Decomposing Welfare Effects of CGE models:
an Exact, Superlative, Path Independent, Second Order Approximation.

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Abstract:

This paper proposes a new decomposition of welfare effects simulated by CGE models. To date, welfare decompositions are based on first order approximations of the CGE specifications. These locally-based decompositions may have a poor explanatory power from an empirical standpoint or may be path dependent. Our approach overcomes these issues and is based on Taylor series approximations of CGE specifications. Then it is a generalization of current ones which still allows to attribute changes in welfare to sources corresponding to the alleviation, or exacerbation, of existing market imperfections and distortions. Our decomposition approach is also attractive because that i) it can be applied to any globally regular representation of preferences, ii) it can be implemented in both level and linearized CGE models and iii) it eases the comparisons of welfare effects across individuals. We implement our approach to a widely used CGE model and show empirically that it performs well in most cases.

Keywords: Welfare Decomposition, Computable General Equilibrium, Equivalent Variation, Second Best Theory

JEL Classifications: D58, F11, F14

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Introduction: Problem Statement and Literature Review

Welfare is very likely the favorite notion of economists ; accordingly it has always received considerable attention, both theoretically and empirically. A significant part of the economic literature deals with the issue of defining the good measure of economic welfare. Most frequently used indicators are real wages, real GDP, real income, consumer surplus as well as the Hicksian Compensating Variation (CV) and Equivalent Variation (EV) measures. All these indicators have both pros and cons ; then one must start choosing a particular one. This choice obviously depends on the problem at hand. In this paper, we adopt the last one (EV) as our main objective is to explain the welfare effects which are simulated using Computable General Equilibrium (CGE) models. Given the on-going creativity on this research area, our opinion is that it may be useful to defend this choice. In addition to be the most widely reported welfare indicator by CGE modelers, we choose EV because all other mentioned welfare measures either suffer from some pitfalls, either are not best-suited in the context of this paper.

In particular, changes in real wages are often used to measure welfare changes but, as demonstrated by Robinson and Thierfelder (1999) a problem arises when analyzing the impact of changes in taxes in the presence of many distortions/taxes. For instance, a policy reform with increases of indirect taxes and decreases of direct taxes may reduce real wages while being revenue neutral and without any effects on aggregate absorption and hence on agent well-being. Because policy reforms are always contemplated in a second-best world, thus we do not retain real wages.

In a similar vein, changes of real GDP are often reported in country comparisons but Kohli (2004) demonstrates that this welfare indicator may be in fact misleading as it underestimate the increase of real domestic income and welfare when one country experiences some terms of trade improvements. Real GDP focuses only on production possibilities and thus is unable to capture the beneficial effect for an economy of an improvement in its terms of trade (say a decrease of import prices). Because policy reforms may translate into terms of trade changes, we also discard the real GDP indicator.

Real income index number and consumer surplus are distinct but quite close concepts (Hicks, 1942) ; both will be disregard for the same reason in our paper. We focus the discussion on the latter one (for a very recent paper on real income index, see Neary, 2005). Originally proposed by Bennet (1920), the consumer surplus has been for a long time the most widely used measure, partly thanks to the strong support offered by Harberger (1971) in his open letter to the economic profession. Its main advantages come from its simplicity and relative sparse data requirements, namely only prices and quantities consumed in the two periods of comparison (be they observed or simulated). But it has been criticized by Chipman and Moore (1976) on the ground that it is a valid/exact measure of consumer well-being only for homothetic preferences. This is clearly a serious problem since now almost 150 years of empirical evidence demonstrates that demand patterns are inconsistent with homotheticity. Diewert (1976) also greatly reduces the relevance of the consumer surplus concept by demonstrating that it can be made positive or negative simply by scaling price in either period. Finally McKenzie and Pearce (1976) argue that consumer surplus is empirically unsatisfactory because it is only a second-order approximate measure of welfare. Subsequent works try to resurrect consumer surplus with slightly modified version of the original version but this proves again controversial. For instance, Willig (1976) shows that, in the case of a single price change, observable estimates of consumer's surplus can be used to provide a good approximation to the theoretically superior EV and CV measures but Hausman (1981) did find that Willig's approach is inaccurate in case of large income effects. Weitzman (1988) also tries to bring back to life the consumer surplus approach by offering a device (a "Paasche normalization" of price in period one) to the normalization issue raised by Diewert (1976) but it finally appears that this device requires the assumption of homothetic preferences (Diewert, 1992). More recently, Chambers (2001) shows that another normalized version of the Bennet consumer surplus measure is an exact and superlative cardinal welfare indicator if preferences are of translation-homothetic generalized quadratic form (the associated Engel curves are linear in expenditures without necessarily departing from the origin as with homothetic preferences). Without doubts, this represent a new support for the consumer surplus but the present proliferation of econometric estimations of rank three demand systems may attenuate this contribution.

