Volume preserving CES and CET formulations

Dominique van der Mensbrugghe and Jeffrey C. Peters*
The Center for Global Trade Analysis, Purdue University

June 5, 2016

Abstract

Two of the most widely used functional forms in quantitative economic analysis are the constant-elasticity-of-substitution (CES) and constant-elasticity-of-transformation (CET) functions. The CES functional form is oft-used to represent production functions—for example, in macro growth models or in multi-sector CGE models—or preference functions—for example the Armington specification for allocating domestic absorption across purchases of domestic and imported goods. In other contexts, the CET functional form is used to allocate production or supply across different markets—for example to allocate domestic production between domestic and export markets or to allocate land supply across various land uses.

As the CES and CET specifications are now increasingly implemented in broader contexts, notably with economic models integrating more and more engineering and bio-physical properties, one potential drawback of their use is that they do not preserve additivity, i.e. the sum of the volume components do not add up to the total volume. In the case of land-use, for example, the sum of hectares devoted to different crops do not necessarily add up to total crop-land. The volume discrepancies can potentially be small and 'adjusted' away, though a paper by Fujimori et al. (2014) suggest that in the case of long-term land-use allocation, the discrepancies can become quite large, particularly at the more (spatially) disaggregated level. Even if the volume discrepancies are small, deviations from strict additivity are often confusing to scientific colleagues from other disciplines.

This paper explores the use of a modified CES/CET function that has virtually the same attributes of the standard CES/CET function but preserves additivity. The additive form of the CET has been introduced in labor allocation decisions by Dixon and Rimmer (2003) and Dixon and Rimmer (2006) and land-use allocation by Giesecke et al. (2013). This paper provides the analytical formulation of both the standard and additive forms of the CES/CET function. It compares the implementation of the additive CET using a small numerical example to highlight some key potential differences between the two formulations and then introduces the additive form of the CET function in the land-use allocation module of the Envisage Model and assesses the impacts of the two formulations on agricultural output, prices and land-use in a long-term global scenario.

1 Introduction

Two of the most widely used functional forms in quantitative economic analysis are the constant-elasticity-of-substitution (CES) and constant-elasticity-of-transformation (CET) functions. The CES functional form is oft-used to represent production functions—for example, in macro growth

*Paper being presented at the 19th Annual Conference on Global Economic Analysis, The World Bank, Washington, DC, June 15-17, 2016. The authors would like to acknowledge useful discussions with Peter Dixon and Thomas Hertel. Correspondence should be sent to Dominique van der Mensbrugghe (vandermd@purdue.edu).

models or in multi-sector CGE models—or preference functions—for example the Armington specification for allocating domestic absorption across purchases of domestic and imported goods. In other contexts, the CET functional form is used to allocate production or supply across different markets—for example to allocate domestic production between domestic and export markets or to allocate land supply across various land uses.

As the CES and CET specifications are now increasingly implemented in broader contexts, notably with economic models integrating more and more engineering and bio-physical properties, one potential drawback of their use is that they do not preserve additivity, i.e. the sum of the volume components do not add up to the total volume.[1] In the case of land-use, for example, the sum of hectares devoted to different crops do not necessarily add up to total crop-land. The volume discrepancies can potentially be small and 'adjusted' away, though a paper by Fujimori et al. (2014) suggest that in the case of long-term land-use allocation, the discrepancies can become quite large, particularly at the more (spatially) disaggregated level. Even if the volume discrepancies are small, deviations from strict additivity are often confusing to scientific colleagues from other disciplines.[2]

One of the key alternatives to the CES/CET functions has been the use of the logit specification, see for example Edmonds and Reilly (1985), Kyle et al. (2011) and Fujimori et al. (2014). Edmonds and Reilly (1985) use the logit to determine market shares of various energy technologies. Kyle et al. (2011) and Fujimori et al. (2014) use the logit to determine the allocation of land across uses. One clear advantage of the logit specification is the preservation of volume additivity. The paper by Fujimori et al. (2014) compares the use of the logit and CET specifications for allocating land in the AIM CGE model.

In this paper we introduce an alternative to the standard CES/CET specification, which we will call the additive CES/CET specification, or ACES/ACET. Examples of this specification come from Dixon and Rimmer (2006) and Giesecke et al. (2013). The former paper uses the additive CET to allocate the supply of workers (in a given category) across different activities, including the decision to be unemployed. The sum across all activities (including unemployment) sums to the total labor supply. The paper by Giesecke et al. (2013) uses the additive CET for land-use allocation.

The first two sections focus on the analytical properties of the standard CES/CET specification and its additive variant. Subsequently, a small numerical example of land-use is used to elucidate some of the key differences between the two specifications and under what conditions the two could lead to significantly different responses to shocks. The additive specification is then incorporated in the Envisage model that is a global recursive dynamic computable general equilibrium (CGE) model. The model has been used to derive projections for food and agriculture through 2050 in the context of the so-called Share Socio-Economic Pathways (SSPs). Impacts in 2050 on agricultural output, prices and land-use are compared across the two different CET specifications for land allocation.

2 The standard CET/CES formulas

2.1 The CET specification

We start with describing the CET specification—even if the CES is more widely used for nested production structures and/or utility/sub-utility specifications (for example the ubiquitous Arming-

[1] In many contexts the notion of additivity is not relevant, for example in the case of CES bundle composed of capital and labor. Additivity is clearly relevant when the components of the CES/CET are being measured in identical units.

[2] Bowles (1970) explored the deviations from additivity in the context of labor demand across different skill types. He concludes that the discrepancies, in this case, is not particularly large.

ton assumption in specifying import demand). However, the most widely used applications to date of the volume preserving CET/CES specifications are on the supply side.[3] Dixon and Rimmer (2003) and Dixon and Rimmer (2006) introduce the additive form of the CET to allocate labor supply decisions across multiple activities (including unemployment). And in a different context Giesecke et al. (2013) introduce the same functional form to allocate land supply across multiple uses. This harks back to the above-cited references to land-use specification using the logit function that also has the additive property.

The standard CET function is often used to allocate supply of a good across different destinations so as to maximize total revenue. It is used for example to allocate land supply across different potential uses and to allocate domestic production across destination markets—domestic and foreign—analogously to the use of the CES to determine commodity demand by region of origin (the Armington assumption). The basic setup is given by equation (1)—the revenue function to maximize, subject to equation (2) that represents the transformability across markets.

$$\max_{X_i} \sum_i P_i X_i \tag{1}$$

subject to

$$V = A\left[\sum_i g_i \left(\lambda_i X_i\right)^{\nu}\right]^{1/\nu} \tag{2}$$

where V is the aggregate volume (e.g. aggregate supply), X_i are the relevant components (market-specific supply), P_i are the corresponding prices, g_i are the CET (primal) share parameters, and ν is the CET exponent. The parameters A and λ_i are shifters that can be used to implement changes in preferences or technology, where A is a global shifter, for example an overall decrease or increase in 'quality' adjusted supply of land, and the λ parameters are changes that are component specific, for example a preference shift to supplying a specific type of land. The CET exponent is related to the CET transformation elasticity, ω via the following relation:

$$\nu = \frac{\omega+1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu-1}$$

The transformation elasticity is assumed to be positive. Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i \left(A\lambda_i\right)^{-1-\omega}\left(\frac{P_i}{P}\right)^{\omega} V \tag{3}$$

and

$$P = \frac{1}{A}\left[\sum_i \gamma_i \left(\frac{P_i}{\lambda_i}\right)^{1+\omega}\right]^{1/(1+\omega)} \tag{4}$$

where the γ_i parameters are related to the primal share parameters, g_i, by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i}\right)^{1/\omega}$$

[3] Peters (2016) introduces the additive CES in determining power demand from various sources—thermal, nuclear, hydro, etc.

The interpretation of equation (3) is that supply to market i is a share of aggregate supply, V, where the share depends positively on the market price in market i relative to the aggregate price (given by P). The price sensitive part of the market share depends on the degree of transformability across markets, i.e. on ω. In one extreme case, when ω is 0, the shares are fixed. In the other extreme case, when ω is infinite, allocation is based on perfect mobility (i.e. perfect homogeneity) and the law-of-one-price must hold: $P_i = P$.[4]

Note that the resulting formulas only depend on the dual share parameters (γ_i) and the transformation elasticity. Hence, the implementation of the CET does not require the primal parameters including the exponent. Calibration is straightforward given initial values for P, V, P_i, X_i and the transformation elasticity ω and assuming the technology (or preference) parameters are initialized at unit values. We have:

$$\gamma_i = \frac{X_i}{V}\left(\frac{P}{P_i}\right)^{\omega}$$

If initial prices are all equal, then the dual revenue share parameters, s_i, are equal to the base year value shares, i.e.:

$$s_i = \frac{P_i X_i}{P.V} = \gamma_i$$

In the case of the CET, it can be shown that the aggregate price index, calculated from equation (4), is the same as the aggregate price calculated from the zero profit condition, i.e.:

$$P.V = \sum_i P_i X_i$$

As we will see below, this condition does not hold in the case of the additive CET. It is also easy to show that P is equal to the shadow price of the Lagrangian function.

