

Applying Meta-modeling for extended CGE-modeling: Sampling techniques and potential application

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Abstract

Apart from the computational time and expenses of the CGE model, the discussion of elasticity parameter estimation and various closure rules as well as the difficulty of combining the results with other analysis approaches always poses obstacles ahead of us, therefore we are motivated to apply the meta-modeling technique in order to tackle these problems from a new perspective, test its applicability and performance in the framework of the Senegal-CGE model and even compare the CGE models. The meta-modeling technique includes three essential components, which are simulation models, meta-models, and design of experiments. Our findings show that the meta-models possess a decent prediction capacity and the marginal effects differ distinctly among the sectors. However, we have not detected significant variability of the marginal effects within each sector separately. We plan to include closure rules in the follow-up research.

Keywords: CGE modeling; Elasticities; Closure Rules; Meta-modeling; Meta-models; DOE.

1 Introduction

CGE-applications are workhorse models in applied economic policy analysis, i.e. the development economic literature or modeling climate and energy policies. However, beyond its prominent application CGE-model approaches are also heavily criticized. On the one hand, while the general equilibrium model has the advantages in terms of internal consistency and allowing for clearer identification of causality, the application of a CGE model requires simplifying assumptions that are open to challenge. Moreover, empirical results derived from the CGE-model application are very sensitive to specific model specifications, that are often only weakly empirically justified, e.g. assumed closure rules and assumed elasticity parameters. Thus, many results, e.g. growth-poverty linkages, that are derived from a CGE model are in fact plagued by high model uncertainty implying a limited potential to generate robust policy-relevant messages. A drawback of existing approaches is that they focus on growth-poverty linkages and neglect policy-growth linkages, i.e. from the viewpoint of a government economic growth does not fall from heaven, but rather has to be generated using scarce budget resources. A good case in point are analyses of development policies, where policy impacts on poverty are modeled via induced policy-growth and growth-poverty linkages. However, the former is generally modeled following an ad hoc approach assuming exogenous policy impacts on sectoral technical progress. To overcome theoretical shortcomings of ad hoc CGE-approaches we suggest a combined approach incorporating econometric approaches to assess policy-growth linkages that are integrated into the CGE-approach modeling growth-poverty linkages. However, estimation of integrated econometric and CGE-modeling approaches are often tedious. Finally, CGE-model approaches are often applied to provide scientific expertise to advise the government in political practice. Hence, it would be necessary to incorporate general equilibrium models into overall decision-making models. However, given the size and complexity of CGE-models integration of these approaches into an overall decision-making modeling approach is rather difficult and often numerically not tractable.

Therefore, in the context of such a situation, we suggest application of meta-modeling as a potential solution of the application problems of standard CGE-models in advanced policy modeling frameworks and we are motivated to begin with tackling the problem of elasticity parameters and closure rules.

Meta-models, compared with standard simulation models such as the CGE model, incorporate several charming attributes: meta-models are fast to analyze. A probable property shared by all simulation models might be that they require a large amount of computational time and expenses, which rise exponentially with the growth of model complexity. Meta-models, on the other hand, are by nature mathematical approximation equations, therefore after they are fit and validated, it's easy and fast for users to exploit them and combine them with other approaches; meta-models are relatively easy to understand. Problem understanding is also an overarching goal of implementing meta-models. The interpretation of CGE model results is always tricky as there exist usually

transmitting channels which distribute the impact to a number of aspects and complicate the understanding and interpretation of the CGE model. In comparison, meta-models such as lower-order polynomial meta-model, have explicit forms which are readily comprehensible; Meta-models are flexible to construct. We can treat the discussion of elasticity parameter estimation and closure rules from a totally new perspective by integrating the elasticity parameters into the meta-models and estimating them under each possible closure rule; Meta-models enable us to combine them with other approaches directly. It is extremely problematic and sometimes even impossible to integrate CGE models into other analysis approaches. For example, we want to apply the Bayesian model selection method in order to select which CGE model has the highest probability of being the true one and this process is impossible to be carried out by using CGE models. However, under such circumstances, using meta-models as surrogates of CGE models provides us an opportunity to solve this problem.

The meta-modeling methodology aims at generating the valid meta-models which are considered to be the surrogate models of the CGE models such that the comparison between various meta-models is equivalent to the comparison between various CGE models.

All in all, the application of the meta-modeling technique enables us to understand the CGE model better, analyze it faster and easier, tackle the problem of elasticity parameters and closure rules with a new method and use it for more research purposes.

2 Literature Review

CGE-modeling is a common workhorse in development economics and policy analysis. It has been widely used to model climate and energy policies as well. (Bourguignon [2003]; Lofgren et al. [2002]; Fan [2008]). However, in spite of its prominent applications, CGE-modeling has been heavily criticized because empirical results derived from CGE-models are very sensitive to specific model specifications, that are often only weakly justified, e.g. assumed closure rules and assumed elasticity parameters. (Lofgren and Robinson [2008]; Hazledine et al. [1992]; Arndt et al. [2002]) Thus, many results, e.g. growth-poverty linkages, that are derived from a CGE model are in fact plagued by high model uncertainty implying a limited potential to generate robust policy-relevant messages. (Lofgren and Robinson [2008]). Meta-modeling technique has been extensively used in field such as engineering, natural science, production design and etc. (Srivastava et al. [2004]; Noordegraaf et al. [2003]; Kleijnen and Standridge [1988]) The basic approach is to construct approximation models of the simulation models in order to generate surrogate models that are accurate and reliable enough to replace the original ones with the purpose of understanding the simulation models better and combining the simulation models with other analysis methods.(Kleijnen and Sargent [2000]; Kleijnen [2008]) Building approximation models include two essential components: design of experiments and meta-models, the former is used to produce the simulation sample (Kleijnen

et al. [2005]; Giunta et al. [2003]; Eriksson et al. [2000]) while the latter is used to determine the form of the surrogate models. (Simpson et al. [2001]; Wang [2007]). The application of meta-modeling technique enables us to circumvent the discussion about the rationality of closure rules and elasticity parameters and go directly into the exploration of true CGE model and integrate CGE models into other research analyses. To the best of our knowledge, despite the rich applications of the meta-modeling technique in other areas, it has not been used in company with CGE models. Thus, this paper aims at building a bridge between the two methods.