To summarize, the main reason why the consumer surplus is criticized in the literature lies in the fact that it is only exact and superlative for some restricted structures of preferences. This is not our reason for rejecting this measure because many CGE evaluations are still performed with these restricted structures. Our concern is different and may be explained as follows. By and large, previous efforts on the consumer surplus try to define an accurate measure of welfare changes that only depends on the observed price and quantity vector for two periods. In other words, they want to avoid the arbitrary specification of household preferences/ demand functions / substitutability patterns between goods which are required for Hicksian welfare measures. However CGE models are built on these arbitrary specifications and accordingly the computation of Hicksian welfare measures can be made without additional costs (if we have only implicit representation of preferences, they can still be evaluated using Vartia's approach). Thus it is no longer useful to limit oneself to these indicators when theoretically superior ones are already available.

Finally, we are left with the different versions of Hicksian welfare measures (Martin, 1997) and must choose between i) the compensated versus uncompensated notion of surplus (also labeled direct or money metric) and ii) CV and EV. As we want our approach to be applicable to multi-country CGE models, we prefer to adopt the most common money metric versions. Finally, because we usually want to compare different policy reforms to a same benchmark, we concentrate on the EV measure. That is, we compare the impact of policy reforms using the same (initial) price vector.

Once one particular welfare measure has been adopted, the critical challenges are to analyze the welfare impacts of policy changes as well as to define optimal policies. These challenges may be much simplified if it would be possible to identify and quantify the sources of welfare and their respective contributions with respect to a given policy scenario. This is basically the purpose of a growing literature on the decomposition of welfare effects.[1] To date, several studies have proposed ways of attributing changes in welfare to sources corresponding to the alleviation, or exacerbation, of existing market imperfections and 'distortions'. In general, they extend the pioneer decomposition proposed by Harberger (1971) in a highly simplified economy. Recent contributions include: Coady and Harris (2004) who evaluate transfer programs in Mexico and decomposes welfare effects into three components (redistribution, reallocative and distortionary) ; Diewert and Woodland (2004) who show in particular how the introduction of new goods into the economy generate welfare gains ; Kohler (2004) who assesses impacts of Eastern enlargement of the EU and decomposes welfare effects into three

[1] It is worthy to note in parallel the growing literature on the decomposition of index numbers (see for instance Hallerbach, 2004).

components too (static gains from trade, dynamic effects following capital accumulation with growth externalities and employments effects in labor markets characterized by unemployment due to costly search) and Kreickmeier (2005) who derives the welfare effects of tariffs and import quotas in the presence of involuntary unemployment and highlights the crucial production elasticities.

As far as we are aware of, all these decompositions are based on first-order approximations of underlying CGE specifications. Without doubts, these decompositions are extremely useful in order to understand some economic mechanisms at work. However, these locally-based decompositions may have a rather poor explanatory power from a empirical standpoint in the case of “multiple and large shock” experiments, which is the traditional purpose of CGE models. In other words, linearized representation of multiple non-linear relations may be prone to erroneous conclusions (Hertel et al., 1992). In order to cope with this potential empirical issue, current practice is to split the original experiment into many smaller ones, so as to update welfare elasticities in the decomposition equation. While this seems a priori crafty, this solution leads to another problem: the welfare decomposition becomes path dependent (Fane et Ahammad, 2003). Appendix 1 presents this issue in a very simplified setting using a graphical analysis. The consequence is that the decomposition is not unique and thus may be of little value to explain the welfare impacts of any given policy experiment.

