag{S.10}$$

Next, tariff revenues can be written as:

$$\begin{aligned}
\frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\widetilde{\varphi}_{ki} \right) &= \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\widetilde{\varphi}_{ki} \right) \\
&= \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\varphi_{ki}^{*} \right) \left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}} \right)^{\sigma-1} \\
&= \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \sigma f_{ki} ta_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu} \frac{\theta}{\theta-\sigma+1} \\
&= \left(ta_{ki} - 1 \right) N_{ki} f_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu} \frac{\theta\sigma}{\theta-\sigma+1}
\end{aligned}$$

Substituting this into equation (S.10) gives:

$$\sum_{k=1}^{J} N_{ki} f_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu} + \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\widetilde{\varphi}_{ki} \right) \frac{\theta-\sigma+1}{\theta\sigma} = \sum_{ag=\{s,p,f\}} \frac{E_i^{ag}}{ta_i^{ag}} \frac{\theta-\sigma+1}{\theta\sigma}$$
$$\sum_{k=1}^{J} N_{ki} f_{ki} p_{Z_k}^{\mu} p_{Z_i}^{1-\mu} = \left(\sum_{ag=\{s,p,f\}} \frac{E_i^{ag}}{ta_i^{ag}} - \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\widetilde{\varphi}_{ki} \right) \right) \frac{\theta-\sigma+1}{\theta\sigma}$$

Using $p_{Z_i} Z_i + \sum\limits_{k=1}^{J} \frac{ta_{ki}-1}{ta_{ki}} N_{ki} \bar{r}_{ki} \left(\widetilde{\varphi}_{ki} \right) = \sum\limits_{ag=\{s,p,f\}} \frac{E_i^{ag}}{ta_i^{ag}}$ therefore leads to:

$$\sum_{k=1}^{J} N_{ki} f_{ki} \frac{p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}} = Z_i \frac{\theta-\sigma+1}{\theta\sigma} \tag{S.11}$$

Substituting (S.11) into (S.9) then gives:

$$Z_i = NE_i en_i \frac{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1} + (1-\mu) Z_i \frac{\theta-\sigma+1}{\theta\sigma} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}} \tag{S.12}$$

Rearranging then leads to:

$$Z_i \left(\frac{\theta\sigma - (1-\mu)(\theta-\sigma+1)}{\theta\sigma} \right) = NE_i en_i \frac{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

And solving for NE_i:

$$\begin{aligned} NE_i &= \frac{\sigma-1}{\theta\sigma} \frac{\theta\sigma - (1-\mu)(\theta-\sigma+1)}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \frac{Z_i}{en_i} \\ &- \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}{b_i te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \\ &= \frac{\sigma-1}{\theta\sigma} \frac{\theta\sigma-\theta+\sigma-1+\mu(\theta-\sigma+1)}{\theta\sigma-\theta+\sigma-1+\mu(\theta-\sigma+1)} \frac{Z_i}{en_i} \\ &- \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \\ &= \frac{\sigma-1}{\theta\sigma} \frac{Z_i}{en_i} - \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{b_i te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \end{aligned}$$

Imposing $\mu = 1$ gives:

$$NE_i = \frac{\sigma-1}{\theta\sigma} \frac{1}{en_i} \left(Z_i - \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}} \right) \tag{S.13}$$

Equation (B.19)

Substituting equations (B.12) and (B.17) into equation (B.16) gives:

$$\begin{aligned}
\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta} \frac{NE_i}{\delta}\left(\frac{ta_{ij} ta_j^{s,ag} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi_{ij}^*}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{\theta \kappa_i^{\theta_i}}{\theta-\sigma+1} \frac{NE_i}{\delta} \frac{\left(ta_{ij} ta_j^{s,ag} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Substituting next equation (B.18) leads to:

$$\begin{aligned}
\frac{p_{ij} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\frac{\theta \kappa_i^{\theta}}{\theta-\sigma+1} \frac{\sigma-1}{\sigma\theta} \frac{\widetilde{Z}_i}{\delta en_i} \frac{\left(ta_{ij} ta_j^{s,ag} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)} \frac{\kappa_i^{\theta} \widetilde{Z}_i}{\delta en_i} \frac{\left(ta_{ij} ta_j^{s,ag} \tau_{ij}\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Equation (B.20)