If we log-differentiate the two expressions above, we can derive a convenient intuitive interpretation of the CET formulas.[5] The log-differentiated forms are given by, where a dot over a variable represents percentage change of the variable in levels:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \omega\left(\dot{P}_i - \dot{P}\right) - (1+\omega)\left(\dot{\lambda}_i + \dot{A}\right)$$

$$\dot{P} = -\dot{A} + \sum_j s_j\left[\dot{P}_j - \dot{\lambda}_j\right]$$

If we substitute the second expression in the first, we have the following:

$$\dot{X}_i = \dot{V} + \omega\dot{P}_i - \omega\sum_j s_j\left[\dot{P}_j - \dot{\lambda}_j\right] - (1+\omega)\dot{\lambda}_i - \dot{A}$$

Ignoring the technology parameters, supply to market i is the sum of three components. The first is the scale effect, the second is the own price effect and the third reflects cross price effects. Note that the percent change in the aggregate price index, P, is simply the weighted average of the percent change in the component prices (ignoring technology), where the weights are the value shares, s_i. This expression is independent of the transformation elasticity.

[4] A more general formulation of the law-of-one-price would allow for fixed price wedges, but all prices would exhibit the same percentage change in case of a shock.

[5] The log-differentiated forms are typical in GEMPACK implementations of CGE models.

2.2 The CES function

The CES function is widely used as a production function to combine X_i inputs to form a composite good V using the so-called CES aggregation function given by equation (6).[6] The producer's (or consumer's) optimization problem is to minimize the the cost of inputs subject to the CES primal constraint:

$$\min_{X_i} C = \sum_i P_i X_i \tag{5}$$

subject to the constraint:

$$V = A\left[\sum_i a_i\left(\lambda_i X_i\right)^{\rho}\right]^{1/\rho} \tag{6}$$

The price of the inputs, P_i are assumed to be given. The parameters a_i are the primal share parameters and the primal exponent, ρ, is linked to the substitution of elasticities across inputs. The parameter A is a global shifter (for example an input neutral technology shifer) and the λ_i parameters are input specific shifters (or preference shifters in the case of consumer demand).

There is a closed form solution to the optimization problem given by equations (7) and (8). Equation (7) states that the demand for input i is a share of the aggregate volume V, where the share depends negatively on the price of the input relative to the aggregate price P. The higher the substitution elasticity, the more the share depends on the relative price. At one extreme, with a zero substitution elasticity, demand for input i is a strict proportion of the aggregate volume.[7] The aggregate price expression, equation (8), is sometimes referred to as the dual price expression. It can be shown that the dual price expression is equivalent to the zero profit equation, i.e. $P.V = \sum_i P_i X_i$.

$$X_i = \alpha_i\left(A\lambda_i\right)^{\sigma-1}\left(\frac{P}{P_i}\right)^{\sigma} V \tag{7}$$

$$P = \frac{1}{A}\left[\sum_i \alpha_i\left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{8}$$

where we made the following substitutions:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma-1}{\sigma}$$

and

$$\alpha_i = a_i^{1/(1-\rho)} = a_i^{\sigma} \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

Similar to the derivation above for the CET, an intuitive interpretation of the CES specification is to log-linearize the demand and aggregate price expressions. This leads to the following two expressions:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \sigma\left(\dot{P} - \dot{P}_i\right) + (\sigma - 1)\left(\dot{A} + \dot{\lambda}_i\right)$$

[6] It is often also used as a preference function, for example as a consumer utility or sub-utility function, for example the Armington specification for import demand.

[7] Also referrred to as a Leontief technology.

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_j s_j \left(\dot{P}_j - \dot{\lambda}_j\right)$$

Ignoring the technology terms, the change in the demand for X_i is a function of the two other terms—the change in the aggregate volume, sometimes referred to as the scale effect, and the relative change of the component price adjusted by the degree of substitutability, i.e. the σ parameter.

The CES is ubiquitous in CGE models, and, in the case of production nestings, Perroni and Rutherford (1995) and Perroni and Rutherford (1998), have shown that a nested CES production specification can provide the same degree of flexibility as more generic flexible functional forms. However, as models are now integrating more than ever physical quantities, such as energy, the one potential drawback of CES functions is that they don't preserve volume additivity, i.e. $V \neq \sum_i X_i$. Depending on the nature of the simulation, the deviation from additivity may not be very significant and 'adjusted' away, but this is clearly not always the case.

The key difference between the CES and CET functions, given that both σ and ω are assumed to be positive is that the component price is in the denominator in the case of the CES function, i.e. demand goes down as the component price increases (relative to the aggregate price), whereas in the case of the CET the component price is in the numerator, i.e. supply to a given market increases as the price on that market increases (relative to the aggregate market price.) The CET, similar to the CES, does not preserve volume additivity. Fujimori et al. (2014) compared the use of the CET to the volume-preserving logit in land allocation specification and found that there were significant deviations to additivity in land allocation—particularly at the spatially disaggregated level.

3 The additive forms of the CET/CES specification

3.1 The additive CET formulation

The additive form of the CET function starts with maximization of a utility function subject to volume additivity. The utility function, equation (9), is a CET aggregation of the revenues associated with the supply to all markets. Equation (10) reflects the additivity condition.

$$\max U = A \left[\sum_i g_i \left(\lambda_i P_i X_i\right)^{\nu}\right]^{1/\nu} \tag{9}$$

$$V = \sum_i X_i \tag{10}$$

The analytical solution is similar to the standard CET solution.

$$X_i = \gamma_i \left(A\lambda_i\right)^{\omega} \left(\frac{P_i}{P^c}\right)^{\omega} V \tag{11}$$

$$P^c = A \left[\sum_i \gamma_i \left(\lambda_i P_i\right)^{\omega}\right]^{1/\omega} \tag{12}$$

Equation (11) is the reduced form demand equation for component X_i, that, ignoring the technology (or preference) shifters, A and λ_i, is virtually identical to the standard CET expression. The one subtle difference is the price index for the composite (or aggregate) bundle, P^c. This is defined in equation (12). Again, ignoring the preference shifters, the price index equation differs in that the substitution elasticity enters as ω and not as $1+\omega$. And in fact the composite price index is no longer equal to the average price:

$$P.V = \sum_i P_i X_i \neq P^c.V$$

The additive form of the CET expression requires an additional equation to find the aggregate price (i.e. the one that implements the zero profit condition) because of the non-equivalence between the composite price index, P^c and the aggregate price defined by P. There are also differences between the primal and dual parameters as described in the expressions below:

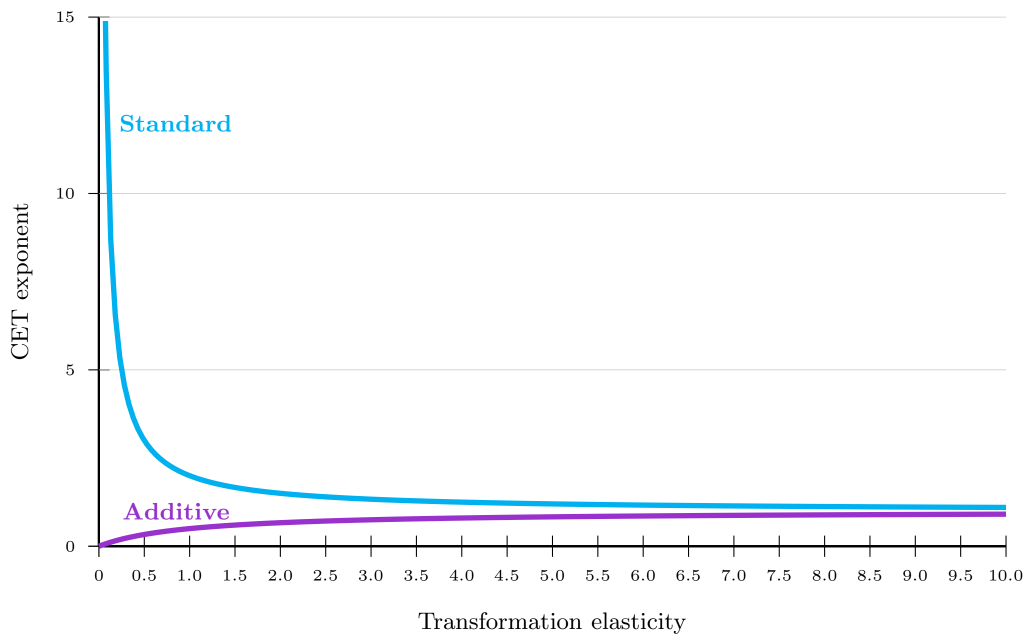
$$\omega = \frac{\nu}{\nu - 1} \Leftrightarrow \nu = \frac{\omega}{\omega + 1}$$

and

$$\gamma_i = g_i^{1/(1-\nu)} = g_i^{1+\omega} \Leftrightarrow g_i = \gamma_i^{1/(1+\omega)} = \gamma_i^{1-\nu}$$

This leads to a different relationship between the transformation elasticity and the primal exponent as illustrated in Figure 1. Under the standard CET, the relationship is continuous from $+\infty$ to 1. With the additive form, the relationship is continuous from 0 to 1. The primal exponent converges towards 1 as the substitution elasticity increases under both forms.

