HOW MANY HOUSEHOLDS DOES A CGE MODEL NEED AND HOW SHOULD THEY BE DISAGGREGATED?

INCOMPLETE DRAFT

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ABSTRACT

In this paper we analyze how the impact of shocks (in terms of changes in aggregate welfare, poverty, size distribution on income, and functional distribution of income) are influenced by the number of representative households (RHs) that are included and the criteria according to which they are disaggregated ("strategically" on the basis of sources of income or, alternatively, on the basis of levels of per-capita income or consumption).

By varying the number of production factors, it also tests the sensitivity of the results to the functional disaggregation. The hypotheses are that (a) starting from a single RH, initial increases in the number of RHs has a strong impact on the results when the disaggregation is strategic but that the impact quite soon becomes miniscule; (b) the larger the number of income sources, the larger the payoffs from household disaggregation; and (c) there is a sharp contrast between the results from disaggregation by quantile and strategic disaggregation, reflecting more limited sensitivity to changes in the functional distribution when households are disaggregated on the basis of per-capita incomes. In short, it is hypothesized that there is a strong case for strategic disaggregation of households and that the payoffs from fine household disaggregation are limited.

To study these issues, we built a simple static CGE model that works with alternative disaggregations of households and income sources. Specifically, our CGE model is applied to several variants – in terms of factor and/or household disaggregation – of a 2011 dataset for Guatemala. In its most disaggregated form, the dataset has 8 factors (unskilled salaried labor, skilled salaried labor, unskilled non-salaried labor, skilled non-salaried labor, capital, land, and two other natural resources), 24 sectors, and 13,100 households. In addition, households receive transfer incomes from the government and abroad

INTRODUCTION

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By varying the number of production factors, it also tests the sensitivity of the results to the functional disaggregation. The hypotheses are that (a) starting from a single RH, initial increases in the number of RHs has a strong impact on the results when the disaggregation is strategic but that the impact quite soon becomes miniscule; (b) the larger the number of income sources, the larger the payoffs from household disaggregation; and (c) there is a sharp contrast between the results from disaggregation by quantile and strategic disaggregation, reflecting more limited sensitivity to changes in the functional distribution when households are disaggregated on the basis of per-capita incomes. In short, it is hypothesized that there is a strong case for strategic disaggregation of households and that the payoffs from fine household disaggregation are limited.

The literature provides various CGE applications with multiple RHs. An early example in the literature is the analysis of the UK tax policy conducted by Pigott and Whalley (1985) using a

CGE with 100 RHs. More recently, Cockburn (2006) and Cockburn et al. (2008) developed models for Nepal and the Philippines with 3,373 and 24,797 RHs, respectively. In the same vein, other recent applications are Boccanfuso et al. (2013) for Senegal (3,278 RHs), Boccanfuso and Savard (2007) for Mali (4,966 RHs), and Rutherford and Tarr (2008) for Russia (? RHs); the latter developed a decomposition algorithm be able to solve their model.

To study these issues, we built a simple static CGE model that works with alternative disaggregations of households and income sources. Specifically, our CGE model is applied to several variants – in terms of factor and/or household disaggregation – of a 2011 dataset for Guatemala. In its most disaggregated form, the dataset has 8 factors (unskilled salaried labor, skilled salaried labor, unskilled non-salaried labor, skilled non-salaried labor, capital, land, and two other natural resources), 24 sectors, and 13,100 households. In addition, households receive transfer incomes from the government and abroad

A household disaggregation is defined as strategic if the RHs that are included are as heterogeneous as possible in terms of how they earn their incomes. We try a large number of strategic disaggregations, ranging from 2 to 13,100 RHs. In addition, we run simulations using databases with a single RH and with 10 RHs disaggregated by decile. For the calculation of poverty and inequality results, we will assume that every RH has the same consumption distribution as the subset of the 13,100 survey households that are mapped to it.

Two simulations are carried out: (1) an increase in world export and import prices for selected agricultural sectors, and (2) an increase in remittances. Thus, given our interest in assessing the sensitivity of distributive results to alternative household disaggregations, the two simulations are designed to change the distribution of income in different ways. To be more precise, we address the following questions:

- How many RHs does a CGE model need if its disaggregation is strategic? In this case, we start with one RH and gradually disaggregating into more and more RHs that are clustered to be as heterogeneous as possible in terms of how they earn their incomes. The end point is 13,100 RHs (i.e., one RH for every household in the survey). How do the results differ in terms of changes in macro indicators, wages, prices, welfare, poverty and inequality? We anticipate that, beyond a relatively small number of RHs (< 20), adding more RHs will make little or no difference to the results.

- Does it make any difference whether the household disaggregation is strategic or not? In order to address this question, we will contrast results for prices, wages, poverty and inequality for two databases with 10 RHs: (a) disaggregation on the basis of decile (common practice), and (b) the strategic disaggregation from above into 10 RHs
- Do you really need multiple RHs? Isn't it best to follow the common practice of having a single RH in the model, generate wage and price results, and analyze distribution via micro simulations based on these results? This question is addressed by comparing results for two of these databases, with a single RH and with our "optimal" number of RHs.[1]

In outline, the paper proceeds as follows. Section 2 provides a non-technical description of our CGE model and the disaggregation of its current database. The paper appendix has a detailed mathematical presentation of our CGE model.

METHOD AND DATA

CGE Model

In this section, we provide a non-technical overview of our CGE model. (Appendix A has a detailed mathematical model statement.)

The simulations in Section 3 use a simple static CGE model. Specifically, the CGE model is typical of models of small open economies with optimizing behavior for households and producers, domestic markets for commodities and factors cleared by flexible prices and wages, respectively, and a government that consumes, saves, taxes, and both receives and pays transfers. In Section 3 we present the assumptions that are applied in the simulations.

[1] A related question is address in Rutherford and Tarr (2008).

Data

The bulk of the dataset is for the simulation base-year – a social accounting matrix (SAM); stocks and sectoral employment levels for production factors (including different types of labor and capital), as well as a set of elasticities (for production, consumption, and trade).