3 Meta-modeling

The meta-modeling technique includes three essential components: the simulation model, the meta-model, and the experimental design.(Kleijnen and Sargent [2000]) The meta-model is a mathematical approximation equation that we assume and use to approximate the Input/Output behavior of the simulation model (the Senegal-CGE model in this paper). The experimental design, also known as Design of Experiments or DOE for short, is a method to produce the simulation sample (simulation inputs) from the design space. The simulation model, the foundation of this technique, is used to take in the simulation inputs (I) and generate the simulation outputs (O). In practice, it is treated as a black box, in other words, it is used as a simulation machine and we focus mainly on the Inputs/Outputs instead of what is happening inside. To sum up, the simulation inputs will be absorbed by the black box and the simulation outputs will be produced, thus we can use them to fit and validate the meta-model.

3.1 Meta-models

Meta-models aim at approximating the Input/Output relationships of simulation models. The term meta-model was popularized and developed by Jack Kleijnen(Kleijnen [1975]), but the term and concept were both originated by Robert Blanning(Blanning [1974]; Blanning [1975]). Meta-models are usually used to model the behavior of another model and they are also termed surrogate models or response surface models. In the history of meta-models, they are applied to approximate both the stochastic simulation and the deterministic simulation.

A meta-model is a mathematical function that takes some simulation model design parameters as inputs and produces an approximation of simulation outputs. Examples of model design parameters are actually any parameter of interest which are considered to have the possibility to exert an impact on the output. In the Senegal-CGE model, for example, we have three types of model design parameters: policy indicators(e.g., technical progress shocks), production elasticities and trade elasticities. Examples of simulation outputs in our case are the welfare of small-scale farmers, urban consumer welfare and etc.

There are many types of meta-models in the literature and for our current

research purpose, we will focus on two types of them, which are the lower-order polynomial and kriging meta-models.

3.1.1 Lower-Order Polynomial Metamodel

Lower-order polynomial meta-models are originally developed for the analysis of physical experiments (Box and Wilson [1951]) and they have been used effectively for building approximations in a variety of applications. There are different forms of this type but the most commonly used forms are first-order polynomial and second-order polynomial meta-models.

A second-order polynomial meta-model has the functional form:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_i \sum_j \beta_{ij} x_i x_j + \epsilon, \quad (1)$$

where x_i and x_j are the model design parameters and β' s are the corresponding coefficients and ϵ is the error term which is often assumed to be a white noise process.

The corresponding coefficients β' s are estimated using the ordinary least-squares regression and the estimates are computed as follows:

$$\hat{\beta} = [X'X]^{-1}X'y, \quad (2)$$

where X is the model design matrix and y is the simulation output. We can also perform other standard statistical analysis of the estimates.

Lower-order polynomial meta-models are attractive because they are easy to construct, understand and analyze. Besides, they work well in modeling local and linear behavior of the simulation model but if the simulation model is nonlinear or irregular, they might fail in approximating the behavior and we must resort to other meta-model types.

3.1.2 Kriging Metamodel

Kriging meta-models are originally developed for applications in geostatistics (Cressie and Chan [1989]), a kriging model postulates a combination of a polynomial model and departures of the form:

$$y = \sum_{i=1}^k \beta_i f_i(x) + Z(x), \quad (3)$$

where $f_i(x)$ is the polynomial model and $Z(x)$ is assumed to be a realization of a stochastic process with mean zero and spatial correlation function given by:

$$Cov[Z(x_i), Z(x_j)] = \sigma^2 R(x_i, x_j), \quad (4)$$

where σ^2 is the variance of this process and R is assumed to be the correlation function of this process. A variety of correlation functions can be chosen, such

as linear correlation function, exponential correlation function and Gaussian correlation function. Besides, as $Z(x)$ is also assumed to be a stationary process so the covariances $R(x_i, x_j)$ are dependent only on the distance between the input combinations x_i and x_j . The corresponding coefficients are estimated using the maximum likelihood estimation method.

The Kriging meta-models are more flexible and can be used to model nonlinear or irregular behaviors of the simulation model.

3.2 Design of Experiments(DOE)

Design of experiments (Eriksson et al. [2000]), or DOE for short, is a sampling technique which we can apply to sample the model design space in order to generate the simulation sample. For example, we have k quantitative design parameters and each of them has n different values, which means that if we want to run all the possible scenarios, we would end up with n^k simulation runs and it could probably be a number that we are not able to handle. Therefore, we need a technique with which we can generate a workable sample while at the same time this sample must possess desirable properties and enough information for the follow-up analysis.

There is a large number of experimental designs in the literature, but for our current purpose we will discuss two types of DOE, the Central Composite Design and the Latin Hypercube Sample Design.

3.2.1 Central Composite Design

The Central Composite Design or CCD is a classical fractional factorial experimental design which spreads the sample points at three different places of the design space: (i) the vertices of the design space; (ii) the center of the design space; (iii) the star points which are placed along the axes but outside the design space (Giunta et al. [2003]).

A two-variable CCD contains the following sample points:

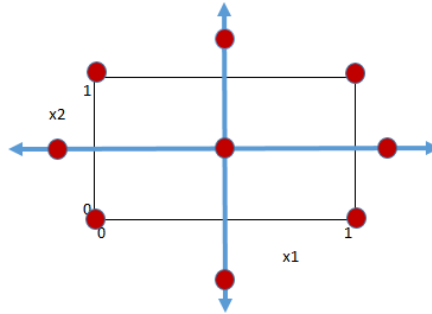


Figure 1: A Central Composite Design for $n = 2$.