Figure 1: **The CET exponent (ν) as a function of the transformation elasticity**



Log-differentiation of the additive CET leads to the following expressions:

$$\dot{X}_i = \dot{V} + \omega\left(\dot{P}_i - \dot{P}^c\right) + \omega\left(\dot{\lambda}_i + \dot{A}\right)$$

$$\dot{P}^c = \dot{A} + \sum_j r_j\left[\dot{P}_j + \dot{\lambda}_j\right]$$

If we substitute the second expression in the first, we have the following:

$$\dot{X}_i = \dot{V} + \omega\left(\dot{P}_i + \dot{\lambda}_i\right) - \omega \sum_j r_j \left[\dot{P}_j + \dot{\lambda}_j\right]$$

where the weights, r_j, represent the volume shares, i.e.:

$$r_j = \frac{X_j}{V}$$

Thus, a few other subtle differences are that the percentage change in the composite price index uses the volume weights and not the value weights as in the standard CET and the technology/preference shifters have a positive impact on supply and the composite price index—not a negative impact. We can also show that the following relationship holds:

$$\lambda = u/V = P^c$$

where λ is the Lagrange multiplier. Since u is an ordinal concept, we have an extra degree of freedom in calibration. One could initialize P^c to the aggregate price (P) and then calculate u using the expression above.

3.2 The additive CES formulation

The additive form of the CES function minimizes the utility that is a CES aggregation of the costs of the components and not the usual CES aggregation of the components themselves. Hence we minimize equation (13) subject to the additivity constraint, i.e. equation (10):

$$\min U = A\left[\sum_i a_i \left(\lambda_i P_i X_i\right)^{\rho}\right]^{1/\rho} \tag{13}$$

We can derive the following solution:

$$X_i = \alpha_i \left(A\lambda_i\right)^{-\sigma}\left(\frac{P^c}{P_i}\right)^{\sigma} V \tag{14}$$

$$P^c = A\left[\sum_i \alpha_i \left(\lambda_i P_i\right)^{-\sigma}\right]^{-1/\sigma} \tag{15}$$

Equation (14) is the reduced form demand equation for component X_i, that, ignoring the technology (or preference) shifters, A and λ_i, is virtually identical to the standard CES expression. As in the case of the additive CET, the price index for the composite (or aggregate) bundle, P^c, is no longer equal to the average price, P, and the substitution elasticity enters as $-\sigma$ and not as $1-\sigma$.

There are also differences between the primal and dual parameters as described in the expressions below:

$$\sigma = \frac{\rho}{\rho - 1} \Leftrightarrow \rho = \frac{\sigma}{\sigma - 1}$$

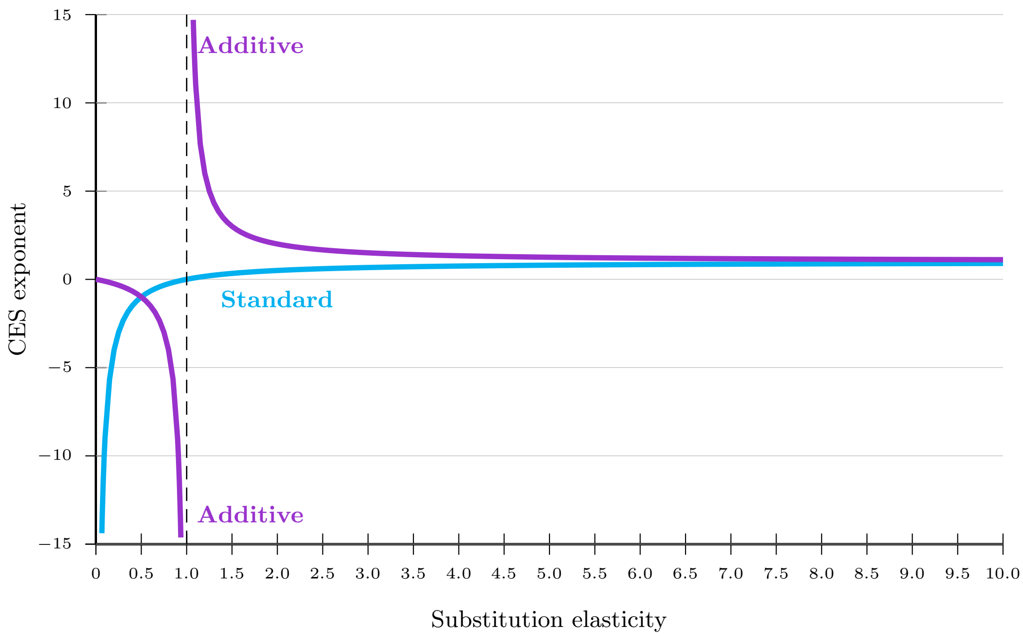
and

$$\alpha_i = a_i^{1/(1-\rho)} = a_i^{1-\sigma} \Leftrightarrow a_i = \alpha_i^{1/(1-\sigma)} = \alpha_i^{1-\rho}$$

This leads to a different relationship between the substitution elasticity and the primal exponent as illustrated in Figure 2. Under the standard CES, the relationship is continuous from $-\infty$ to 1.

With the additive form, there is a discontinuity at $\sigma = 1$. The primal exponent converges towards 1 as the substitution elasticity increases under both forms.

Figure 2: **The CES exponent (ρ) as a function of the substitution elasticity**



Log-differentiation of the additive CES leads to the following expressions:

$$\dot{X}_i = \dot{V} + \sigma\left(\dot{P}^c - \dot{P}_i\right) - \sigma\left(\dot{\lambda}_i + \dot{A}\right)$$

$$\dot{P}^c = \dot{A} + \sum_j r_j\left[\dot{P}_j + \dot{\lambda}_j\right]$$

If we substitute the second expression in the first, we have the following:

$$\dot{X}_i = \dot{V} - \sigma\left(\dot{P}_i + \dot{\lambda}_i\right) - \sigma\sum_j r_j\left[\dot{P}_j + \dot{\lambda}_j\right]$$

As in the additive CET, the percent change in the aggregate CES price index uses volume weights (r_j) instead of the value weights (s_j).

Finally, as with the CET, utility is equal to the product of the price index and the volume, i.e. $U = P^c V$.

4 Analytical differences between CET and ACET allocations

The clear parallels between the CET and ACET formulations allow for a simple analytical comparison in the context of a substitution between two different allocations (e.g. crop land allocation of wheat and maize). To focus on substitution let us allow only a single price shift, $\dot{P}_1$, and assume that there are no changes in the price of the second allocation, $\dot{P}_2$, total land supply, $\dot{V}$, productivity, $\dot{A}$, or preferences, $\dot{\lambda}_i$. By substituting the price index into the volume equations and ignoring the variables that are assumed fixed, we can then rewrite the log-linearized versions of the CET and ACET as:

$$\dot{X}_i = \omega(\dot{P}_i - s_1\dot{P}_1) \tag{16}$$

$$\dot{X}_i^A = \omega^A(\dot{P}_i - r_1\dot{P}_1) \tag{17}$$

where $\dot{X}_i$ and $\dot{X}_i^A$ are the CET and ACET allocations, respectively. We can then write an equation for the relative difference between the volume results for allocations 1 and 2 as:

$$\frac{\dot{X}_1^A}{\dot{X}_1} = \frac{\omega^A}{\omega} \cdot \frac{(1-r_1)}{(1-s_1)} \tag{18}$$

$$\frac{\dot{X}_2^A}{\dot{X}_2} = \frac{\omega^A}{\omega} \cdot \frac{r_1}{s_1} \tag{19}$$

Notice that the relative percentage change in allocation between formulations, is independent of the price shock and proportional to the difference in substitution parameters. Assuming substitution parameters are identical this difference measure is also independent of the substitution parameter. However, it is important to note that absolute discrepancy (i.e. $\dot{X}_i^A/\dot{X}_i$) will increase with the substitution parameter, ω. The relative percentage change in allocation only depends on the difference between the volume and value shares, r_1 and s_1, respectively. In other words, it is only dependent on whether P_1 is greater or less than P_2, where P_1 and P_2 are initial unit prices for 1 and 2, respectively. We can make the following conclusions about sensitivity of the discrepancy to the initial prices:

- If the original shocked $P_1 <$ unshocked price P_2 then $r_1 > s_1$ and the sensitivity of the ACET relative to the CET is greater for the unshocked allocation 2 and less for shocked allocation 1.
- If the original shocked $P_1 >$ unshocked price P_2 then $r_1 < s_1$ and the sensitivity of the ACET relative to the CET is less for the unshocked allocation 2 and greater for shocked allocation 1.