Like other CGE models, our CGE uses a base-year SAM for Guatemala to define base-year values for the bulk of the model parameters, including production technologies, sources of commodity supplies (domestic output or imports), demand patterns (for household and government consumption, investment and exports), transfers between different institutions, and tax rates. The disaggregation of the Guatemala SAM coincides with that of the rest of the model database. As shown in Table 2.1, it is disaggregated into 24 sectors (activities and commodities) – 3 in agriculture, 1 in mining, 7 in manufacturing, and 13 in services – with each activity producing one or more commodities. The factors are split into 4 labor categories, private capital, government capital, and natural resources (4 types: agricultural land, forestry land, fishing resources, and a natural resource used in mining). The institutions are split into 4 RHs, enterprises, government, and the rest of world. As explained below, in this paper we consider a variety of disaggregations. A set of auxiliary accounts cover the different tax instruments as well as trade and transport margins on domestic sales, imports and exports. Finally, investment is split into private, government, and change in stocks.

Table 2.1: Disaggregation of Guatemala CGE and SAM

Category - #	Item	Category - #	Item
Primary (4)	Crops, traditional	**Factors (9)**	Labor, salaried unskilled
	Crops, non-traditional		Labor, salaried skilled
	Livestock, forestry and fishing		Labor, non-salaried unskilled
	Mining		Labor, non-salaried skilled
Manufacturing (7)	Food, beverages and tobacco		Capital, private
	Textiles and wearing apparel		Land
	Wood and paper		Nat res, forestry
	Petrochemical		Nat res, fishing
	Rubber, plastic, non-met min prod		Nat res, mining
	Metals, met prod and mach and eq	**Taxes (7)**	Tax, activities
	Other manufactures		Subsidy, activities
Services (13)	Electricy and water		Tax, value added (VAT)
	Construction		Tax, imports (tariffs)
	Trade		Tax, commodities
	Hotels and restaurants		Subsidy, commodities
	Transport and communications		Tax, income
	Financial services	**Intitutions (13)**	Households, by decile
	Real estate		Enterprises
	Business services		Government
	Public adminsitration		Rest of the world
	Education	**Savings and Investment (4)**	Savings
	Health		Private (non-government)
	Other services		Government
	Domestic services		Stock change
Distribution margins (3)	Dist marg, domestic		
	Dist marg, imports		
	Dist marg, exports		

Source: Authors' elaboration.

On the basis of SAM data, Table 2.2 summarizes the sectoral structure of Guatemala's economy in 2011: sectoral shares in value-added, production, employment, exports and imports, as well as the split of domestic sectoral supplies between exports and domestic sales, and domestic sectoral demands between imports and domestic output. For instance, while (primary) agriculture represents a significant share of employment (around 43.4 percent), its shares of value added (VA), production, and exports are much smaller (in the range of 12-20 percent). For traditional crops (i.e., coffee, banana, and cardamom), the share of output that is exported is around 83 percent. In turn, for non-traditional crops such as cereals, some 17 percent of domestic demands are met via imports.

Table 2.2: Sectoral structure of Guatemala's economy in 2011 (percent)

Sector	VAshr	PRDshr	EMPshr	EXPshr	EXP-OUTshr	IMPshr	IMP-DEMshr
Crops, traditional	3.2	2.2	6.5	13.5	83.1	0.0	0.2
Crops, non-traditional	5.4	4.0	28.2	3.1	10.6	5.4	17.2
Livestock, forestry and fishing	3.2	3.1	8.6	3.2	14.3	0.4	2.9
Mining	3.0	2.0	1.1	11.7	78.9	0.6	28.0
Food, beverages and tobacco	11.1	14.3	6.7	15.6	14.9	9.6	14.8
Textiles and wearing apparel	2.5	4.0	5.6	13.2	44.6	8.1	44.3
Wood and paper	0.9	1.3	0.8	2.5	26.4	4.1	50.6
Petrochemical	1.4	2.3	0.5	8.2	47.7	31.6	85.6
Rubber and plastic	1.6	2.6	1.0	3.1	16.8	4.5	32.8
Metals, met prod and mach and eq	1.0	1.8	0.8	5.9	44.4	27.7	86.7
Other manufactures	0.8	1.2	0.9	1.1	12.1	2.1	31.4
Electricy and water	2.0	2.3	0.3	0.2	1.3	0.4	3.9
Construction	4.1	6.4	4.9	0.1	0.3	0.0	0.1
Trade	18.6	14.4	14.2	0.0	0.0	0.0	0.0
Hotels and restaurants	2.4	3.7	2.5	14.5	53.7	3.4	35.2
Transport and communications	8.1	8.5	2.1	2.3	3.7	0.6	2.0
Financial services	1.8	2.9	0.7	0.0	0.1	0.6	5.4
Real estate	10.3	6.7	1.5	0.0	0.0	0.0	0.0
Business services	6.0	5.6	3.1	1.7	4.1	0.7	3.6
Public adminsitration	4.2	3.8	1.8	0.0	0.0	0.0	0.0
Education	3.5	2.4	3.1	0.0	0.0	0.0	0.0
Health	2.8	2.7	1.8	0.0	0.0	0.0	0.0
Other services	1.0	1.0	0.4	0.0	0.0	0.0	0.5
Domestic services	1.0	0.6	3.0	0.0	0.0	0.0	0.0

where VAshr = value-added share (%); PRDshr = production share (%); EMPshr = share in total employment (%); EXPshr = sector share in total exports (%); EXP-OUTshr = exports as share in sector output (%); IMPshr = sector share in total imports (%); IMP-DEMshr = imports as share of domestic demand (%).

Source: Authors' calculations based on 2011 Guatemala SAM and employment data.

Table 2.3 shows the factor shares in total sectoral value added. For example, the table shows that agriculture is relatively intensive in the use of unskilled non-salaried labor; this information will be useful to analyze the results from the CGE simulations.