The central composite design guarantees that the estimates of the coefficients of a second-order polynomial metamodel are unbiased. The number of sample points of a CCD follows the formula $2^n + 2n + C_0$, where n is the number of variables, 2^n is the number of sample points at the vertices, $2n$ is the number of star points and C_0 is the number of center points. In a central composite design, we can not control the number of sample points once n is fixed except that C_0 is an arbitrary number which we can alter. This means when the number of variables n grows, the sample points that we need to estimate the metamodel also increases exponentially.

3.2.2 Latin Hypercube Sample Design

Latin Hypercube Sample Design, or LHS for short, is a space-filling design which arranges the sample points as spread-out as possible across the design space in order to collect information inside the design space. Besides, LHS has another attribute that we can control the number of sample points based on practical concerns.

The Latin hypercube sample design works as follows (McKay et al. [1979]; Stocki [2005]): suppose we have n variables and we need p sample points to fit our metamodel. Then the intervals of every variable are divided into p subintervals and one value is chosen out of every subinterval based on the probability density within that subinterval for each variable. Next, the p values of x_1 is paired randomly with the p values of x_2 , then this established pair of x_1 and x_2 is again paired at random with the p values of x_3 , and this process will be continued until the p n -tuplets are formed which are exactly the p sample points that we need for the simulation.

We can have a look at the following example with $n = 2$ and $p = 4$:

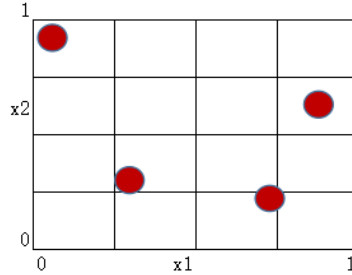


Figure 2: A Latin Hypercube Sample Design.

As a member of the space-filling design family, the latin hypercube design aims at placing the sample points as spread-out as possible across the design space. There are many criteria and optimality rules regarding generation of nice latin hypercube samples from which we list the following ones, such as (R package “lhs”):

1. Random LHS: draws a latin hypercube sample from a set of uniform distributions for use in creating a latin hypercube design. This sample is taken in a random manner without regard to optimization.
2. Improved LHS: draws a latin hypercube sample from a set of uniform distributions for use in creating a latin hypercube design. This sample is drawn based on the idea of optimizing the sample with respect to an optimum euclidean distance between design points.
3. Maximin LHS: draws a latin hypercube sample from a set of uniform distributions for use in creating a latin hypercube design. This sample is drawn based on the idea of optimizing the sample by maximizing the minimum distance between design points (maximin criteria).
4. Genetic LHS: draws a latin hypercube sample from a set of uniform distributions for use in creating a latin hypercube design. This sample is drawn based on the idea of optimizing the sample with respect to the S optimality criterion through a genetic type algorithm. S optimality seeks to maximize the mean distance from each design point to all the other points in the design space, so the points are as spread-out as possible.
5. Optimum LHS: draws a latin hypercube sample from a set of uniform distributions for use in creating a latin hypercube design. This sample is drawn using the Columnwise Pairwise (CP) algorithm to generate an optimal design with respect to the S optimality criterion.

3.3 CGE Models

The simulation model we use in this paper is a standard dynamic CGE model using the Senegal data for calibration. For a detailed description of CGE model, see Lofgren et al. [2002].

3.4 Methodology

The general meta-modeling flow (Figure 3) can be described as follows:

1. The simulation model (the Senegal-CGE model) is treated as a black box and we assume a meta-model for it.
2. The design of experiments is applied to generate the inputs and they are used to produce the outputs.
3. The simulation inputs and outputs are collected in order to fit and validate the meta-model. If the criteria are met, we can use the meta-model for other research purposes.

Table 1 lists the detailed meta-modeling process which contains the following 7 steps (Barton [2015]):

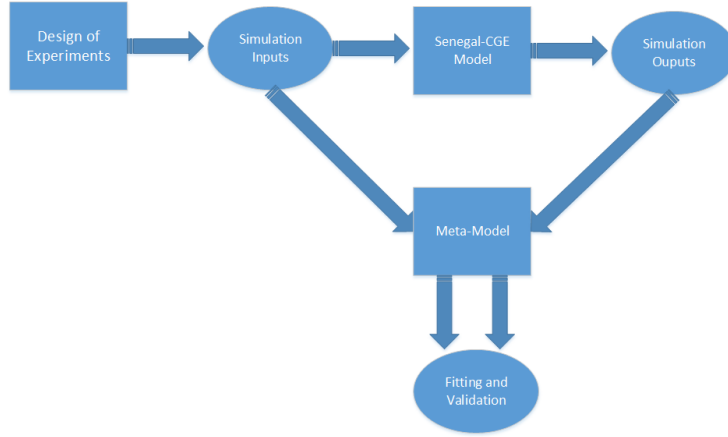


Figure 3: The Meta-modeling Flow.

Step	Activity
1	Meta-modeling Purposes
2	Inputs and Outputs Identification
3	Meta-model Types
4	Experimental Design based on Meta-model Types
5	Conducting Simulation Runs and Collecting Outputs
6	Fitting and Validation of Meta-model Adequacy
7	Use Meta-model for Other Research Purposes

Table 1: Meta-modeling Process

3.4.1 Meta-modeling Purposes

Our purpose of implementing meta-models is to understand the Senegal-CGE model better and use meta-models in the follow-up research.

3.4.2 Inputs and Outputs Identification

The Senegal-CGE model contains a number of variables (inputs) and for our current purpose, we take three categories of them into consideration (in total we have 20 variables of interests):

1. Policy indicators. In our case it is the technical progress shock in eight aggregated sectors: crop, export, livestock, fish, agribusiness, industry, service and public. All of them range from 1% to 10%.
2. Production elasticities (factor substitution) in eight aggregated sectors:

crop, export, livestock, fish, agribusiness, industry, service and public. All of them range from 1.5 to 6.

3. Trade elasticities (Armington transformation) in agricultural and non-agricultural sectors. The trade elasticities in agricultural sector range from 0.5 to 3.3 and the trade elasticities in nonagricultural sector range from 0.9 to 4.1.