In a similar fashion, we can explore the discrepancy between the total allocation in non-volume preserving CET and the volume-preserving ACET using the following equation constructed from the initial equations, assuming identical substitution parameters (i.e. $\omega^A = \omega$).

$$\frac{\dot{V}}{\dot{V}^A} = \frac{r_1\omega(\dot{P}_1 - s_1\dot{P}_1) + r_2\omega(-s_1\dot{P}_1)}{r_1\omega(\dot{P}_1 - r_1\dot{P}_1) + r_2\omega(-r_1\dot{P}_1)} = \frac{r_1 - s_1(r_1 - r_2)}{r_1 - r_1(r_1 - r_2)} \tag{20}$$

where only $\dot{P}_1$ is shocked. We can make the following conclusions:

- For $P_1 > P_2$ (i.e. $s_1 > r_1$) and $r_1 > r_2$, then $\frac{\dot{V}}{\dot{V}^A} < 1$ and the CET loses land
- For $P_1 > P_2$ (i.e. $s_1 > r_1$) and $r_1 < r_2$, then $\frac{\dot{V}}{\dot{V}^A} > 1$ and the CET gains land
- For $P_1 < P_2$ (i.e. $s_1 < r_1$) and $r_1 > r_2$, then $\frac{\dot{V}}{\dot{V}^A} > 1$ and the CET gains land
- For $P_1 < P_2$ (i.e. $s_1 < r_1$) and $r_1 < r_2$, then $\frac{\dot{V}}{\dot{V}^A} < 1$ and the CET loses land

Table 1: Initial values

	Price	Volume	Value
Wheat	1	25	25
Maize	1	75	75
Total	1	100	100

Finally, we can conclude that the direction of the individual CET and ACET allocation results ($\dot{X}_i$ and $\dot{X}_i^A$) will be identical (i.e. the relative difference in allocation must be positive).

Let us turn to a case where price shocks are present for both allocation prices; that is, $\dot{P}_1$ and $\dot{P}_2$ are different from zero. Again, total allocation, productivity, and preferences remain fixed. In this case, the same relative difference measure can be written for allocation 1 as:

$$\frac{\dot{V}^A}{\dot{V}} = \frac{\dot{P}_1 - r_1\dot{P}_1 - r_2\dot{P}_2}{\dot{P}_1 - s_1\dot{P}_1 - s_1\dot{P}_2} = \frac{\dot{P}_1 - r_1\dot{P}_1 - \dot{P}_2 + r_1\dot{P}_2}{\dot{P}_1 - s_1\dot{P}_1 - \dot{P}_2 + s_1\dot{P}_2} = \frac{(\dot{P}_1 - \dot{P}_2) - r_1(\dot{P}_1 - \dot{P}_2)}{(\dot{P}_1 - \dot{P}_2) - s_1(\dot{P}_1 - \dot{P}_2)} \tag{21}$$

The relationship above shows that in the case of dual shocks the direction of both CET and ACET are identical, independent of the shock. Further, direction is preserved as long as the substitution parameters (ω and ω^A) have the same sign. While these properties are convenient in the case where total land allocation, productivity, and preferences are not changed, these convenient properties do not necessarily hold when these variables are changed. Still, these properties help us explain some of the results in the simple illustrative example and the application in Envisage that follow.

5 A numerical example of the additive CET

This section illustrates the additive CET with a small numerical example. Table 1 depicts a land supply choice between two sectors, called wheat and maize. The 'known' data is the total remuneration to land, i.e. the column under 'Value'. This is the data that would be incorporated in a Social Accounting Matrix (SAM). In the absence of additional information—either prices or volumes—the split of the value between price and volume is somewhat arbitrary and the standard approach is to set prices to 1 and thus volumes and values coincide. In the example, wheat and maize have respectively shares of 25 percent 75 percent share. In the sensitivity analysis below, we show that making different assumptions about the price/volume split has no impact on a counterfactual exercise because in the case of the standard CET, only value shares matter. However, in the case of the ACET, counterfactual shocks are sensitive to price initialization.

Table 2 reflects a limited set of sensitivity experiments using different assumptions on relative prices and the elasticity of transformation. The top panel assumes that initial prices are identical (and equal to 1). The middle panel assumes that the initial price of land in maize is twice the price of land in wheat. And the bottom panel assumes the reverse, i.e. that the price of wheat land is twice the price of land. The table reflects a single experiment which is a doubling of the price of wheat land—under three different assumptions of the land transformation elasticity: 0.5, 1.0 and 2.0. Under each of the elasticity assumptions, the columns represent percent changes relative to specific variables under the standard CET (labeled 'CET') and the additive CET (labeled 'ACET'). The table reports the percent change in land supply to the wheat and maize markets, the discrepancy between the sum of land supply to the individual markets and aggregate land supply, and the percent change in the aggregate price of land and the land (dual) price index.

Table 2: Sensitivity analysis from a doubling of the price of wheat land, (percent)

	$\omega = 0.5$		$\omega = 1.0$		$\omega = 2.0$	
	CET	*ACET*	*CET*	*ACET*	*CET*	*ACET*
Unit prices initially						
Wheat supply	24.7	28.2	51.2	60.0	103.8	128.6
Maize supply	-11.8	-9.4	-24.4	-20.0	-49.1	-42.9
Land Discrepancy	2.7	0.0	5.5	0.0	10.8	0.0
Price	28.5	32.0	32.3	40.0	40.1	57.1
Price index	28.5	21.8	32.3	25.0	40.1	32.3
Initial price of maize = 2 × price of wheat						
Wheat supply	24.7	21.3	51.2	42.9	103.8	81.8
Maize supply	-11.8	-14.2	-24.4	-28.6	-49.1	-54.5
Land Discrepancy	-2.8	0.0	-5.8	0.0	-12.1	0.0
Price	28.5	25.0	32.3	25.0	40.1	25.0
Price index	28.5	35.9	32.3	40.0	40.1	48.3
Initial price of wheat = 2 × price of maize						
Wheat supply	24.7	33.5	51.2	75.0	103.8	180.0
Maize supply	-11.8	-5.6	-24.4	-12.5	-49.1	-30.0
Land Discrepancy	6.6	0.0	13.6	0.0	27.2	0.0
Price	28.5	37.6	32.3	53.1	40.1	92.5
Price index	28.5	12.2	32.3	14.3	40.1	19.5

Notice that the price initialization assumption has no impact on the results down the standard CET column, i.e. the results only depend on the value shares and are independent of the volume shares. This is not the case with the additive CET formulation where the results are sensitive to the price/volume splits.[8]

With the transformation elasticity set to 0.5, land supply for wheat increases by 25 percent and land supply for maize decreases by 12 percent. The land discrepancy row measures the deviation of the sum of land supply to each sector relative to aggregate land (which is invariant), i.e. the land discrepancy measure is $100(1 - \sum_i X_i/V)$. If it is positive, there is a 'loss' of land, i.e. the sum of the components is less than total land. The discrepancy of land supply does vary with the price initialization assumption. With the transformation elasticity at 0.5, the discrepancy is almost 3 percent with the default initialization and 6.6 percent when the initial price of wheat land is twice the price of maize land. In the opposite case, when maize land is initially twice the price of wheat land the discrepancy is nearly -2.8 percent, in other words, land supply to the two sectors is greater than total land supply. The supply responsiveness using the additive CET is roughly comparable with the standard CET with the low transformation elasticity, though the differences can be important depending on price initialization (and the nature of the shock)—see for example the bottom panel.