Table 2.3: Sectoral factor intensity in 2011 (percent)

Sector	Lab, sal unsk	Lab, sal sk	Lab, non-sal unsk	Lab, non-sal sk	Capital	Natural resources	Total
Crops, traditional	14.5	2.0	35.1	3.0	21.4	24.0	100.0
Crops, non-traditional	11.6	1.6	59.0	5.0	10.8	12.1	100.0
Livestock, forestry and fishing	14.0	1.9	46.1	3.9	22.3	11.8	100.0
Mining	17.8	9.9	4.6	0.1	40.1	27.6	100.0
Food, beverages and tobacco	10.9	16.1	15.5	20.7	36.8	0.0	100.0
Textiles and wearing apparel	32.5	23.4	22.2	5.1	16.8	0.0	100.0
Wood and paper	15.1	21.1	16.0	7.3	40.5	0.0	100.0
Petrochemical	12.2	26.0	0.6	0.2	61.0	0.0	100.0
Rubber, plastic, non-met min prod	8.7	18.6	4.5	1.9	66.3	0.0	100.0
Metals, met prod and mach and eq	20.7	19.3	3.7	3.2	53.0	0.0	100.0
Other manufactures	19.6	8.0	23.8	9.3	39.3	0.0	100.0
Electricy and water	4.6	18.3	0.0	0.6	76.5	0.0	100.0
Construction	29.9	11.4	23.9	4.9	29.9	0.0	100.0
Trade	8.1	17.4	17.9	11.0	45.6	0.0	100.0
Hotels and restaurants	14.4	20.0	22.5	9.9	33.2	0.0	100.0
Transport and communications	9.3	11.2	1.8	8.2	69.5	0.0	100.0
Financial services	6.7	53.0	0.3	2.3	37.8	0.0	100.0
Real estate	0.0	0.0	0.8	11.8	87.4	0.0	100.0
Business services	4.9	29.1	0.5	15.3	50.1	0.0	100.0
Public adminsitration	19.8	80.2	0.0	0.0	0.0	0.0	100.0
Education	4.6	81.2	0.0	1.2	13.0	0.0	100.0
Health	3.4	51.2	2.9	23.9	18.6	0.0	100.0
Other services	9.8	24.9	6.8	10.9	47.6	0.0	100.0
Domestic services	87.7	12.3	0.0	0.0	0.0	0.0	100.0
Total	11.5	20.6	14.1	9.2	42.0	2.6	100.0

Source: Authors' calculations based on 2011 Guatemala SAM.

Table 2.4 shows household income sources in the Guatemala SAM with the 10 household split based on decile according to the level of per capita consumption. For the poorest decile, the main income source is unskilled labor, followed by remittances and transfers from government. In turn, for the richest decile, transfers from enterprises and skilled labor income are its main sources of income.

Table 2.4: Household income sources by decile (percent)

Income source	HHD decile 1	HHD decile 2	HHD decile 3	HHD decile 4	HHD decile 5	HHD decile 6	HHD decile 7	HHD decile 8	HHD decile 9	decile 10
Transfers from government	7.7	5.4	4.5	3.5	2.6	2.1	1.3	1.6	1.4	1.1
Transfers from RoW	11.5	9.7	20.5	18.1	13.8	13.8	11.9	13.2	6.4	5.6
Transfers from insdng	0.1	10.9	4.8	8.6	8.4	14.2	23.9	25.7	37.6	40.4
Labor, salaried unskilled	41.6	31.4	28.2	25.1	22.0	16.9	11.1	7.0	2.7	0.4
Labor, salaried skilled	6.7	9.9	12.6	18.1	24.7	23.5	21.4	26.8	20.4	19.6
Labor, non-salaried unskilled	31.6	27.0	25.2	20.6	22.2	19.3	17.0	11.7	10.2	3.3
Labor, non-salaried skilled	0.7	2.3	2.6	3.5	3.7	5.9	6.2	6.3	9.9	17.4
Capital	0.0	3.1	1.4	2.4	2.4	4.1	6.8	7.3	10.7	11.5
Land	0.0	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.4	0.5
Forestry res	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Fishing res	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Extractive res	0.0	0.1	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2
total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Source: Authors' calculations based on 2011 Guatemala SAM.

In addition to the SAM, our CGE model also requires (a) base-year estimates for sectoral employment levels, and (b) a set of elasticities (for production, consumption and trade). In order to estimate sectoral employment we combined employment data from the national accounts with estimates for sectoral employment shares in broad sectoral categories from the ENCOVI 2011 household survey. In turn, elasticities were given a value based on the available evidence for comparable countries. For elasticities, the following values were used: (a) the elasticity of substitution among factors is in the 0.2–1.15 range, relatively low for primary sectors and relatively high for manufactures and services (see Narayanan et al. 2015); (b) the expenditure elasticities for household consumption were obtained from Seale et al. (2003); and (c) trade elasticities are 0.9 and 1.1 for Armington and CET elasticities, respectively. Given the uncertainty with respect to our elasticity values, in Appendix B we conduct a systematic sensitivity analysis of our simulation results with respect to their values.

SIMULATIONS

Scenarios

Results and Analysis

CONCLUDING REMARKS

REFERENCES

Boccanfuso, D. and L. Savard, 2007, Poverty and Inequality Impact Analysis Regarding Cotton Subsidies: A Mali-based CGE Micro-accounting Approach, Journal of African Economies 16, pp 629-659.

Boccanfuso, D. L. Savard and A. Estache, 2013, The Distributional Impact of Developed Countries' Climate Change Policies on Senegal: A Macro-Micro CGE Application, Sustainability 2013 (5), pp 2727-2750.

Cockburn, J., 2006, Trade Liberalisation and Poverty in Nepal: A Computable General Equilibrium Micro Simulation Analysis, in M. Bussolo and J. Round, (eds.), Globalisation and Poverty - Channels and Policy Responses, Chapter 7, Routledge.

Cockburn, J., E.L. Corong and C.B. Cororaton, 2008, Poverty Effects of the Philippines' Tariff Reduction Program: Insights from a Computable General Equilibrium Analysis, Asian Economic Journal 22 (3), pp 289-319.

Piggott, J.R. and J. Whalley, 1985, Economic Effects of U.K. Tax-Subsidy Policies: A General Equilibrium Appraisal, Cambridge University Press, Cambridge.