The Senegal-CGE model has seven outputs, which are: $z1$ (Small Household Income), $z2$ (Poverty Reduction Index), $z3$ (General Public Services), $Z4$ (Welfare of Agribusiness), $z5$ (Urban Consumer Welfare), $z6$ (Welfare of Agricultural Export Sectors), $z7$ (Environmental Protection). In this paper, we will analyze $z1$, $z2$, and $z5$ in order to test the performance of meta-models and the impacts of reduced-form meta-models.

3.4.3 Meta-model Types

In section 3.1, we have introduced two commonly used meta-model types: the lower-order polynomial meta-model and the Kriging meta-model. In addition to these two types, there are many other meta-model types to choose from, including radial basis functions, neural networks, and regression trees. See Chen et al. [2006] for a review. Because of the simplicity of construction and comprehension, lower-order polynomial meta-models are always a good place to start.

3.4.4 Experimental Design

In section 3.2, we have introduced two experimental designs: the central composite design and Latin hypercube design. There have been researches on the connection between the meta-models and experimental designs as well as the connection between the nature of simulations and experimental designs. (Simpson et al. [2001]) In this paper, because of the practical considerations such as computational time and the fact that we come across a deterministic Senegal-CGE model, LHS is a proper beginning point.

3.4.5 Fitting and Validation

We collect the simulation inputs using the DOE and the simulation outputs by running the simulation scenarios. Then we use them to fit the meta-model. This process is not tricky because we use the standard OLS approach to estimate the coefficients. If the fitting is satisfactory, usually it is determined by the R^2_{adj} , we can move forward to validating the meta-model. There are various kinds of criteria which can determine the validation adequacy and we use the root mean squared prediction error(rmse) and the maximal absolute relative error(mare):

$$rmse = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (5)$$

$$mare = \max \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad \forall i, \quad (6)$$

where n is the number of observations, y_i is the true output and $\hat{y}_i$ is the corresponding predicted output.

Besides, in order to be more certain of the validation adequacy, we apply both the in-sample validation and out-of-sample validation, therefore we need to split the sample into two parts. We have a sample with the size of 2000 and divide it into two subsamples, which are the training sample and the test sample, each with the size of 1000. The training sample is used to perform the in-sample validation while the test sample is used to perform the out-of-sample validation.

With respect to the in-sample validation, we use the cross-validation method. Firstly, the first observation is deleted and the meta-model is fitted using the rest 999 observations. Then the output of the deleted observation is predicted using the newly fitted meta-model. Secondly, the second observation is deleted and the meta-model is fitted using the rest 999 observations. Then the output of the deleted observation is predicted using the newly fitted meta-model. The same process will be applied to each observation of the training sample and 1000 predicted outputs will be computed. With respect to the out-of-sample validation, we use the training sample to fit the meta-model and then use the fitted meta-model to predict the outputs using the observations of the test sample. Thus we can calculate the two statistics for both the in-sample validation and out-of-sample validation.

3.4.6 Use Meta-model for Other Research Purposes

The CGE models are criticized for many reasons from which we will discuss two prominent ones: elasticity parameters and closure rules.

On the one hand, the CGE models are calibrated by the elasticity parameters in order to fit the data, namely, for each different set of elasticities, we have a completely different CGE model. In one case, some elasticity parameters cannot be estimated and researchers usually solve this problem by assuming them to be certain values, which leads to the fact that the results become very sensitive. In another case, the estimation and determination of some elasticities are not trivial because they require a large amount of econometric modeling, therefore in practice, most researchers recycle the estimates of others, though often modifying them for one reason or another or “.....selecting these values by consulting econometric and other model-based studies.....” (Lofgren and Robinson [2008]). Besides, empirically, researchers will perform the sensitivity analysis by altering the elasticities either to a high value or to a low value and comparing the result with that of the original value.

On the other hand, the CGE models aim to capture all the impacts, which might be distributed through the whole economy, of a shock such as tax rise or tariff cut. And the models are ‘closed’ by requiring that the total supply of an object must be equal to the total demand, with whatever adjustments needed to achieve it. In practice, however, there does not exist a true general model and the choice of closure rules is usually arbitrary, which will definitely affect the results.

All in all, both the elasticity parameters and the closure rules have impacts on the final results but we normally don’t have information on which of them are correct or rational, therefore in this paper, we would like to tackle this problem from a completely new perspective with the help of the meta-modeling technique.

With respect to the elasticity parameters, the simulation sample at our hand contains a sample size of 2000 and each of them contains a set of elasticity parameters. Therefore the simulation sample is equivalent to 2000 CGE models in the background. For each CGE model, we generate a reduced-form meta-model for it. From such an angle, we free ourselves from the discussion on the rationality of elasticity parameters, instead, we accept the fact that we have no idea which CGE model is the true one but we can use our data to find it (this is the topic of our follow-up paper). In this paper, we want to test if the policy indicators are sensitive to different CGE models because if the answer was yes, it would make a great sense that we endeavor to find the true CGE model but if the answer was no, that means the option of CGE models (elasticities parameters) in our case does not merit high attention and efforts.

With respect to the closure rules, we use the same approach to generate the reduced-form meta-models under each closure rule and apply other methods to find the best one that matches the data.

In this paper, we will only discuss the analysis of elasticity parameters. The study of closure rules will be left to another paper.

The general polynomial meta-model in our case takes policy indicators, production elasticities and trade elasticities as independent variables:

$$\begin{aligned}
\hat{y} = & \alpha_0 + \sum_i \alpha_i tp_i + \sum_j \beta_j \theta_j \\
& + \sum_i \sum_{i'=i+1} \alpha_{ii'} tp_i tp_{i'} + \sum_i \sum_j \beta_{ij} tp_i \theta_j + \sum_j \sum_{j'=j+1} \gamma_{jj'} \theta_j \theta_{j'} \\
& + \sum_i \alpha_{ii} tp_i^2 + \sum_j \gamma_{jj} \theta_j^2,
\end{aligned} \tag{7}$$

where tp_i ’s are the technical progress shocks and θ ’s are the elasticity parameters. α ’s, β ’s and γ ’s are the corresponding coefficients.