Substituting equation (B.7) into equation (B.19) gives:

$$
\begin{aligned}
\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\frac{\left(ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\frac{\frac{\sigma}{\sigma-1}\left(te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)ta_{ij}\tau_{ij}}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{1}{1-\sigma}}}\left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{1}{1-\sigma}}\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1+\frac{1}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\frac{\left(\theta-\sigma+1\right)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}}\left(\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\left(ta_j^{ag} ta_{ij}\tau_{ij}\right)^{1-\sigma}\left(te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{1-\sigma-\theta+\sigma-1}\left(f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{\theta-\sigma+1}{1-\sigma}}\left(ta_{ij}\tau_{ij}\right)^{-(\theta-\sigma+1)}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma+\theta-\sigma+1}{\sigma-1}}\frac{\left(\theta-\sigma+1\right)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}}\left(\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}\left(te_{ij}b_i p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\left(\left(ta_j^{ag}\tau_j^{ag}\right)^{\frac{\sigma-1}{\theta}}\left(ta_{ij}\tau_{ij}\right) f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\left(ta_{ij}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} p_{Z_j}^{-(1-\mu)\frac{\theta-\sigma+1}{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma+\theta-\sigma+1}{\sigma-1}} \frac{(\theta-\sigma+1)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}} \left(\frac{\kappa_i^{\theta} Z_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta} p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu} \left(ta_{ij}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\left(ta_{ij}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} \left(ta_j^{ag}\tau_j^{ag}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \\
&= \left(\frac{\gamma_m\kappa_i^{\theta}\widetilde{Z}_i\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta} p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}}{\delta en_i} \left(ta_{ij}^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} \left(ta_j^{ag}\tau_j^{ag}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}}
\end{aligned}
$$

With γ_m defined as:

$$\gamma_m = \psi\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}$$

Equations (D.29)-(D.30)

From equation (D.27) we can write θ as:

$$\theta = \widetilde{e} - \frac{1}{d}$$

We can rewrite the expression for $\widetilde{e}$ in equation (D.27) as follows:

$$
\begin{aligned}
\widetilde{e} &= \frac{(\theta+1)(\sigma-1)}{\sigma-1} + \frac{\theta-\sigma+1}{\sigma-1} \\
&= \frac{\theta\sigma+\sigma-\theta-1+\theta-\sigma+1}{\sigma-1} \\
&= \frac{\theta\sigma}{\sigma-1}
\end{aligned}
$$

Therefore we can write σ as:

$$\widetilde{e} = \frac{\sigma}{d}$$

$$\sigma = d\widetilde{e}$$

Equation (B.25)

Substituting the expressions for t_{ij}, c_i and e_j^s into equation (A.1) gives:

$$q_{ij}e_j^s = \left(\frac{ta_{ij}\left(\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} ta_{ij}^{\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)\tau_{ij}\left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\frac{\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}} \right)^{-\sigma}$$

$$* \sum_{ag\in\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}$$

$$= \left(\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}} \tau_{ij}^{\frac{\theta}{\sigma-1}} ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \left(\frac{\gamma_m\kappa_i^\theta\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}}\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}$$

$$* \left(\sum_{ag\in\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}$$

Equation (B.30)

Elaborating on equation (B.6) gives:

$$\sum_{ag=\{s,p,f\}}\frac{\left(ta_j^{ag}p_{ij}^o\left(\varphi_{ij}^*\right)\right)^{1-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}}{ta_j^{ag}ta_{ij}} = \sigma f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}}\frac{\frac{ta_j^{ag}\frac{\sigma}{\sigma-1}ta_{ij}\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi_{ij}^*}\left(\frac{ta_j^{ag}\frac{\sigma}{\sigma-1}ta_{ij}\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi_{ij}^*}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}}{ta_j^{ag}ta_{ij}} = \sigma f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

Rearranging:

$$\sum_{ag=\{s,p,f\}}\left(\frac{\sigma}{\sigma-1}\frac{ta_j^{ag}ta_{ij}\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi_{ij}^*}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag} = (\sigma-1)\frac{f_{ij}}{\tau_{ij}\left(b_ite_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)}\varphi_{ij}^*\frac{p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}}$$

$$o_{ij}\left(\varphi_{ij}^*\right) = (\sigma-1)\frac{f_{ij}}{\tau_{ij}\left(b_ite_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)}\varphi_{ij}^*\frac{p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}}$$

Equation (B.31)