Rutherford, R.F. and D.G. Tarr, 2008, Poverty Effects of Russia's WTO Accession: Modeling "Real" Households with Endogenous Productivity Effects, Journal of International Economics 75 (1), pp 131–150.

APPENDIX: CGE MODEL MATHEMATICAL STATEMENT

This appendix provides a detailed mathematical statement used for this paper. The model equations are divided into four blocks:

A. Production activities and factor markets;
B. Domestic and foreign trade;
C. Domestic institutions; and
D. System constraints and price indices.

Tables A.1-A.5 explain notational principles and define model sets, variables, and parameters. Drawing on these tables and set of tables with the model equations (Tables A.6-A.9), we subsequently provide a detailed presentation of the model equations with one section for each block. In the mathematical presentation, the settings for closure rules (for government budget, savings-investment payments, and factor and commodity markets) and other assumptions match what was used in the simulations of this paper.

Table A.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	exemplified on the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	QG_c
Exogenous variables	Upper-case Latin letters with a bar*	$\overline{QFS}_f$
Parameters	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$; ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and wages (rents) start with *P*, *Q*, and *WF*, respectively.

Table A.2. Sets

Name	Description
$a \in A$	activities (production sectors or industries)
$c \in C$	commodities (i.e., goods and services)

$c \in CD(\subset C)$	commodities with domestic sales of domestic output
$c \in CE(\subset C)$	exported commodities
$c \in CM(\subset C)$	imported commodities
$c \in CT(\subset C)$	transactions commodities (trade and transport services paid for under distribution margins)
$f \in F$	factors
$i \in INS$	institutions
$i \in INSD(\subset INS)$	domestic institutions
$i \in INSDNG(\subset INSD)$	domestic non-government institutions
$h \in H(\subset INSDNG)$	representative households (RHs)

Table A.3. Variables

Name	**Description**
CPI	consumer price index
DPI	domestic producer price index (PDS-based)
EG	total current government expenditure
EH_h	household consumption expenditure
EXR	exchange rate (local currency per unit of foreign currency
MPS_i	marginal propensity to save for domestic non-government institutions *i* (in *INSDNG*)
$MPSSCAL$	*MPS* scaling factor
PA_a	output price of activity *a*
PDD_c	demand price for commodity *c* produced and sold domestically
PDS_c	supply price for commodity *c* produced and sold domestically
PE_c	export price for *c* (domestic currency)
PM_c	import price for *c* (domestic currency)
PQ_c	composite commodity price for *c*
PX_c	producer price for commodity *c*
QA_a	level of activity *a*
QD_c	quantity sold domestically of domestic output *c*
QE_c	quantity of exports of commodity *c*
$QF_{f,a}$	quantity demanded of factor *f* by activity *a*
QFS_f	supply of factor *f* (in *FNPROX*)
QG_c	quantity of government consumption of commodity *c*
$QGSCAL$	government consumption scaling factor
$QH_{c,h}$	quantity consumed of commodity *c* by household *h*

$QINT_{c,a}$	quantity of commodity *c* as intermediate input to activity *a*
$QINV_c$	quantity of investment demand for commodity *c*
$QINVSCAL$	investment scaling factor
QM_c	quantity of imports of commodity *c*
QQ_c	quantity of composite supply and demand of commodity *c*
QT_c	quantity of transactions demand for commodity *c*
QX_c	quantity of domestic output of commodity *c*
$RSAVG$	real government savings (CPI-indexed)
$SAVF$	foreign savings (FCU)
$SAVG$	government savings
$SAVINS_i$	savings of domestic non-government institution *i* (in *INSDNG*)
$TRII_{i',i}$	transfers to institution *i* (in *INS*) from domestic non-government institution *i'* (in *INSDNG*)
TY_i	rate of direct tax on domestic non-government institution *i* (in *INSDNG*)
$TYSCAL$	scaling variable for direct tax on domestic non-government institutions
$WALRAS$	variable checking satisfaction of Walras' law
WF_f	economy-wide wage of factor *f* (in FNPROX)
$WFDIST_{f,a}$	wage distortion factor for factor *f* in activity *a*
YF_f	income of factor *f*
YG	government current revenue
YI_i	income of (domestic non-government) institution *i* (in *INSDNG*)
$YIF_{i,f}$	income of institution *i* (in *INS*) from factor *f*

Table A.4. Latin letter parameters

Name	Description
$cwts_c$	weight of commodity *c* in the CPI
$dwts_c$	weight of commodity *c* in the DPI
$ica_{c,a}$	quantity of intermediate input c per unit of activity *a*
$icd_{c,c'}$	transactions input of *c* per unit of commodity *c'* produced and sold domestically
$ice_{c,c'}$	transactions input of *c* per unit of commodity *c'* that is exported
$icm_{c,c'}$	transactions input of *c* per unit of commodity *c'* that is imported
$mpsb_i$	base-year marginal propensity to save for domestic non-government institution *i* (in *INSDNG*)
pwe_c	export price for *c* (foreign currency)
pwm_c	import price for *c* (foreign currency)

$qdstk_c$	change in stock (inventories) of *c*
qgb_c	base-year quantity of government consumption of commodity *c*
$qinvb_c$	base-year quantity of investment (GFCF) demand for *c*
$shif_{i,f}$	share for institution *i* (in *INS*) in the income of factor *f*
$shii_{i',i}$	share of institution *i'* (in *INS*) in the disposable income of domestic non-government institution *i* (in *INSDNG*)
$trnsfr_{ac,i}$	transfers from institution i (government or rest of world) to institution *i* (in *INS*) or factor *f* (in *F*)
ta_a	rate of tax on gross output value for activity *a*
te_c	rate of export tax on commodity *c*
tf_f	rate of direct tax on factor *f*
tm_c	rate of import tariff on commodity *c*
tq_c	rate of sales tax on commodity *c*
tyb_i	base rate of direct tax on domestic non-government institution *i* (in *INSDNG*)