Our main objective in this paper is to propose a new approach for decomposing welfare effects of CGE models. The main idea is to develop Taylor series approximations to CGE specifications rather than relying on first order marginal conditions only. This idea was already present in Harberger (1971) but, as far as we are aware of, have never been exploited in empirical analysis. This way, we want to avoid the arbitrary sharing out of policy experiments (hence the qualifier path independent). As will be apparent below, the attractive feature of our approach is that it encompasses previous decompositions and still retains the suggestion of Harberger to express welfare in terms of distortions. It may also be applied to any representation of globally regular preferences (hence the qualifiers exact and superlative). From a practical point of view, it can be implemented in both level and linearized models. Finally comparisons of welfare across individuals are made easier if one makes use of multiple household CGE model.

On the other hand, we must acknowledge that our decomposition includes a residual term which captures Taylor series approximation terms of order greater than 2 (hence the qualifier second order). Depending on the degree of non-linearity, second-order Taylor series approximations may still be insufficient to provide an accurate decomposing of a particular function. For instance, McKenzie et Pearce show with a simple quadratic utility function that even a fifth order approximation to one given welfare measure is still an approximation. However they also show that the error, expressed in percentage of expenditure, significantly declines from the first (1.54%) to the second order approximation (0.08%). Consequently our approach remains an approximation that must be empirically appreciated in real situations. In this paper, we offer some numerical examples using a widely used CGE model.

The paper is organized as follows. The first section presents the basics of our Taylor based approach in a highly simplified CGE model with few distortions. We also contrast it with those currently used. In the second section, we derive a new welfare decomposition equation for a widely used CGE model. Finally we perform various policy reforms in order to illustrate the empirical benefits of the proposed decomposition.

1. Analytical framework.

CGE models are now extensively employed in order to evaluate the market and welfare impacts of policy reforms. Many theoretical improvements have been introduced in such models, including dynamic behavior, imperfect competition, risk aversion, … It is beyond the scope of this paper to provide a welfare decomposition that incorporates all these possible features. In this section, we prefer to adopt a quite standard CGE model in order to ease the comparison between our decomposition and previous ones. In fact, we adopt the synthetic model described in Fane and Ahammad (2003) where they compare different welfare indicators. We first briefly present the notations, then provide the usual first-order decomposition of the EV and finally detail our Taylor based approach.

1.1. Notations

We consider a static open economy model with only one representative consumer, I goods (indexed by i) and mono-product activities and F primary factors of production (indexed by f). Extension to multiple household CGE models is described in appendix 2. Producers are assumed to maximize their profit subject to constant return to scale production technologies. Likewise, the representative consumer maximizes utility subject to budget constraint. His income is given by the primary factor returns and the net product of specific taxes/subsidies on production, primary factor use, consumption and trade. Primary factors of production are perfectly mobile between activities and are in fixed supply. Perfect competition prevails in all markets. Finally we assume an uncompensated setting where transfer from/to abroad is fixed, that domestic and foreign goods are perfect substitutes and that our economy is potentially large in world markets. This is clearly a simplified CGE model, where we abstract from savings, investment and intermediate use in production technologies. Nevertheless it is a useful setting to derive our welfare decomposition and contrasts it to current ones. Mathematically, such a model is represented by the following eleven equations:

$$X_{i,f} = X_{f,i}\left(Y_i, W + tf\right) \tag{1}$$

$$P_i.Y_i = \sum_{f=1}^{F}\left(W_f + tf_{i,f}\right).X_{i,f} \tag{2}$$

$$C_i = C_i\left(Q, R\right) \tag{3}$$

$$R = E\left(Q, U\right) \tag{4}$$

$$PW_i = PW_i\left(M, z\right) \tag{5}$$

$$P_i = PW_i + tm_i - ty_i \tag{6}$$

$$Q_i = PW_i + tm_i + tc_i \tag{7}$$

$$\sum_i X_{i,f} = x_f \tag{8}$$

$$C_i = Y_i + M_i \tag{9}$$

$$R = \sum_{f=1}^{F} W_f.x_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.X_{i,f} + \sum_{i=1}^{I} ty_i.Y_i + tc_i.C_i + tm_i.M_i + b \tag{10}$$

$$b = \sum_{i=1}^{I} PW_i.M_i \tag{11}$$

with the following notations for the endogenous variables (always written with upper case letters):
$X_{i,f}$ the use of primary factor f by activity i, W_f the market price of primary factor f, P_i the producer price of good i, Y_i the domestic production of good i, C_i the domestic consumption of good i, Q_i the consumer price of good i, R the total income or expenditure, U the consumer utility, PW_i the world price of good i, M_i the trade volume of good i,
and the following notations for the exogenous variables (always written with lower case letters):
$tf_{i,f}$ the specific tax on primary factor f used by activity i, ty_i the specific tax on production i, tc_i the specific tax on consumption i, tm_i the specific tax on trade i, b the balance of trade deficit and x_f the fixed endowment of factor.