Substituting equation (B.30) and also the expressions for N_{ij} and NE_i in equations (B.17)-

(B.18) into equation (B.29) leads to:

$$
\begin{aligned}
q_{ij}^{ag} e_j^s &= N_{ij}^{\frac{\sigma}{\sigma-1}} \left(\sigma - 1\right) \varphi_{ij}^* \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)} \left(\frac{\theta}{\theta - \sigma + 1}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta} \frac{\sigma - 1}{\sigma\theta} \frac{\widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \left(\sigma - 1\right) \varphi_{ij}^* \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)} \left(\frac{\theta}{\theta - \sigma + 1}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{\sigma - 1}{\sigma \left(\theta - \sigma + 1\right)}\right)^{\frac{\sigma}{\sigma-1}} \left(\sigma - 1\right) \left(\frac{\kappa_i^{\theta} \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{1}{\left(\varphi_{ij}^*\right)^{\frac{\theta\sigma - \sigma + 1}{\sigma - 1}}} \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}
\end{aligned}
$$

Equivalence Equation (B.25)

Substituting the expression for φ_{ij}^* in equation (B.7) into equation (B.31) gives:

$$q_{ij}e_j^s = \left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{1}{\left(\varphi_{ij}^*\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}} \frac{f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}\tau_{ij}\left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}$$

$$= \frac{\left(\frac{\sigma-1}{\sigma(\theta-\sigma+1)}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{\tau_{ij}\left(b_i p_{Z_i} te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}}{\left(\frac{\frac{\sigma}{\sigma-1} ta_{ij}\tau_{ij}p_{Z_i}\left(te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)}{\left(\sigma f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{1}{1-\sigma}}\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}}$$

$$= \frac{\left(\theta-\sigma+1\right)^{-\frac{\sigma}{\sigma-1}}\left(\sigma-1\right)}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}+\frac{\theta\sigma-\sigma+1}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{1-\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1+\frac{\theta\sigma-\sigma+1}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \frac{\left(\theta-\sigma+1\right)^{-\frac{\sigma}{\sigma-1}}}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta\sigma+1}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}-1} \frac{\sigma}{\sigma-1}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\sigma^2-2\sigma+1-(\theta\sigma-\sigma+1)}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta\sigma}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \frac{\left(\theta-\sigma+1\right)^{-\frac{\sigma}{\sigma-1}}}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta\sigma+\sigma}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1-\left(\sigma^2-2\sigma+1\right)}{(\sigma-1)^2}}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\sigma^2-\sigma-\theta\sigma}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta\sigma}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \left(\left(\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1} \frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \left(\tau_{ij}\left(b_i p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta}{\sigma-1}} \left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} ta_{ij}^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

Using the definition for γ_m in equation (22) this expression is identical to the expression in equation (*B.25*):

$$q_{ij}e^s_j = \left(\left(te_{ij}b_i p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}} \right)^{\frac{\theta}{\sigma-1}} \tau_{ij}^{\frac{\theta}{\sigma-1}} ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}} \left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i}{\delta en_i} \right)^{\frac{1}{1-\sigma}} \left(f_{ij} p^{\mu}_{Z_i} p^{1-\mu}_{Z_j} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \right)^{-\sigma}$$
$$* \left(\sum_{ag\in\{p,g,f\}} \left(\frac{P^{ag}_j}{ta^{s,ag}_j} \right)^{\sigma-1} \frac{E^{ag}_j}{ta^{s,ag}_j} \right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}$$

With:

$$\gamma_m = \psi \left(\frac{\sigma}{\sigma-1} \right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}$$

Equation (C.15)

Substituting equation (C.14) into equation (C.13), we can solve for f_x/f_d as follows:

$$\begin{aligned}
\lambda &= \frac{1}{1 + ef \left(\frac{f_x}{f_d} \right)^{\frac{\theta}{\sigma-1}} \frac{f_x}{f_d}^{-\frac{\theta-\sigma+1}{\sigma-1}}} \\
&= \frac{1}{1 + ef \frac{f_x}{f_d}} \\
\lambda \left(1 + ef \frac{f_x}{f_d} \right) &= 1 \\
\lambda ef \frac{f_x}{f_d} &= 1 - \lambda \\
\frac{f_x}{f_d} &= \frac{1-\lambda}{\lambda ef}
\end{aligned}$$