The reduced-form meta-model takes only the policy indicators as independent variables while the production and trade elasticities are considered to be

fixed values in the reduced-form metamodel:

$$\begin{aligned}
\hat{y} = & (\alpha_0 + \sum_j \beta_j \bar{\theta}_j + \sum_j \sum_{j'=j+1} \gamma_{jj'} \bar{\theta}_j \bar{\theta}_{j'} + \sum_j \sum_j \gamma_{jj} \bar{\theta}_j^2) \\
& + \sum_i (\alpha_i + \sum_j \beta_{ij} \bar{\theta}_j) tp_i \\
& + \sum_i \sum_{i'=i+1} \alpha_{ii'} tp_i tp_{i'} + \sum_i \alpha_{ii} tp_i^2,
\end{aligned} \tag{8}$$

where the general settings remain the same with the only exception that in the reduced-form meta-models θ 's are fixed values and thus they are grouped into other coefficients.

In such a sense, we are using the general meta-model we have fitted and validated to construct the reduced-form meta-models. Furthermore, we also validate these reduced-form meta-models by using the reduced-form meta-models to predict the outputs based on all observations and determine their performance via the two aforementioned criteria *rmse* and *mare*. The fitting and validation process is the first barrier on our way to using this technique, if we had good results of it, we are ready to move to the second barrier, which is, to what extent can meta-models affect the variables that we are interested in. We quantify this impact by the marginal effects of the sector-specific technical progress shocks on the outputs:

$$\begin{aligned}
\frac{\partial \hat{y}}{\partial tp_i} = & \sum_i (\alpha_i + \sum_j \beta_{ij} \bar{\theta}_j) tp_i \\
& + \sum_{i'=i+1} \alpha_{ii'} tp_{i'} + \sum_i \alpha_{ii} tp_i
\end{aligned} \tag{9}$$

We expect to detect significant deviations of the marginal effects for different sectors because that could lead us to the conclusion that reduced-form meta-models are impacting the variables of interests (technical progress shocks) and it is thus meaningful to make selections among them by using other methods such as Bayesian model selection.

4 Results

4.1 Fitting and Validation

4.1.1 General Meta-model

We apply the methodology to three outputs $Z1$ (welfare of small-scale farmers), $Z2$ (poverty reduction) and $Z5$ (urban consumer welfare).

Firstly, let's have a look at the fitting performance. The R_{adj}^2 are 0.998, 0.9998 and 0.998 respectively, which means that the fitting works quite well for the three outputs.

Secondly, let's have a look at the validation performance which is summarized in Table 2.

	In-Sample rmse	In-Sample mare	Out-of-Sample rmse	Out-of-Sample mare
Z1	0.821	0.0035	0.808	0.0034
Z2	0.008	0.0004	0.012	0.0014
Z5	2.792	0.0027	2.750	0.0051

Table 2: Validation Performance

In practice, there is not a lower threshold for *rmse* but the value of both the in-sample and out-of-sample validation can be viewed as very low, meaning a good validation performance. While for *mare*, a recommended lower threshold is 0.1 and we can easily see that the values for both cases are much lower than this threshold, which gives us an interpretation that our meta-model has done a good job in modeling the relationship between the inputs and outputs.

Besides, we can have a look at Figure 4 which shows the predicted outputs versus true outputs for both the training sample and the test sample. The fact that there are not clear deviations of the predicted values from the true values (almost a perfect fit) leads us again to the conclusion that the meta-model works well in modeling the behavior of the simulation model (the Senegal-CGE model) and can be used as a surrogate of the simulation model in the following analysis. In other words, if we intend to achieve other research purposes with the help of the simulation model, we can now use the meta-model to replace the simulation model. This will not only make the analysis faster and easier but will also make some previously-not-feasible approach feasible, such as the Bayesian Model Selection approach.

4.1.2 Reduced-form Meta-model

Since the general meta-model has been validated, we can furthermore validate all the reduced-form metamodels by using each reduced-form meta-model to predict the outputs based on each set of technical progress shocks. Moreover, we can compare the *rmse* and *mare* of the predictions from all reduced-form meta-models in order to quantify the performance.

	mean rmse	min rmse	mean mare	min mare
Z1	4.477	3.308	0.014	0.010
Z2	0.052	0.039	0.002	0.001
Z5	14.611	10.628	0.012	0.008

Table 3: Reduced-form Meta-model Performance

With respect to Z1, the average and minimal *rmse* are 4.477 and 3.308 while the average and minimal *mare* are 0.014 and 0.010. In comparison with