The illustrative sensitivity analysis highlights a number of other points:

- The discrepancy rises with the transformation elasticity. In the final panel, with an elasticity of 2.0, the discrepancy is nearly 30 percent. Note that in practice land transformation elasticities tend to be relatively low, see for example Figure 6.2 in Hertel et al. (2009) where

[8] The results of the additive CET would be insensitive if we hold the volume shares constant and adjust the value shares.

the tiered transformation elasticities vary from 0.25 to 1. The default land transformation elasticity in the standard GTAP model is 1.[9]

- With low values for the transformation elasticities, both forms of the CET specification provide broadly similar orders of magnitudes in terms of the percent deviation from baseline values. The deviations become much greater as the transformation elasticity increases. In the third panel, land for wheat increase by 180 percent in the case of a transformation elasticity of 2 with the additive version of the CET, whereas the increase is only 104 percent with the standard CET. This is also where the discrepancy on land volumes is the highest.

- In the standard CET, there is no difference between the aggregate price and the aggregate (dual) price index. That is not the case with the additive CET where there can be sharp deviations between the two.

In summary, with ”standard” transformation elasticities, the two formulations of the CET exhibit a behavior that is roughly comparable–though within limits if the price/volume splits are assumed to be important and plausible. In the bottom panel, with a transformation elasticity of 1, the differences in the supply response are relatively significant as is the discrepancy in total land use. The next section assesses the relative importance of the two formulations using the Envisage model—comparing the land use implications in a baseline underlying current analysis of the economics of climate change.

6 Implications of the additive CET on land-use in Envisage

The sharp rise in agriculture and food prices in 2007/08 reinvigorated a policy focus on future food security after a relatively long period of dormant prices. The renewal of anxiety about food security also ties in with the increasing evidence of the impacts of climate change that will lead to higher temperatures, changes in precipitation patterns and more frequent extreme weather events. A new network of scientists and economists focused on agriculture has been formed—known as the Agriculture Model Intercomparison and Improvement Project (AgMIP[10])—that includes climate scientists, crop modelers and economic modelers. Their collective effort is intended to lead to improved analysis within their respective disciplines and to promote more cross fertilization across disciplines. One notable feature of the economic models of global agriculture is the wide dispersion in long-term projections for agricultural prices and land-use.[11]

Within the AgMIP context, the global economic modeling teams harmonized some of the exogenous elements of the model in order to narrow the list of reasons for model differences. Harmonization centered on three key drivers—population and GDP growth and exogenous changes in crop yields. The first two were harmonized to the relatively new shared socio-economic pathways (SSPs)[12] that have been developed for use by the Integrated Assessment Modeling (IAM) community. For the Phase 1 activities, the exogenous yield changes were provided by the International Food Policy Research Institute (IFPRI) based on the Decision Support System for Agrotechnology

9 Dixon and Rimmer (2006) use a transformation elasticity in the case of labor markets of 2. In the case of CES elasticities, there are many examples of relatively high elasticities, for example Armington trade elasticities.

10 See `www.agmip.org`

11 Phase 1 of the collective efforts of the global economic modeling teams is summarized in von Lampe et al. (2014).

12 There are a number of references to the SSPs and the SSP process. See for example Moss et al. (2010), Kriegler et al. (2012), O'Neill et al. (2012), van Vuuren et al. (2012) and O'Neill et al. (2014). The quantification of the GDP and population projections for the SSPs can be downloaded from `https://secure.iiasa.ac.at/web-apps/ene/SspDb`.

Transfer (DSSAT) crop model[13] coupled with climate models and an economic model linking yield growth to R & D expenditures and GDP growth.

Phase 1 of the modeling intercomparison work of the global economic models was detailed in a special issue of *Agricultural Economics* that included an overview of the comparison exercise (von Lampe et al. (2014)) and chapters focused on specific modules of the global models, for example demand (Valin et al. (2014)), supply (Robinson et al. (2014)) and land-use (Schmitz et al. (2014)). The comparison project did lead to many improvements of the individual models and also to a narrowing of the initial dispersion in the long-term projections—though the remaining dispersion was nonetheless significant.

The remainder of this section will focus on one of the model's used in the AgMIP exercise—known as Envisage. The model was initially developed while the author was at the World Bank and was subsequently used by the Food and Agriculture Organization of the United Nations (FAO). Though still in use at both agencies, model development is mostly done at the Center for Global Trade Analysis (GTAP) at Purdue University. Version 8 of the model will be used for the analysis herein and uses as a starting point work undertaken by a subset of the AgMIP models in a USDA funded project that resulted in a paper in *Environmental Research Letters*, see Wiebe et al. (2015). Compared to the Phase 1 simulations, those done for the *ERL* paper relied on a somewhat updated GTAP database, the latest version of the SSP GDP and population projections and new projections for the exogenous yield trends.

6.1 Land-use in Envisage

The land-use module in Envisage uses the land data available in the standard GTAP database—in this version land is only used in agriculture.[14] Land is part of a nested production structure with land demand substitutable with other factors of production (labor and capital) and also substitutable with key inputs such as chemicals in crop production and feed in meat and dairy production.[15] On the supply side, aggregate land for agriculture uses a logistic supply curve that is calibrated to an initial elasticity of supply with an asymptote that provides a limit to the amount of usable land for agriculture. Aggregate agricultural land is then allocated to different uses using a nested CET structure as depicted in figure 3.[16] The top nest differentiates land-use for vegetables and fruits and other crops from other uses—with a relatively low transformation elasticity of 0.4. The second nest differentiates sugar, cattle and raw milk from land used for cereals with an elasticity of 0.6. The final bundle includes cereals and oil seeds with a somewhat higher elasticity of 0.8. Land demand by sector is equated to land supply by sector and determines a sector-specific return to land. In the standard model, there is no guarantee that the sum of land-use across sectors will equal aggregate land use.

13 See `http://dssat.net/`.

14 Note that land use in the pork and poultry sectors is zeroed out and the land rents are imputed to capital payments. We are also not using the AEZ satellite accounts that has land by activity broken out by AEZ, of which there are up to 18 available per GTAP region.

15 See van der Mensbrugghe (2016) for a full specification of the Envisage model.

16 This structure has been adapted from LEI's MAGNET model.

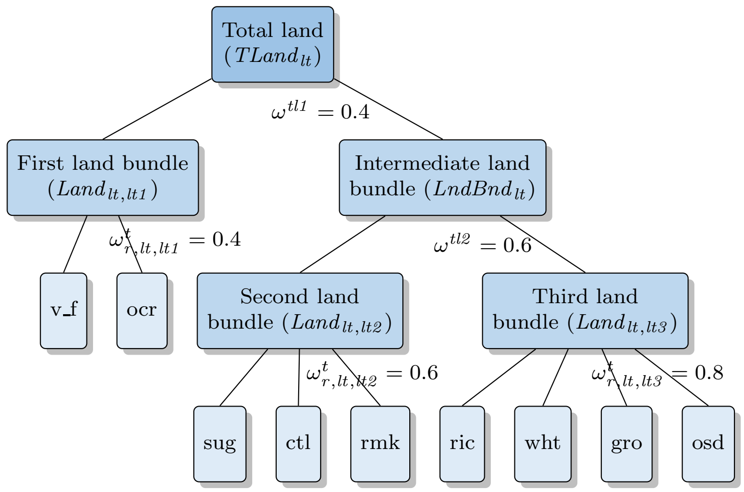


Figure 3: **Land allocation nesting**

6.2 Key results with standard CET implementation

The baseline implements the growth drivers for SSP2 for population and GDP, where we use the OECD baseline projections for GDP. Figure 4 highlights the growth in agricultural production at the world level. Under the SSP2 assumptions there would be relatively high growth in sugar (sug), vegetables and fruits (v_f), other crops (ocr), the production of beef and sheep meat (ctl) and other livestock (oap), i.e. pork and poultry. Oil seeds (osd) would be pulled along with meat production and to a somewhat lesser extent other grains (gro) that includes maize. Production of rice and wheat would lag behind.[17] This projection is probably somewhat higher than the FAO's projection of a 60 percent increase in agricultural production in 2050 from 2005/07 levels[18]—in part because a different aggregation methodology makes exact comparison difficult. In addition, the SSP2 scenario has somewhat higher population growth that is concentrated in developing countries with higher income elasticities and the SSP2 GDP growth projections are similarly significantly higher than the ones used by the FAO, again even more so for developing countries.

[17] Need to look into the sluggishness of raw milk production as dairy products are normally associated with a relatively high income elasticity.