Table A.5. Greek letter parameters

Name	Description
$\beta_{c,h}$	share parameter in LES function for household consumption of commodity *c*
$\gamma_{c,h}$	subsistence quantity in LES function for household consumption of commodity *c*
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity *c*
δ_c^{ds}	share parameter for domestic sales in CET function for commodity *c*
δ_c^{e}	share parameter for exports in CET function for commodity *c*
$\delta_{c,h}^{les}$	marginal share on commodity *c* in consumption of household *h*
δ_c^{m}	share parameter for imports in Armington function for commodity *c*
$\delta_{f,a}^{va}$	share parameter for factor *f* in CES VA function of activity *a*
$\theta_{a,c}$	yield of output c per unit of activity *a*
ρ_c^q	exponent in Armington function for commodity *c*
ρ_a^{va}	exponent in CES VA function for activity *a*
ρ_c^x	exponent in CET function for commodity *c*
σ_c^q	elasticity of substitution between purchases of domestic output and imports in Armington function for *c*
σ_a^{va}	elasticity of substitution between factors in CES VA function of activity *a*
σ_c^x	elasticity of transformation between domestic sales and exports in CET function

	for *c*
φ_c^q	shift parameter in Armington function for commodity *c*
φ_a^{va}	shift parameter for CES VA function of activity *a*
φ_c^x	shift parameter in CET function for commodity *c*

Table A.6. Equations for production activities and factor markets

PRD1	$QA_a = \varphi_a^{va}\left(\sum_f \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{va}}\right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A$	Value-added
PRD2	$\overline{QF}_{f,a} = \left(\frac{PVA_a}{\overline{\overline{WF}}_f \cdot WFDIST_{f,a}}\right)^{\sigma_a^{va}} \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} \left(\varphi_a^{va}\right)^{\sigma_a^{va}-1} \cdot QA_a$	$f \in FNLAB$ $a \in A$	Factor demands (non-labor)
PRD3	$QF_{f,a} = \left(\frac{PVA_a}{WF_f \cdot \overline{\overline{WFDIST}}_{f,a}}\right)^{\sigma_a^{va}} \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} \left(\varphi_a^{va}\right)^{\sigma_a^{va}-1} \cdot QA_a$	$f \in FLAB$ $a \in A$	Factor demands (labor)
PRD4	$QINT_{c,a} = ica_{c,a} \cdot QA_a$	$c \in C$ $a \in A$	Intermediate demands
PRD5	$QX_c = \sum_{a \in A} \theta_{a,c} \cdot QA_a$	$c \in C$	Output
PRD6	$PVA_a = PA_a\left(1 - ta_a\right) - \sum_{c \in C} PQ_c \cdot ica_{c,a}$	$a \in A$	Value-added price
PRD7	$PA_a = \sum_{c \in C} \theta_{a,c} \cdot PX_c$	$a \in A$	Activity price
PRD8	$QFS_f = \sum_{a \in A} \overline{QF}_{f,a}$	$f \in F$	Factor markets
PRD9	$YF_f = \sum_{a \in A} \overline{\overline{WF}}_f \cdot WFDIST_{f,a} \cdot \overline{QF}_{f,a} + trnsfr_{f,row} \cdot EXR$	$f \in FNLAB$	Factor income (non-labor)
PRD10	$YF_f = \sum_{a \in A} WF_f \cdot \overline{\overline{WFDIST}}_{f,a} \cdot QF_{f,a} + trnsfr_{f,row} \cdot EXR$	$f \in FLAB$	Factor income (labor)

Production activities and factor markets

The equations for this block are presented in Table A.6. Equations PRD1-PRD3 are the first-order conditions for the optimization problem solved by the representative firm in each industry or activity (i.e., cost minimization/profit maximization).[2] The value added production technology is CES (Constant Elasticity of Substitution; equation PRD1). In the demand functions

[2] To make it easier to read the equations, multiplication signs are inserted between multiplied items unless one or both are parenthesized.

for factors *f*, the (unit) wage in activity *a* is computed as $WF_f \cdot WFDIST_{f,a}$, where $WFDIST_{f,a}$ is a "distortion" factor that allows modeling cases in which the factor remuneration differs across activities.[3] As discussed in Lofgren et al. (2002), this formulation facilitates implementing alternative closures (i.e., mechanisms to equalize quantities supplied and demanded) in the factor markets.[4] In this application, the factors in *FNLAB* are all activity-specific and $WFDIST_{f,a}$ is the variable that clears these activity-specific markets.

For factors in *FNLAB* -- i.e., non-labor factors that are sector-specific --, equation PRD2 provides the demand functions for this factor subset. Given that factor supplies are exogenous by activity (as indicated by the bar on the factor employment variable *QF*), their wage "distortion" variables, $WFDIST_{f,a}$, clear the markets. Equation PRD3 shows demand functions for labor factors, which are assumed perfectly mobile among sectors. Equation PRD4 defines intermediate input demands using Leontief coefficients, which represent exogenous quantities of intermediate input by commodity and activity ($ica_{c,a}$).

Equation PRD5 computes the production of each commodity on the basis of the $\theta_{a,c}$ parameter, which is a fixed output coefficient (the production of *c* per unit of activity *a*). Thus, like the supply-and-use tables, our model differentiates between activities and commodities (or products): an activity can produce more than commodity and the same commodity may be produced by more than one activity.

Equations PRD6 and PRD7 define prices related to each production activity. In equation PRD6, the price of value added (the payment to value added per unit of each activity) is defined as the difference between the activity price (the payment received for outputs per unit of the activity net of the tax on activity revenue) minus the cost of intermediate inputs per unit of the activity.

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[3] In this presentation we assume that its value is constant only for labor factors, the only factor in FLAB (see below).

[4] Besides, for the factors considered as specific, equation (PRD3) is interpreted as an equilibrium condition between factor supply and demand.

The latter cost is the product of the above-mentioned intermediate input coefficient ($ica_{c,a}$) and the price of each intermediate, summed over all intermediates. Equation PRD7 defines the activity price as the product of output yields per unit of the activity, $\theta_{c,a}$, and related producer output prices, summed over all outputs.