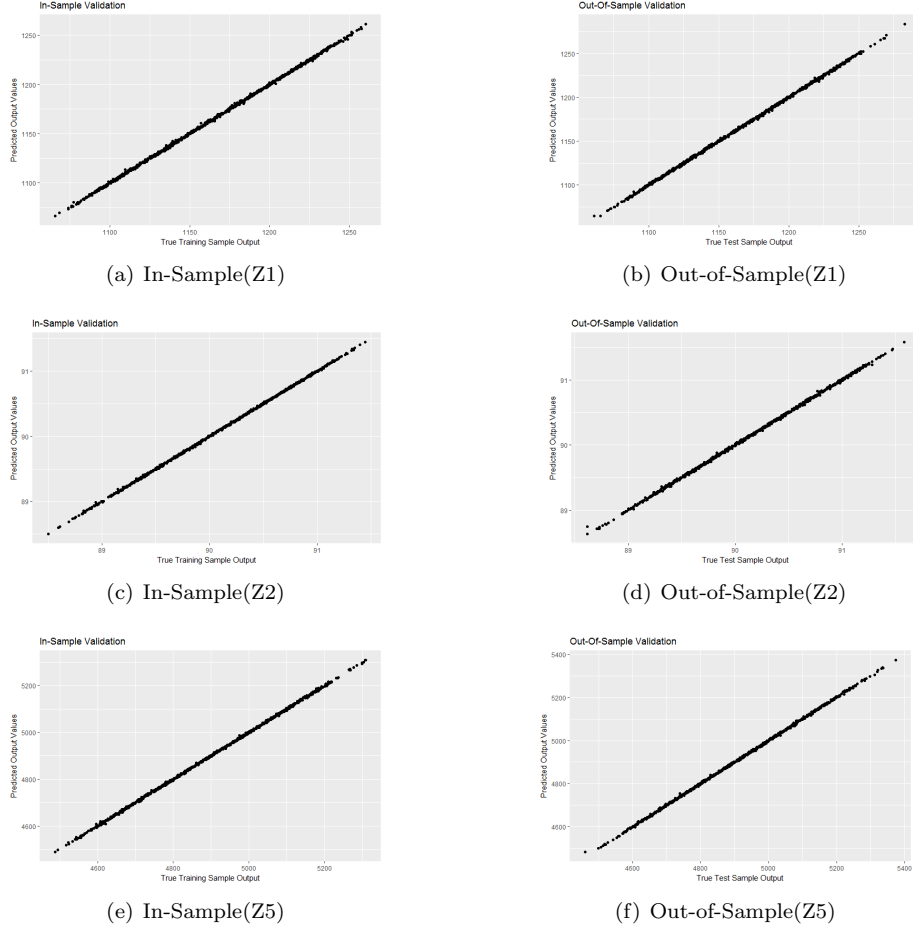


Figure 4: Prediction Performance

the validation performance of the general meta-model, we can readily see that the reduced-form meta-models have much larger $rmse$ and $mare$ than that of the general meta-model meaning that the general meta-model has better prediction capability. However, we could have foreseen this result because we have constructed the reduced-form meta-models on the basis of the general meta-model such that they have less explanatory variables which decrease their capacity of prediction. Nevertheless, we can still come to the conclusion that the reduce-form meta-models are accepted based on the validation results. Figure 5 displays the distribution of $rmse$ which proves that the prediction performance of reduced-form meta-models are in general still quite good.

With respect to Z2 and Z5, we arrive at similar conclusions which can also be shown by Figure 6 and 7 that the reduced-form meta-models work well in

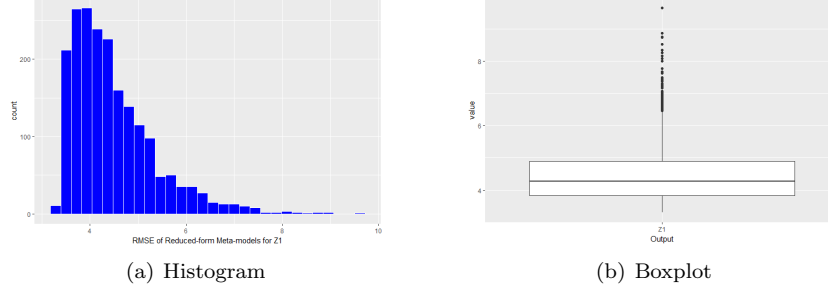


Figure 5: RMSE of Reduced-form Meta-models for Z1

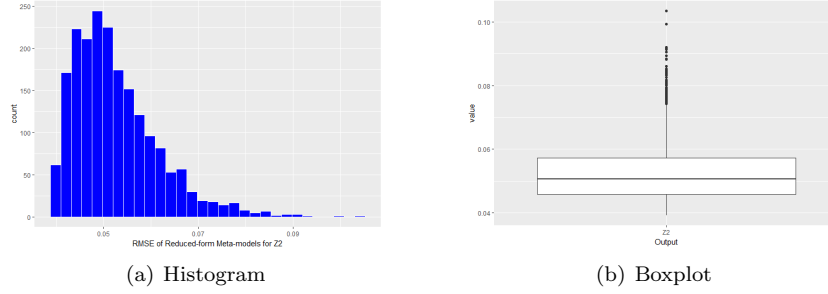


Figure 6: RMSE of Reduced-form Meta-models for Z2

predicting the outputs meaning that we can use the reduced-form meta-models to perform the analysis in the next step.

4.2 Testing the Impact of Reduced-form Meta-models

In practice, we find out that the coefficients of the main effects are much larger than the coefficients of the interaction effects and quadratic effects, therefore, to simplify the calculation, we measure the marginal effects of technical progress shocks using only the coefficients of the main effects. Figure 8,10 and 12 show the distribution of the sector-specific marginal effects of technical progress shocks on the outputs $Z1$, $Z2$ and $Z5$ respectively and figure 9, 11 and 13 are the corresponding histogram of the marginal effects. Thus, we want to use two measures to determine if there is variability in the sector-specific marginal effects, namely, the interquartile range and standard deviation. By definition, the interquartile range of a box-whisker plot includes the middle 50% of the data and the larger the interquartile range, the more variable the data set is. From figure 8,10 and 12 we can come to the conclusion that for all the three outputs, the interquartile ranges of marginal effects of every sector are extremely narrow meaning that the sector-specific marginal effects are quite stable in this case, however, we can see that for different sectors, the marginal effects differ obviously, which means

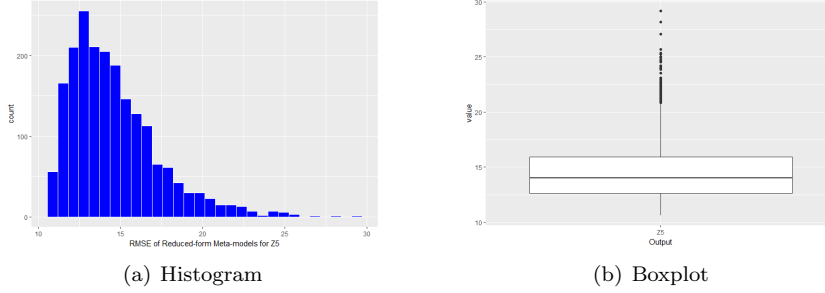


Figure 7: RMSE of Reduced-form Meta-models for Z5

that sectors have quite distinct impacts on the outputs. Moreover, from figure 9, 11 and 13 we can detect that the marginal effects of each sector for all the three outputs is distributed normally or approximately normally, and thus the 95% of the distribution is within two standard deviations from the mean. In our case, the standard deviation of the marginal effects of each sector for all the three outputs are relatively low, mostly varies between 0.5 and 2, therefore, we can conclude that the marginal effects are not variable. To summarize, the impacts from elasticity parameters in this case are stable and not obvious.