[18] See Alexandratos and Bruinsma (2012)

Figure 4: Global output changes in baseline

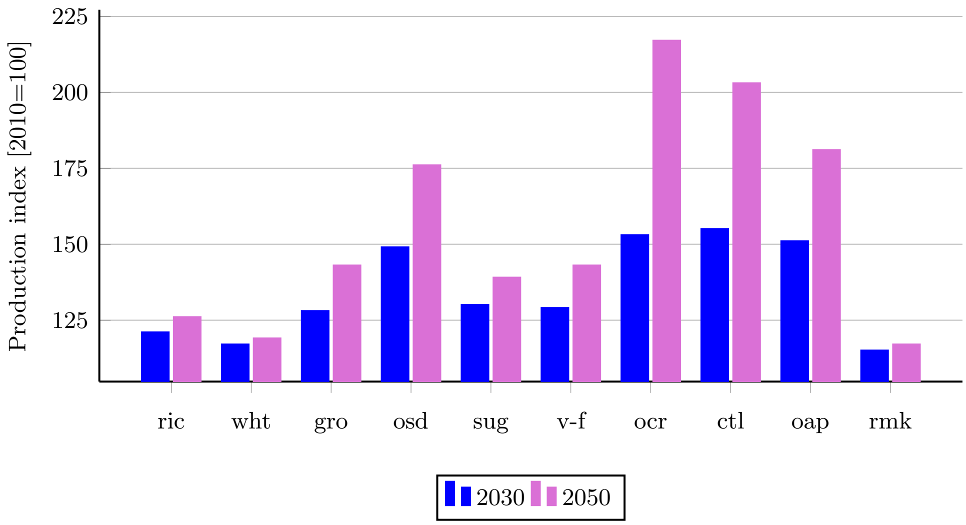
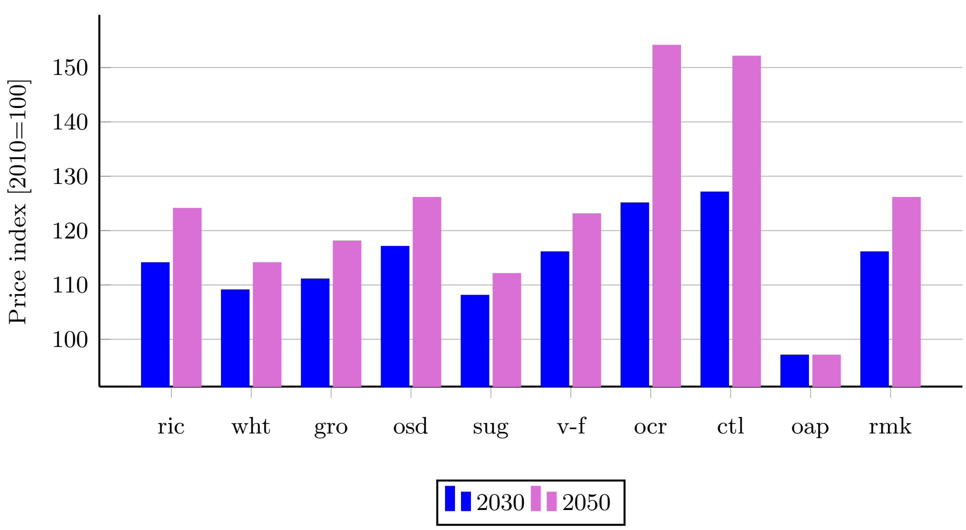


Figure 5 highlights the ENVISAGE Model's projection for prices. These are measured as the average producer price weighted by regional production shares. The model's numéraire is the average price of manufactured exports from high-income countries and thus the price increase is relative to that index. With the exception of other crops and beef and sheep production, the price increase is relatively modest—though nonetheless positive. This would signify a reversal in the decades long trend of moderately declining agricultural prices—notwithstanding the 2007/08 price spike.[19]

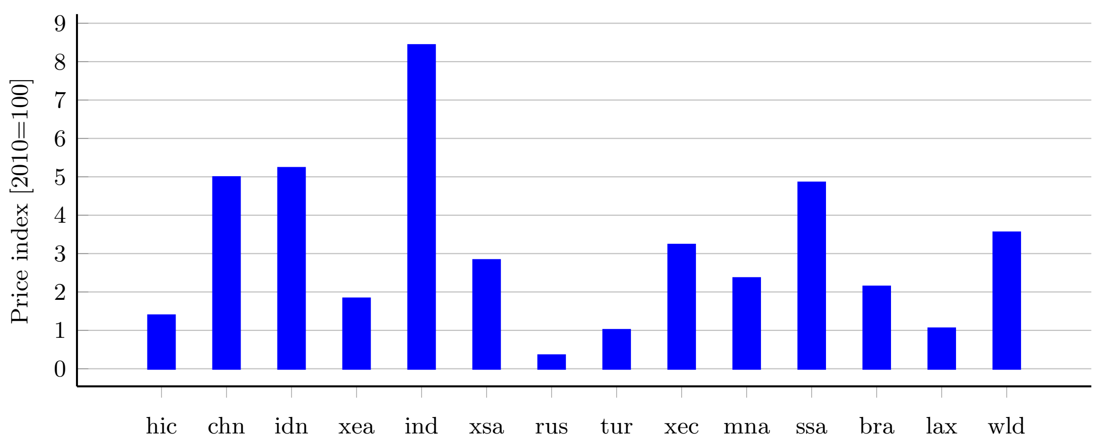
Figure 5: Global price changes in baseline



[19] Data on commodity prices is available from the World Bank's so-called *Pink Sheet* that can be downloaded from http://www.worldbank.org/en/research/commodity-markets.

Figure 6 depicts the deviations in land supply generated by the CET in the year 2050, i.e. it measures $100(1 - V/\sum_i X_i)$ where V is the aggregate amount of land and X_i is land supplied to activity i. The deviation at the global level is 3.5 percent a relatively small amount that could be 'adjusted' away without causing too much concern, particularly given the overall noise in projecting over a 40-year horizon. The deviations at the country level can be substantially greater, as highlighted in Fujimori et al. (2014). In the case of India (IND) the deviation is over 8 percent and it is around 5 percent for China (CHN), Indonesia (IDN) and Sub Saharan Africa (SSA). As alluded to in the previous section, it is possible that the deviations could be significantly higher: 1) if the land transformation elasticities are greater; 2) a different set of shocks is simulated; and 3) under a different initialization of the land variables, i.e. the price/volume split initialization. Sensitivity analysis would be the recommended path for assessing the quantitative significance of point 1. Different baseline assumptions could help elucidate the role of point 2, for example how sensitive are the deviations to changes in exogenous yield growth, differences in substitution elasticities in the production function and/or the role of income and price elasticities. The role of variable initialization could also be readily assessed with the availability of a satellite data base with land-use in volumes.

Figure 6: Land deviations



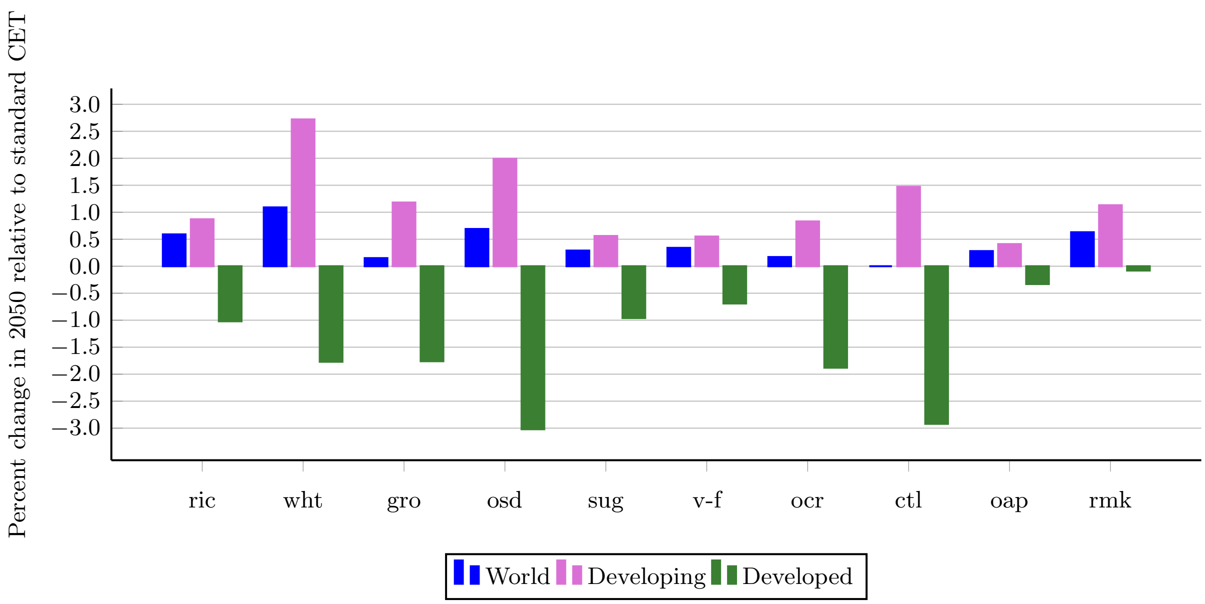
6.3 Introduction of the additive CET

This section describes the key results from replacing the standard CET for land allocation with the additive form of the CET. From the point of view of implementation—the switch is relatively painless for the standard version of Envisage. In practical terms, we assume the same initialization of prices and volumes (with all initial prices set at 1). We use the same nesting for the allocation of land supply—see figure 3 and the same CET transformation elasticities. Hence the initialization and calibration code are identical with an additional initialization of the CET (dual) price index variable that is now differentiated from the aggregate price variable. The dual price index is initialized to the initial aggregate price.[20] The model specification requires two changes. The dual price equation uses ω as the exponent rather than $\omega + 1$. And the model needs an additional equation to calculate the aggregate price derived from the zero profit condition.