Equations 1 and 2 form the production block and include the primary factor derived demands and the zero profit condition. Equations 3 and 4 form the consumption block and include the final demands as well as the expenditure function, which implicitly define utility. Note that we still do not restrain the structure of preferences. The only restriction is to be globally regular. Equations 5 to 7 form the price block where all taxes on good are introduced. Equations 8 and 9 are the market equilibrium conditions. Finally equations 10 and 11 are macro-economic conditions expressing the economy budget constraint and the balance of payments. As expected, one equation in this model is redundant and must be skipped when solving the model. This has no incidence of results and is indeed an ex post powerful way to check the consistency of the CGE model. It may be finally noted that in general modelers also remove equation 4 when solving the model as it only serves to determine the ordinal utility.

1.2. "First-order" decomposition of welfare

First-order decomposition of welfare usually start from total differentiation of the income equation (10) around the initial point (for instance, Huff and Hertel, 2001):

$$\begin{aligned} dR = & \sum_{f=1}^{F} dW_f.x_f + W_f.dx_f + \sum_{f=1}^{F}\sum_{i=1}^{I} dtf_{i,f}.X_{i,f} + tf_{i,f}.dX_{i,f} \\ & + \sum_{i=1}^{I} dty_i.Y_i + ty_i.dY_i + dtc_i.C_i + tc_i.dC_i + dtm_i.M_i + tm_i.dM_i + db \end{aligned} \tag{12}$$

Subtracting $\sum_{i=1}^{I} C_i.dQ_i$ from both sides and arranging terms, we get:

$$\begin{aligned} dR - \sum_{i=1}^{I} C_i.dQ_i = & \; db + \sum_{f=1}^{F} W_f.dx_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.dX_{i,f} + \sum_{i=1}^{I} ty_i.dY_i + tc_i.dC_i + tm_i.dM_i \\ & + \sum_{f=1}^{F} dW_f.x_f + \sum_{f=1}^{F}\sum_{i=1}^{I} dtf_{i,f}.X_{i,f} + \sum_{i=1}^{I} dty_i.Y_i + (dtc_i - dQ_i).C_i + dtm_i.M_i \end{aligned} \tag{13}$$

The left hand side of this equation simply gives the change of real income (measured with initial consumption quantities). The first two elements of the right hand side represent exogenous flows of income to the economy (from the rest of the world or from an increase of primary factor endowment). The fourth following terms measure the alleviation, or exacerbation, of existing distortions. Other terms are much more difficult to interpret but can be much more simplified by proceeding in the following way. Total differentiation of the zero profit condition (equation 2) gives:

$$P_i.dY_i + dP_i.Y_i = \sum_{f=1}^{F} \left(W_f + tf_{i,f}\right).dX_{i,f} + \left(dW_f + dtf_{i,f}\right).X_{i,f} \tag{14}$$

Because production technologies are constant return to scale, we have from the Euler Theorem:

$$dP_i.Y_i = \sum_{f=1}^{F} \left(dW_f + dtf_{i,f}\right).X_{i,f} \tag{15}$$

If we sum this last identity over all production sectors and use the market clearing equation (8), then we get:

$$\sum_{i=1}^{I} dP_i.Y_i = \sum_{f=1}^{F} dW_f.x_f + \sum_{i=1}^{I}\sum_{f=1}^{F} dtf_{i,f}.X_{i,f} \tag{16}$$

We are now in a good position to simplify the terms on the right hand side of equation (13). If we substitute the right hand side of (16) for the left hand side in equation (13), next use first order differentiation of price equations (6) and (7) and make use of product market equilibrium equation (9), the equation 13 becomes:

$$\begin{aligned} dR - \sum_{i=1}^{I} C_i.dQ_i = db + \sum_{f=1}^{F} W_f.dx_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.dX_{i,f} + \sum_{i=1}^{I} ty_i.dY_i + tc_i.dC_i + tm_i.dM_i \\ - \sum_{i=1}^{I} dPW_i.M_i \end{aligned} \tag{17}$$

The last new term reduces to classical terms of trade effect which obviously have some impacts on real income. If the prices of import increase, then real income decreases as expected. This last term can be further decomposed into two terms using equation (5) in order to reflect changes due to the evolution of trade flows and changes due to the evolution of foreign market conditions (for instance, see Fane and Ahammad, 2003).