The elasticity parameters are very important in CGE modeling (Lofgren et al. [2002] and Fan [2008]). Some of them cannot be directly estimated while some of them need complex econometric models in order to be estimated, thus researchers often extract them from literature and use them to start the simulations. Afterward, for the sake of comparison, they usually perform the sensitivity analysis by giving the parameters high and low values so as to compare the results and draw conclusions. In our demonstration, we use a completely different method to test the impact of reduced-form meta-models which is equivalent to the impact of elasticity parameters. Although we have not detected obvious effects, the methodology merits research and study. Besides, in the follow-up research, we plan to incorporate closure rules into the analysis in order to see if they can make a difference.

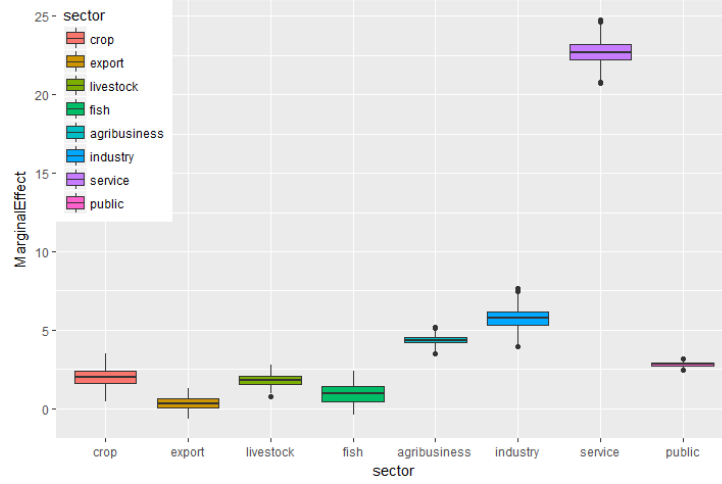


Figure 8: Marginal Effects of Technical Progress Shocks on Output Z1

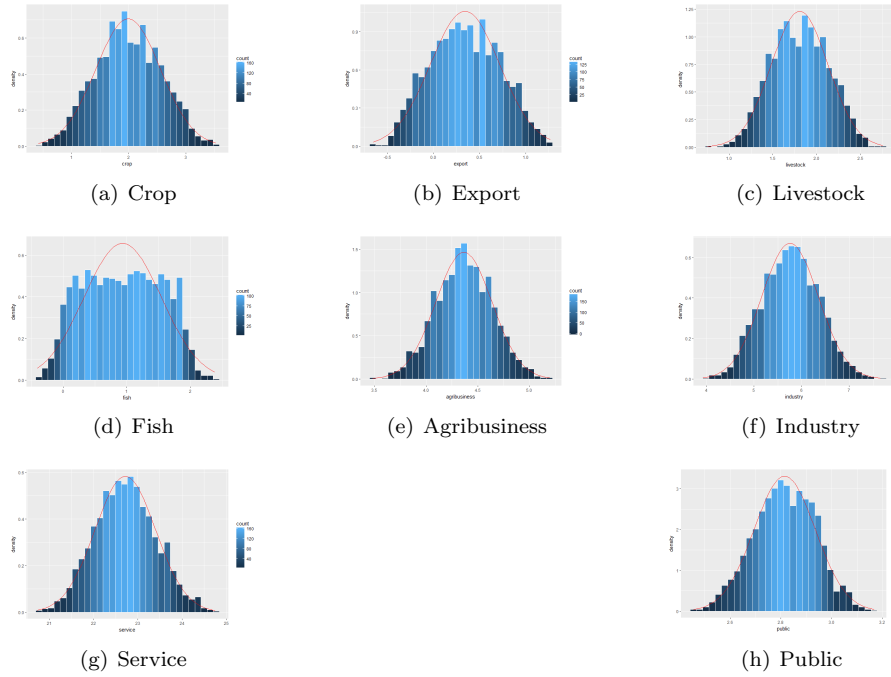


Figure 9: Histogram of Marginal Effects of Technical Progress Shocks on Output Z1