The remaining equations of this block are factor-related, imposing equality between quantities demanded and supplied. As noted, in the current application, all factors in the set *FNLAB* are activity-specific with the variable $WFDIST_{f,a}$ clearing their markets (cf. equation PRD2). Given this, the economy-wide market equilibrium condition, PRD8, becomes superfluous; here it simply defines total factor supply (which is endogenous) as the sum of the exogenous demands in the different activities. To facilitate a switch to closures where this condition matters, this equation is nevertheless retained for all factors.[5] On the other hand, for factors in the set FLAB equation PRD8 is the economy-wide market equilibrium condition.

In equations PRD9 and 10, the total income of each factor *f,* YF_f , is defined for spector-specific and mobile factors, respectively. In both equations, the first term on the right hand side corresponds to the total factor payments from activities while the second term defines transfers to factors from the rest of the world.[6]

Table A.7. Equations for domestic and foreign trade

TRD1	$PE_c = (1-te_c) EXR \cdot pwe_c - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c',c}$	$c \in C$	Export price
TRD2	$PM_c = (1+tm_c) EXR \cdot pwm_c + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c',c}$	$c \in C$	Import price
TRD3	$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c',c}$	$c \in C$	Domestic demander

[5] Different treatments are possible. For example, total supply (employment) may be exogenous while mobility may be free across all activities; for such factors, QFS_f and $WFDIST_{f,a}$ would be exogenous while $QF_{f,a}$ and WF_f would be endogenous, leaving the total number of endogenous variables unchanged. In this case, equation PRD2 is an essential part of the model.

[6] Note that the *trnsfr* parameter is expressed in foreign currency units.

			price
TRD4	$QQ_c = \varphi_c^q \left(\delta_c^m \cdot QM_c^{-\rho_c^q} + \delta_c^{dd} \cdot QD_c^{-\rho_c^q} \right)^{-\frac{1}{\rho_c^q}}$	$c \in CM \cap CD$	Composite supply (tradables)
TRD5	$QQ_c = QM_c + QD_c$	$\left(c \in CM \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CM\right)$	Composite supply (non-tradables)
TRD6	$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \frac{\delta_c^m}{\delta_c^{dd}} \right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$	Import-domestic demand ratio
TRD7	$PQ_c \cdot QQ_c = \left(PDD_c \cdot QD_c + PM_c \cdot QM_c \right)\left(1 + tq_c\right)$	$c \in C$	Composite price
TRD8	$QX_c = \varphi_c^x \left(\delta_c^e \cdot QE_c^{\rho_c^x} + \delta_c^{ds} \cdot QD_c^{\rho_c^x} \right)^{\frac{1}{\rho_c^x}}$	$c \in CE \cup CD$	Output transfor-mation (tradables)
TRD9	$QX_c = QE_c + QD_c$	$\left(c \in CE \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CE\right)$	Output transfor-mation (non-tradables)
TRD10	$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \frac{\delta_c^{ds}}{\delta_c^e} \right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cup CD$	Export-domestic demand ratio
TRD11	$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$	$c \in C$	Producer output price
TRD12	$QT_c = \sum_{c' \in C} icd_{c,c'} \cdot QD_{c'} + \sum_{c' \in C} icm_{c,c'} \cdot QM_{c'} + \sum_{c' \in C} ice_{c,c'} \cdot QE_{c'}$	$c \in CT$	Trans-actions demand

Domestic and foreign trade

Table A.7 shows the equations for the domestic and foreign trade block. Equations TRD1-TRD3 are related to prices. In TRD1, the export price received by producers, PE_c, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient ($ice_{c',c}$) and the input price, summed over all inputs. In analogous fashion, equation TRD2 defines the domestic currency import price for demanders, PM_c, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price; note that this price does not include the sales tax (cf. equation TRD7). In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (pwe_c and pwm_c) are exogenous. Equation TRD3 links the demander and supplier prices for domestic output sold domestically, PDD_c and PDS_c : the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output; as will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation SYS2).

Equations TRD4-TRD7 explain the allocation of domestic demands between imports and domestic purchases and define the related demander price of composite commodities, made up of imported and/or domestically produced commodities. Following the Armington assumption, we assume that products are differentiated on the basis of their origin (domestic or foreign). Consequently, it is possible to account for two-way trade (i.e., the same product may be exported and imported simultaneously; this is in distinction to merely accounting for net trade). The composite commodities that are demanded domestically are a CES aggregation of domestic and imported products (equation TRD4). For commodities that lack either imports or domestic production, TRD4 is replaced by TRD5: their composite quantities are simply identical to the quantity of imports or the quantity of domestic sales of domestic output.[7] For commodities with both imports and domestic purchases of domestic output equation TRD6 is the tangency condition that determines the domestic/imported mix; it is part of the first-order conditions for demander cost minimization. Equation TRD7 implicitly defines the price of the composite product, PQ_c , given the composite quantity, QQ_c , and the sum of the values of imports and domestic purchases, adjusted upward for the sales tax; notice that the tax base excludes the tax.

[7] The elasticity of substitution between domestic purchases and imports is $\sigma_c^q = 1/(1+\rho_c^q)$.

Equations TRD8-TRD11 turn to the supply side, addressing the allocation of domestic output between two destinations: exports and domestic markets: output is viewed as imperfectly transformable between these two destinations. A CET (Constant Elasticity of Transformation) function defines the allocation of output, QX_c, between exports and domestic sales (QE_c and QD_c; equation TRD8).[8] For products that lack exports or domestic sales, equation TRD8 is replaced by equation TRD9: output quantity equals the export quantity or the domestic sales quantity. Equation TRD10 defines the ratio between exports and domestic sales for commodities that have both; it is part of the first-order conditions for producer profit maximization. Equation TRD11 implicitly defines the producer price for commodity *c*, PX_c, given the output quantity and, on the right-hand side, the sum of the values of domestic sales and exports at producer prices.

The last equation in this block, TRD12, defines the total transactions demand, i.e., the demand for commodities (in practice trade and transportation services) that are used in the distribution of commodities between (1) producers and the border (for exports); (2) producers and domestic demanders (for domestic sales); and (3) demanders and the border (for imports). For each category, the transactions demand is defined as the quantity that is distributed (exported, imported, or sold domestically) multiplied by exogenous unit input coefficients.