Equation (C.18)

Solving equation (C.17) for τ leads to:

$$\begin{aligned}
es &= \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \\
\tau^{1-\sigma} &= es \left(1 + x^{1-\sigma} \right) \\
\tau^{1-\sigma} \left(1 - is \right) &= es \\
\ln \tau^{1-\sigma} &= \ln \frac{es}{1-es} \\
\ln \tau &= -\frac{\ln \frac{es}{1-es}}{\sigma-1} \\
\tau &= \exp \left(-\frac{\ln \frac{es}{1-es}}{\sigma-1} \right)
\end{aligned}$$

Equality of total trade flows in GEMPACK from model and from margin decomposition

We can check the correctness of the margin decomposition expressions by comparing the total margin TM in equation (D.34) with the change in trade flows following from the main model. We do that employing GEMPACK notation. The change in the quantity of trade in the main model is given by:

$$qxs(i,r,s) = -ams(i,r,s) + qim(i,s) - \sigma[pms(i,r,s) - ams(i,r,s) - pim(i,s)]$$

In value terms the change in trade flows is given by:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) &= qim(i,s) + pim\,(i,s) \\
&- (\sigma - 1)\,[pms(i,r,s) - ams(i,r,s) - pim(i,s)] \\
&= qim(i,s) + pim\,(i,s) - (\sigma - 1)\,pms(i,r,s) \\
&+ (\sigma - 1)\,ams\,(i,r,s) + (\sigma - 1)\,pim(i,s) \\
&= qim(i,s) + pim\,(i,s) \\
&- (\sigma - 1)\,(tm(i,s) + tms(i,r,s)) - (\sigma - 1)\,pcif\,(i,r,s) \\
&+ (\sigma - 1)\,sext\,(i,r) \\
&- \frac{\sigma\,(\theta - \sigma + 1)}{\sigma - 1}\,(tm\,(i,s) + tms\,(i,r,s)) - \theta itc\,(i,r,s) \\
&- \frac{\theta - \sigma + 1}{\sigma - 1} fex\,(i,r,s) - (\theta - \sigma + 1)\,pcif\,(i,r,s) \\
&+ (\sigma - 1)\,pim(i,s)
\end{aligned}$$

Rearranging gives:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) = {} & qim(i,s) + pim\,(i,s) + (\sigma - 1)\,pim(i,s) \\
& - \left((\sigma - 1) + \frac{\sigma\,(\theta - \sigma + 1)}{(\sigma - 1)}\right)(tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
& + (\sigma - 1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta - \sigma + 1}{\sigma - 1} fex\,(i,r,s)
\end{aligned}$$

And further rearranging we get:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) &= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
&- \left(\frac{\sigma^2 - 2\sigma + 1 + \sigma\theta - \sigma^2 + \sigma}{(\sigma-1)}\right)(tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
&+ (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
&= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
&- \left(\frac{\theta\sigma - \theta + \theta + 1 - \sigma}{(\sigma-1)}\right)(tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
&+ (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
&= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
&- \left(\theta + \frac{\theta-\sigma+1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
&+ (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s)
\end{aligned}$$

We have employed both the expression for ams:

$$\begin{aligned}
ams\,(i,r,s) = sext\,(i,r) &- \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2}\,(tm\,(i,s) + tms\,(i,r,s)) \\
&- \left(1 + \frac{\theta-\sigma+1}{\sigma-1}\right) itc\,(i,r,s) - \frac{\theta-\sigma+1}{(\sigma-1)^2} fex\,(i,r,s) \\
&- \frac{\theta-\sigma+1}{\sigma-1} pcif\,(i,r,s)
\end{aligned} \tag{S.14}$$

And for pms:

$$pms(i,r,s) = tm(i,s) + tms(i,r,s) + pcif(i,r,s)$$

Next, we elaborate on $qim(i,r) + pim\,(i,r) + (\sigma-1)\,pim(i,r) = \sigma pim\,(i,r) + qim(i,r)$:

$$\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) &= pim\,(i,r) + qim(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= pim\,(i,r) + sum(j, PROD_COMM, SHRIFM(i,j,r) * qfm(i,j,r)) \\
&+ SHRIPM(i,r) * qpm(i,r) + SHRIGM(i,r) * qgm(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= pim\,(i,r) \\
&+ sum(j, PROD_COMM, SHRIFM(i,j,r)\,(qf(i,j,r) - \sigma * [pfm(i,j,r) - pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) - \sigma[ppm(i,r) - pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) - \sigma[pgm(i,r) - pg(i,r)]) - Dextm\,(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= -\,(\sigma-1)\,pim\,(i,r) + (\sigma-1)\,Dextm(i,r) + (\sigma-1)\,pim\,(i,r) \\
&+ sum(j, PROD_COMM, SHRIFM(i,j,r)(qf(i,j,r) + pf(i,j,r) \\
&- \sigma tfm(i,j,r) + (\sigma-1)\,pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) + pp(i,r) - \sigma tpm(i,r) + (\sigma-1)\,pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) + pg(i,r) - \sigma tgm(i,r) + (\sigma-1)\,pg(i,r)])
\end{aligned}$$

$$\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) &= (\sigma-1)\,Dextm(i,r) \\
&+ sum(j, PROD_COMM, SHRIFM(i,j,r)(qf(i,j,r) + pf(i,j,r) \\
&- \sigma tfm(i,j,r) + (\sigma-1)\,pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) + pp(i,r) - \sigma tpm(i,r) + (\sigma-1)\,pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) + pg(i,r) - \sigma tgm(i,r) + (\sigma-1)\,pg(i,r)]) \\
&= (\sigma-1)\,Dextm(i,r) + valueD(i,r) + (\sigma-1)\,priceDm(i,r) - \sigma tariffDm(i,r)
\end{aligned}$$

Using:

$$\begin{aligned}
pfm\,(i,j,r) &= tfm(i,j,r) + pim(i,r) - Dextm(i,r) \\
pgm\,(i,r) &= tgm(i,r) + pim(i,r) - Dextm(i,r) \\
ppm\,(i,r) &= tpm(i,r) + pim(i,r) - Dextm(i,r)
\end{aligned}$$

Elaborating on $Dextm\,(i,r)$ gives:

$$\begin{aligned}
Dextm\,(i,r) = & [g(i) * [\sigma-1]/\sigma] * [priceDm(i,r) - pfactwld] \\
& + [g(i)/\sigma] * (valueDm(i,r) - pfactwld) \\
& + g(i) * tariffDm(i,r) \\
& = \frac{\theta-\sigma+1}{\sigma-1}[priceDm(i,r) - pfactwld] + \frac{\theta-\sigma+1}{(\sigma-1)^2}(valueDm(i,r) - pfactwld) \\
& + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} tariffDm(i,r)
\end{aligned}$$

Substituting in gives then:

$$\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) = & (\sigma-1)\,(\frac{\theta-\sigma+1}{\sigma-1}[priceDm(i,r) - pfactwld] \\
& + \frac{\theta-\sigma+1}{(\sigma-1)^2}(valueDm(i,r) - pfactwld) + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} tariffDm(i,r)) \\
& + valueDm(i,r) + (\sigma-1)\,priceDm(i,r) - \sigma tariffDm(i,r) \\
& = \theta priceDm(i,r) + \frac{\theta}{\sigma-1} valueDm(i,r) - \frac{\theta}{\sigma-1}\sigma tariffDm(i,r)
\end{aligned}$$

So, the overall effect becomes:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) = & (\sigma-1)\,sext\,(i,r) - \left(\theta + \frac{\theta-\sigma+1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\
& - \theta\,(itc\,(i,r,s) + pcif\,(i,r,s)) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
& + \theta priceDm(i,r) + \frac{\theta}{\sigma-1} valueDm(i,r) - \frac{\sigma\theta}{\sigma-1} tariffDm(i,r)
\end{aligned}$$

And from the decomposition in equation (D.34) we had:

$$\begin{aligned}
d\ln V_{ij} = & TM = EM + IM + CM \\
& = (\sigma-1)\,sext\,(i,r) - \left(\theta + \frac{\theta-\sigma-1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\
& - \theta\,(itc\,(i,r,s) + pcif\,(i,r,s)) - \frac{\theta-\sigma-1}{\sigma-1} fex\,(i,r,s) \\
& + \theta priceD(i,s) + \frac{\theta}{\sigma-1} valueD\,(i,s) - \frac{\sigma\theta}{\sigma-1} tariffDs\,(i,s)
\end{aligned}$$

So, the two approaches generate identical expressions, which is confirmed by calculating the change in trade flows in GEMPACK in the two alternative ways.