[20] The initialization of the dual price index is arbitrary as it in essence defines the initial utility level that is irrelevant to model results.

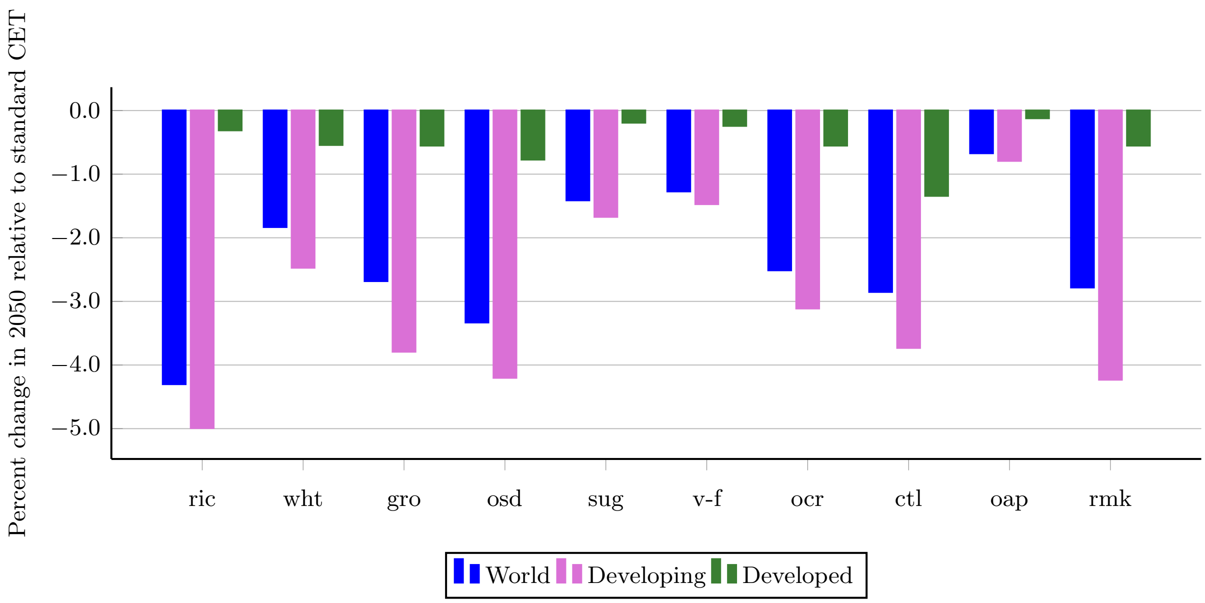
Figure 7 shows the impact on output in 2050 from implementing the additive CET compared with the standard CET. The impacts are shown across the model's agricultural sectors and for broad regional aggregates—global and developing and developed regions. The differences at the global level are relative small—most well within 1 percent. There are greater variations across regions—when comparing between developed and developing and that appear to offset each other. In the case of wheat, production is almost 2.5 percent higher in developing countries with the additive CET, counterbalanced by a reduction of over 1.5 percent for developed countries and an average increase of about 1 percent at the global level.

Figure 7: Impact of the additive CET on output



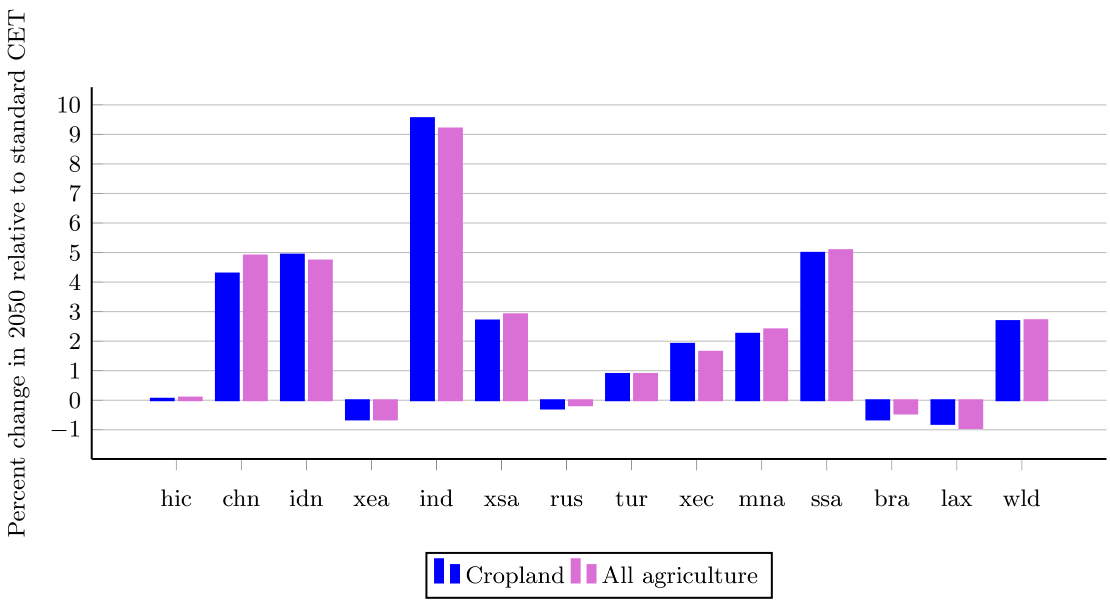
The impact on prices of using the additive CET is generally to lower agricultural prices—globally and across the two broad income regions, figure 8. The impacts on prices are significantly higher for developing countries—often around 4–5 percent and generally below 1 percent for developed countries.

Figure 8: Impact of the additive CET on prices



The impact on land-use can be relatively significant, figure 9. At the global level, the additive CET increases aggregate land-use in 2050 by nearly 3 percent. At the regional level there are sharp variations from negative or no change in the high-income (hic), rest of East Asia (xea), Russia (rus), Brazil (bra) and the rest of Latin America (lax). India (ind) has the highest percent change at 9 percent, followed by China (chn), Indonesia (idn) and Sub Saharan Africa (SSA) at around 5 percent. There is little difference in the impacts between land-use for crops and total agriculture.

Figure 9: Impact of the additive CET on land-use



7 Conclusion

With the increasing use of physical quantities in CGE models, the ubiquitous use of the CET and CES functions in model specification can lead to counter-intuitive results—particularly when sharing results across disciplines. Examples abound including labor and land markets, energy and power, and potentially others such as fertilizers and water. The most widely used alternative to to the CET/CES functions is the logit function that has been applied often in energy models and has been more recently used in land-use models and one of its characteristics is volume additivity.

This paper explores a modified version of the CES/CET function that is volume preserving and requires very modest changes in model implementation (and potentially none in model calibration and elasticity estimates). Beyond the difference in the objective function that is being optimized, the key difference between the standard form and volume preserving forms is the interpretation of the dual price index. In the case of the standard form, the dual price index is equated to the price of the aggregate volume and percent changes in the price index relates back to the initial value shares of the components. In the case of the additive form, there is no equality between the dual price index and the aggregate price, and percentage changes to the dual price index relate to the initial volume shares. Thus in the standard form the results of a shock are invariant to the initial volume/price split. This is not the case for the additive variant. Some of these issues were explored using a simple numerical example that also highlighted the role of the level of the elasticity in determining the deviation from additivity.

The final section contrasted the use of the standard versus additive CET in the land-use module of the Envisage model that has been used to look at future scenarios for agriculture and food. Deviations from additivity in the standard model are relatively small at the global level, but can be significant at the country or regional level. With the minor model modifications cited above, the model results using the additive version of the CET induce some changes, but relatively modest, particularly when considering the 40-year time horizon. They are more variegated across sectors and regions, suggesting the need for additional in-depth analysis as well as more sensitivity analysis. Three clear paths forward suggest themselves. The first is to run the same baseline simulation using the standard and additive forms of the CET but with different and presumably higher elasticities.[21] A second strand would be to introduce a proper price/volume split for land-use. The small numerical exercise suggests that large price differences that would generate large initial deviations between volume and value shares, could have a significant influence on the differences in impacts from using one form over the other. A third direction would be to assess a wide variety of shocks—particularly those that might lead to sharp changes in land-use. These might include some of the drivers in the baseline scenario such as exogenous yield growth.