Table A.8. Equations for domestic institutions

INS1	$YIF_{i,f} = shif_{i,f} \cdot YF_f \left(1 - tf_f\right)$	$i \in INS$ $f \in F$	Institutional factor income
INS2	$YI_i = \sum_f YIF_{i,f} + trnsfr_{i,gov} \cdot \overline{CPI}$ $+ trnsfr_{i,row} \cdot EXR + \sum_{i' \in INSDNG} TRII_{i,i'}$	$i \in INSDNG$	Non-gov't institution income
INS3	$MPS_i = mpsb_i \cdot MPSSCAL$	$i \in INSDNG$	Marginal propensity to save

[8] The elasticity of transformation between domestic sales and exports is $\sigma_c^x = 1/\left(\rho_c^x - 1\right)$.

INS4	$SAVINS_i = MPS_i\left(1-ty_i\right)YI_i$	$i \in INSDNG$	Non-gov't institution savings
INS5	$TRII_{i,i'} = shii_{i,i'}\left(1-MPS_{i'}\right)\left(1-ty_{i'}\right)YI_{i'}$	$i \in INS$ $i' \in INSDNG$	Institutional transfers
INS6	$EH_h = \left(1-\sum_{i\in INS} shii_{i,h}\right)\left(1-MPS_h\right)\left(1-ty_h\right)YI_h$	$h \in H$	Household consumption expenditure
INS7	$QH_{c,h} = \gamma_{c,h} + \frac{\beta_{c,h}}{PQ_c}\left(EH_h - \sum_{c'} PQ_{c'} \cdot \gamma_{c',h}\right)$	$c \in C$ $h \in H$	Household consumption demand
INS8	$YG = \sum_{i\in INSDNG} TY_i \cdot YI_i + \sum_{f\in F} tf_f \cdot YF_f$ $+\sum_{c\in C} tq_c\left(PDD_c \cdot QD_c + PM_c \cdot QM_c\right)$ $+\sum_{c\in C} tm_c \cdot EXR \cdot pwm_c \cdot QM_c$ $+\sum_{c\in C} te_c \cdot EXR \cdot pwe_c \cdot QE_c + \sum_{a\in A} ta_a \cdot PA_a \cdot QA_a$ $+EXR \cdot trnsfr_{gov,row} + \sum_{i\in INSDNG} TRII_{gov,i} + \sum_{f\in F} YIF_{gov,f}$		Government income
INS9	$TY_i = tyb_i \cdot TYSCAL$	$i \in INSDNG$	Direct tax rate
INS10	$QG_c = qgb_c \cdot \overline{QGSCAL}$	$c \in C$	Government consumption demand
INS11	$EG = \sum_{c\in C} PQ_c \cdot QG_c + \sum_{i\in INSDNG} trnsfr_{i,gov} \cdot \overline{CPI} + trnsfr_{row,gov} \cdot EXR$		Government expenditure
INS12	$SAVG = YG - EG$		Government savings
INS13	$\overline{RSAVG} = \frac{SAVG}{\overline{CPI}}$		Real govern-ment savings

Domestic institutions

The equations presented in Table A.8 define current incomes and their uses (consumption, savings, and transfers) for domestic institutions (government and non-government). The first equation, INS1, computes the factor income received by each institution *i* as the total income of the factor (YF_f) net of direct taxes on factor income multiplied the share of the institution in the total endowment of the factor ($shif_{i,f}$).

Equations INS2-INS7 apply to domestic non-government institutions (in the set *INSDNG*), invariably one or more RHs but, depending on the database, potentially also including enterprises and other institutions like NGOs. This set of equations defines their incomes, savings, and spending. As shown by equation INS2, the income of institution *i* is the sum of four elements: (1) factor income; (2) transfers from the government, indexed to the consumer price index (CPI); (3) transfers from rest of the world (i.e., remittances), exogenous in foreign currency and transformed into domestic currency; and (4) transfers from other domestic non-government institutions. Equation INS3 computes the marginal propensity to save as the exogenous base rate, $mpsb_i$, multiplied by a scaling factor, $MPSSCAL$, which here is exogenous; for the base solution, the scaling factor is equal to one but otherwise it may take on other values and, under alternative treatments of the savings-investment balance, it may be endogenous. Equation INS4 computes the value of savings for each institution as its total income net of direct taxes multiplied by its savings rate. Equation INS5 defines transfers from domestic non-government institutions to any other institution (government, non-government, or the rest of the world). These transfer flows are endogenous, defined as exogenous shares of the incomes of domestic non-government institutions net of direct taxes and savings. Equation INS6 computes the consumption spending by domestic non-government institutions as their income net of transfers to other institutions, savings, and direct taxes.[9] Equation INS7 defines consumption expenditure for the subset of consuming units in *INSDNG*, labeled as households – the set *INSDNG* may also include institutions that do not consume, like enterprises. Household consumption expenditure is distributed across commodities on the basis of a Stone-Geary utility function from which linear expenditure system (LES) demand functions are derived.

A parallel set of equations, INS8-INS13, defines incomes, savings, and current spending for the government. Equation INS8 computes government current income as the sum of (1) revenues from direct taxes (on institutions and factors); (2) revenues from indirect taxes (taxes on sales, imports, exports, imports, and producer output value); (3) transfers from the rest of the world and domestic non-government institutions; and (4) factor income. Note that transfers from the

[9] Among the potential domestic non-government institutions, enterprises do not consume by definition whereas households consume. If the database includes NPISHs (non-profit institutions in the service of households, including NGOs), then these may also consume.

rest of the world to the government, like private transfers, are exogenous in foreign currency and transformed into domestic currency. As indicated by Equation INS9, initial direct tax rates are scaled by the variable *TYSCAL*, the variable that clears the current government budget. Equation INS10 computes government consumption of *c* as the product of (a) qgb_c, which is exogenous and initially holds the base-year quantities; and (b) a scaling factor, *QGSCAL*, which here also is exogenous; for the base solution, this scaling factor is equal to one but otherwise it may take on other values and, under alternative treatments of the government budgetary balance, it may be endogenous. Government current spending (equation INS11) is defined as the sum of government consumption and government transfers (domestic and/or to the rest of the world). Nominal government savings is the difference between current income and current spending (equation INS12). *Real* government savings, the ratio between nominal government savings and the CPI (the numéraire), is fixed (equation INS13). [10]