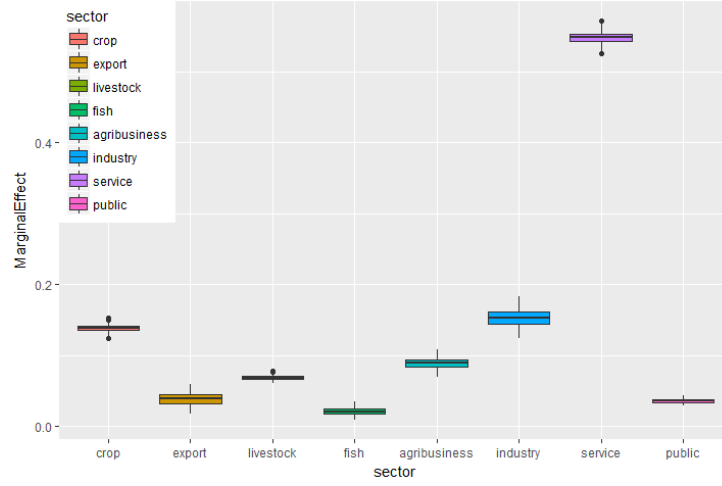


Figure 10: Marginal Effects of Technical Progress Shocks on Output Z2

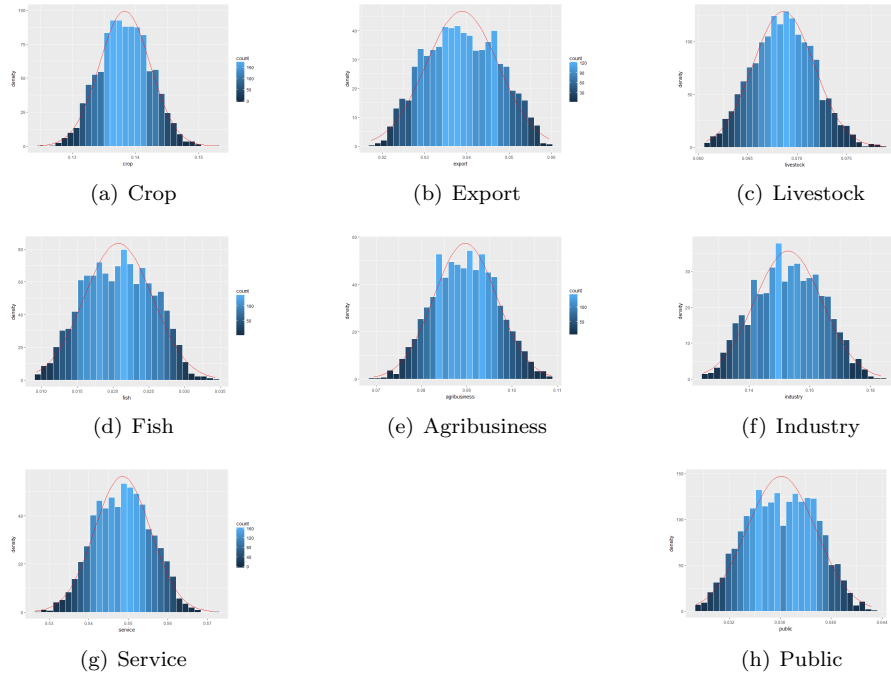


Figure 11: Histogram of Marginal Effects of Technical Progress Shocks on Output Z2

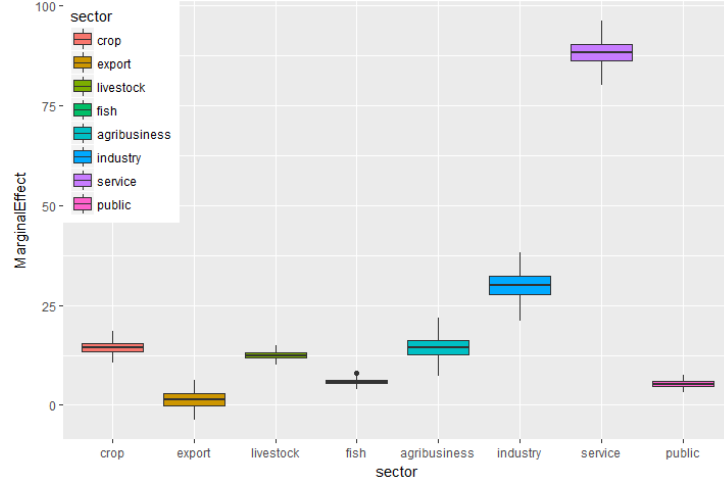


Figure 12: Marginal Effects of Technical Progress Shocks on Output Z5

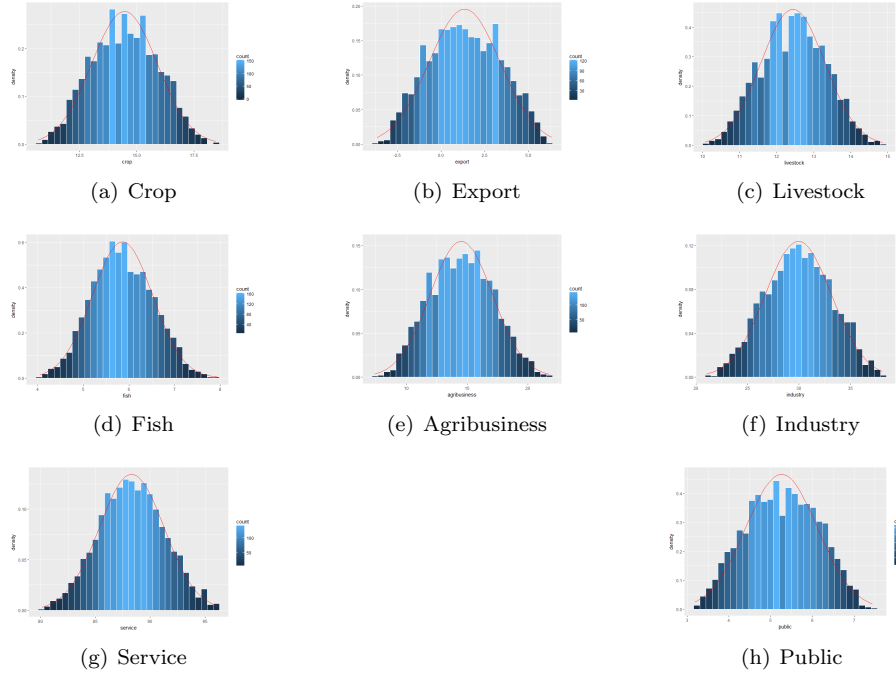


Figure 13: Histogram of Marginal Effects of Technical Progress Shocks on Output Z5

5 Conclusion

A highly discussed and criticized aspect of CGE modeling is the elasticity parameters, researchers usually extract them from literature because of either the complex and tedious estimation process or the impossibility of estimation and use them directly in the simulations. Moreover, due to the complexity of CGE models, it is difficult to combine them with other approaches such as Bayesian Model Selection or incorporate them into other models such as the decision-making model. Therefore, we apply the meta-modeling methodology to generate valid surrogates of the CGE models which are calibrated by various groups of elasticity parameters and test the impact of them on the outputs. In our demonstration, we have shown how to apply the meta-modeling technique. Although the results in our case present that the reduced-form meta-models do not have large impacts with respect to elasticity parameters, the valid surrogates of CGE models open the door to many possibilities. The application of meta-modeling allows us to tackle the problems from a totally new angle and gives us the opportunity to solve them.

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