Despite the limited analysis of the use of the additive CET/CES described in this paper, it appears as a promising alternative to either the standard CET/CES or the logit. In the case of the former, it requires only a slight modification to the existing standard specification and with potentially no change to the input elasticities. A more systematic analysis would be needed to see how it compares with the logit—including ideally some back-casting exercise.

References

Alexandratos, N., and J. Bruinsma. 2012. "World Agriculture Towards 2030/2050: The 2012 Revision." ESA Working Paper No. 12-03, Food and Agriculture Organization of the United Nations (FAO), Rome. http://www.fao.org/docrep/016/ap106e/ap106e.pdf.

Bowles, S. 1970. "Aggregation of Labor Inputs in the Economics of Growth and Planning: Experiments with a Two-Level CES Function." *Journal of Political Economy* 78:68–81. http://www.jstor.org/stable/1829620.

Dixon, P.B., and M.T. Rimmer. 2006. "The Displacement Effect of Labour-Market Programs: MONASH Analysis." *Economic Record* 82:S26–S40. doi:10.1111/j.1475-4932.2006.00330.x.

—. 2003. "A New Specification of Labour Supply in the MONASH Model with an Illustrative Application." *Australian Economic Review* 36:22–40. doi:10.1111/1467-8462.00265.

Edmonds, J., and J.M. Reilly. 1985. *Global Energy: Assessing the Future*. New York, NY: Oxford University Press.

Fujimori, S., T. Hasegawa, T. Masui, and K. Takahashi. 2014. "Land use representation in a global CGE model for long-term simulation: CET vs. logit functions." *Food Security* 6:685–699. doi:10.1007/s12571-014-0375-z.

Giesecke, J.A., N.H. Tran, E.L. Corong, and S. Jaffee. 2013. "Rice Land Designation Policy in Vietnam and the Implications of Policy Reform for Food Security and Economic Welfare." *Journal of Development Studies* 49:1202–1218. doi:10.1080/00220388.2013.777705.

Hertel, T.W., H. Lin, S. Rose, and B. Sohngen. 2009. "Modelling land use related greenhouse gas sources and sinks and their mitigation potential." In T. W. Hertel, S. K. Rose, and R. S. J. Tol, eds. *Economic Analysis of Land Use in Global Climate Change Policy*. Oxon, UK: Routledge, chap. 6, pp. 123–153.

[21] It would also be worth pursuing analysis of other markets—for example the energy and power markets where elasticities are likely to be higher, particularly if trade in these products is considered.

Kriegler, E., B.C. O'Neill, S. Hallegatte, T. Kram, R.J. Lempert, R.H. Moss, and T. Wilbanks. 2012. "The need for and use of socio-economic scenarios for climate change analysis: A new approach based on shared socio-economic pathways." *Global Environmental Change* 22:807–822. doi:10.1016/j.gloenvcha.2012.05.005.

Kyle, P., P. Luckow, K. Calvin, W. Emanuel, M. Nathan, and Y. Zhou. 2011. "GCAM 3.0 Agriculture and Land Use Modeling: Data Sources and Methods." Working paper No. 21025, Pacific Northwest National Laboratory (PNNL), December. http://wiki.umd.edu/gcam/images/2/25/GCAM_AgLU_Data_Documentation.pdf.

Moss, R.H., J.A. Edmonds, K.A. Hibbard, M.R. Manning, S.K. Rose, D.P. van Vuuren, T.R. Carter, S. Emori, M. Kainuma, T. Kram, G.A. Meehl, J.F.B. Mitchell, N. Nakićenović, K. Riahi, S.J. Smith, R.J. Stouffer, A.M. Thomson, J.P. Weyant, and T.J. Wilbanks. 2010. "The Next Generation of scenarios for climate change research and assessment." *Nature* 463:747–756.

O'Neill, B.C., T.R. Carter, K.L. Ebi, J. Edmonds, S. Hallegatte, E. Kemp-Benedict, E. Kriegler, L. Mearns, R. Moss, K. Riahi, B. van Ruijven, and D. van Vuuren. 2012. "Meeting Report of the Workshop on The Nature and Use of New Socioeconomic Pathways for Climate Change Research." Working paper, National Center for Atmospheric Research. https://www.isp.ucar.edu/sites/default/files/Boulder%20Workshop%20Report_0.pdf.

O'Neill, B.C., E. Kriegler, K. Riahi, K.L. Ebi, S. Hallegatte, T.R. Carter, R. Mathur, and D.P. van Vuuren. 2014. "A new scenario framework for climate change research: the concept of shared socioeconomic pathways." *Climatic Change* 122:387–400. doi:10.1007/s10584-013-0905-2.

Perroni, C., and T.F. Rutherford. 1998. "A Comparison of the Performance of Flexible Functional Forms for Use in Applied General Equilibrium Modelling." *Computational Economics* 11:245–263. doi:10.1023/A:1008641723127.

—. 1995. "Regular flexibility of nested CES functions." *European Economic Review* 39:335–343. doi:10.1016/0014-2921(94)00018-U, Symposium of Industrial Oganizational and Finance.

Peters, J.C. 2016. "GTAP-E-Power: An electricity-detailed extension of the GTAP-E model." Presented at the 19th annual conference on global economic analysis, washington dc, usa, Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN. https://www.gtap.agecon.purdue.edu/resources/download/7841.pdf.

Robinson, S., H. van Meijl, D. Willenbockel, H. Valin, S. Fujimori, T. Masui, R. Sands, M. Wise, K. Calvin, P. Havlík, D. Mason d'Croz, A. Tabeau, A. Kavallari, C. Schmitz, J. Dietrich, and M. von Lampe. 2014. "Comparing supply-side specifications in models of global agriculture and the food system." *Agricultural Economics* 45:21–35. doi:10.1111/agec.12087.

Schmitz, C., H. van Meijl, P. Kyle, G.C. Nelson, S. Fujimori, A. Gurgel, P. Havlík, E. Heyhoe, D. d'Croz Mason, A. Popp, R. Sands, A. Tabeau, D. van der Mensbrugghe, M. von Lampe, M. Wise, E. Blanc, T. Hasegawa, A. Kavallari, and H. Valin. 2014. "Land-use change trajectories up to 2050: insights from a global agro-economic model comparison." *Agricultural Economics* 45:69–84. doi:10.1111/agec.12090.

Valin, H., R.D. Sands, D. van der Mensbrugghe, G.C. Nelson, H. Ahammad, E. Blanc, B. Bodirsky, S. Fujimori, T. Hasegawa, P. Havlík, E. Heyhoe, P. Kyle, D. Mason D'Croz, S. Paltsev, S. Rolinski, A. Tabeau, H. van Meijl, M. von Lampe, and D. Willenbockel. 2014. "The future of food demand: understanding differences in global economic models." *Agricultural Economics* 45:51–67. doi:10.1111/agec.12089.

van der Mensbrugghe, D. 2016. "The Environmental Impact and Sustainability Applied General Equilibrium (ENVISAGE) Model. Version 9.0." Unpublished manuscript, Center for Global Trade Analysis (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN.

van Vuuren, D.P., M.T. Kok, B. Girod, P.L. Lucas, and B. de Vries. 2012. "Scenarios in Global Environmental Assessments: Key characteristics and lessons for future use." *Global Environmental Change* 22:884–895. doi:10.1016/j.gloenvcha.2012.06.001.

von Lampe, M., D. Willenbockel, H. Ahammad, E. Blanc, Y. Cai, K. Calvin, S. Fujimori, T. Hasegawa, P. Havlík, E. Heyhoe, P. Kyle, H. Lotze-Campen, D. Mason d'Croz, G.C. Nelson, R.D. Sands, C. Schmitz, A. Tabeau, H. Valin, D. van der Mensbrugghe, and H. van Meijl. 2014. "Why do global long-term scenarios for agriculture differ? An overview of the AgMIP Global Economic Model Intercomparison." *Agricultural Economics* 45:3–20. doi:10.1111/agec.12086.

Wiebe, K., H. Lotze-Campen, R. Sands, A. Tabeau, D. van der Mensbrugghe, A. Biewald, B. Bodirsky, S. Islam, A. Kavallari, D. Mason-D'Croz, C. Müller, A. Popp, R. Robertson, S. Robinson, H. van Meijl, and D. Willenbockel. 2015. "Climate change impacts on agriculture in 2050 under a range of plausible socioeconomic and emissions scenarios." *Environmental Research Letters* 10:085010. `http://stacks.iop.org/1748-9326/10/i=8/a=085010`.