Table A.9. Equations for system constraint and price indices

SYS1	$\sum_{c\in C} pwe_c \cdot QE_c + \sum_{i\in INSD} trnsfr_{i,row} + \sum_{f\in F} trnsfr_{f,row} + \overline{SAVF} =$ $\sum_{c\in C} pwm_c \cdot QM_c + trnsfr_{row,gov} + \sum_{i\in INSDNG} \frac{TRII_{row,i}}{EXR} + \sum_{f\in F} \frac{YIF_{row,f}}{EXR}$		Current account of the balance of payments
SYS2	$\sum_{h} QH_{c,h} + \sum_{a} QINT_{c,a} + QINV_c + QG_c + qdstk_c + QT_c = QQ_c$	$c\in C$	Composite commodity demand-supply balance
SYS3	$QINV_c = qinvb_c \cdot \overline{QINVSCAL}$	$c\in C$	Investment demand

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[10] The model would not be homogeneous (of degree zero) if GSAV were fixed; hence the need to introduce real government savings. Note that, for domestic *non*-government institutions, savings is expressed as a share of income net of direct taxes. By construction, their income is equal to the sum of their spending (consumption, savings, direct tax payments and transfers to other institutions). For the government, income and current spending are specified independently with savings as the difference. Other government closures are possible; for example, fixing *TYSCAL* and flexing *RGSAV* would turn government savings into the clearing variable for the government.

SYS4	$$\sum_{i \in INSDNG} SAVINS_i + SAVG + EXR \cdot \overline{SAVF} =$$ $$\sum_{c \in C} PQ_c \left(QINV_c + qdstk_c \right) + WALRAS$$		Savings-investment balance
SYS5	$$\sum_{c \in C} PQ_c \cdot cwts_c = \overline{CPI}$$		Consumer price index
SYS6	$$\sum_{c \in C} PDS_c \cdot dwts_c = DPI$$		Producer price index

System constraints and price indices

The last equation block, presented in Table A.9, defines system constraints (the current account of the balance of payments, commodity markets, and the savings-investment balance) and two aggregate price indices (the CPI and the producer price index for domestic sales), either of which may be the numéraire of the model. The constraints for factor markets were defined in the first equation block.

The rest of the world is represented by the current account of the balance of payments, expressed in foreign currency (equation SYS1). Except for the exchange rate, *EXR*, which is used to convert terms in domestic currency into foreign currency, the different terms in this balance were covered in the preceding blocks. The left-hand side shows the inflows of foreign exchange: it is the sum of exports, current transfers, factor payments, and foreign savings; the latter is an item that corresponds to the current-account deficit and has as its counterpart a surplus in the capital and financial account. On the right-hand side, foreign exchange outflows out due to imports, current transfers from the government and non-government institutions, and factor payments. As written, it is assumed that foreign savings are exogenous and that external balance is achieved via adjustments in the value of *EXR*. To illustrate, elimination of a balance of payments deficit would be achieved via depreciation (a higher value for *EXR*), which adjusts the real exchange rate, raising the *PE/PDS* ratio for suppliers and reducing the *PDD/PM* ratio for demanders, in this way bringing about higher exports and lower imports (cf. equations TRD1, TRD2, TRD6 and TRD10); through the same mechanisms, appreciation would eliminate a surplus. Alternatively, the exchange rate could be fixed (implicitly indexed to the model numéraire; see discussion below); if so, adjustment in foreign savings would clear the external balance.

Equation SYS2 is the condition for supply-demand equilibrium in commodity markets. The composite supply, made up of domestic and imported varieties, is used for household consumption, intermediate use, investment, government consumption, and changes in inventories. For the domestic component of the commodity market (where quantities demanded for and supplied of domestic output meet – the variable QD_c in equations TRD6 and TRD10, respectively), demanders respond to changes in PDD_c and suppliers to changes in PDS_c, two variables that are linked (equation TRD3). To illustrate, if this market has excess demand for a commodity *c*, an increase in PDS_c would increase quantities supplied (by generating incentives for higher production of *c* and allocation of a larger share of this output to domestic sales as opposed to exports) while the simultaneous increase in PDD_c, would reduce quantities demanded. As stated above, the small-country assumption applies to the imported part of the commodity market: the quantities supplied are infinitely elastic at exogenous international prices and could be viewed as clearing this commodity market constraint.

The only term in equation SYS2 that had not yet been covered, investment demand (by origin), $QINV_c$, is defined in equation SYS3, which computes investment demand for *c* as the product of (a) $qinvb_c$, which is exogenous and initially holds the base-year quantities; and (b) a scaling factor, *QINVSCAL*. Like *QGSCAL*, *QINVSCAL* is exogenous and equal to one for the base solution. In other simulations it may take on other values and, depending on the treatment of the savings-investment balance, the next equation, it may be endogenous.

In the savings-investment balance, SYS4, the left-hand side defines available savings as the sum of savings from domestic non-government institutions, the government, and the rest of the world; the right-hand side defines the total investment value.[11] The variable *WALRAS* must be zero in equilibrium (if not, savings and investment are not equal and the model did not solve

[11] Given that the model is static, there is no need to disaggregate investment into different types (for example government and non-government) or to address how different types of investment are financed.

correctly). In our application, this balance is cleared by adjustments in the variable $MPSSCAL$, which is endogenous (see equation INS4). In order to switch from savings-driven investment to investment-driven savings, the analyst may exogenize $MPSSCAL$ and endogenize $INVSCAL$ (in equation SYS3). Finally, equation SYS5 defines the *CPI*, which is exogenous and the model numéraire, as a weighted average of composite commodity prices (PQ_c); the weights are the shares of each commodity in private (i.e., household) consumption.[12]

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[12] Under alternative closures for the balance of payments, the analyst may wish to fix the real exchange rate, which typically is defined as the ratio between the nominal exchange rate and an index of domestic producer prices; if so, it would be necessary to augment the model with this variable and an equation in which it is defined.