Finally, EV is related to the real income just decomposed in the following manner:

$$EV = E\left(P^0, U^1\right) - E\left(P^0, U^0\right) \tag{18}$$

where superscript 0 refers to the initial situation and superscript 1 to the final situation. At this stage, let's recall that the current practice is to split the whole experiment into smaller ones. Let's assume that there are K sub-experiments (indexed by k). For each sub-experiment, we have:

$$dEV^k = dE\left(P^0, U^k\right) = \frac{\partial E\left(P^0, U^k\right)}{\partial U} dU^k \tag{19}$$

Let's now totally differentiate equation 4 at the same sub-experiment:

$$dR^k = \frac{\partial E(P^k, U^k)}{\partial P} dP^k + \frac{\partial E(P^k, U^k)}{\partial U} dU^k = C^k . dP^k + \frac{\partial E(P^k, U^k)}{\partial U} dU^k \tag{20}$$

The second equality is satisfied only locally because the derivative of the expenditure function with respect to price gives the hicksian demand function and not the marshallian demand function. Combining equations 19 and 20 and summing over all sub-experiments, we finally obtain:

$$\begin{aligned} EV &= \sum_{k=1}^{K} \frac{\frac{\partial E(P^0, U^k)}{\partial U}}{\frac{\partial E(P^k, U^k)}{\partial U}} . \left(dR^k - C^k . dP^k\right) \\ &= \sum_{k=1}^{K} \frac{\frac{\partial E(P^0, U^k)}{\partial U}}{\frac{\partial E(P^k, U^k)}{\partial U}} . \left(\begin{array}{l} db^k + \sum_{f=1}^{F} W_f^k . dx_f^k + \sum_{f=1}^{F} \sum_{i=1}^{I} tf_{i,f}^k . dX_{i,f}^k \\ + \sum_{i=1}^{I} ty_i^k . dY_i^k + tc_i^k . dC_i^k + tm_i^k . dM_i^k - dPW_i^k . M_i^k \end{array} \right) \end{aligned} \tag{21}$$

As Fane and Ahammad (2003) indicate, the issue with the uncompensated version of EV is that the terms of the real income decomposition must be weighted by a term involving the inverse of marginal utility of income. Accordingly, uncompensated EV can not be decomposed simply in terms of changes in the levels of distorted activities, each multiplied by the excess of the marginal social benefit of the activity over its marginal cost. Our main issue here is that this decomposition is based on local approximation and thus may be empirically poor for experiments including large changes, unless many sub-experiments are contemplated. But the decomposition may not be unique, thus not really facilitating the interpretation of welfare effects.

1.3. Taylor based decomposition of welfare

Like Harberger (1971) and Weitzman (1988), we start our procedure by developing Taylor series approximations to the EV and to the direct utility function:

$$EV = E(P^0, U^1) - E(P^0, U^0) = \frac{\partial E(P^0, U^0)}{\partial U} . \Delta U + 0.5 . \frac{\partial^2 E(P^0, U^0)}{\partial U^2} . (\Delta U)^2 + O(\Delta U) \tag{22}$$

where $O(\Delta U)$ stands for all polynomial terms of third order or higher.

Even if preferences are such that it is impossible to express the direct utility function (like the implicit CDE demand system), we always have:

$$\Delta U = \sum_{i=1}^{I} \frac{\partial U}{\partial C_i}.\Delta C_i + 0.5.\sum_{i=1}^{I}\sum_{j=1}^{I} \frac{\partial^2 U}{\partial C_i \partial C_j}.\Delta C_i.\Delta C_j + O(\Delta C)$$
$$= \sum_{i=1}^{I} \frac{\partial U}{\partial C_i}.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta \frac{\partial U}{\partial C_i}.\Delta C_i + O(\Delta C) \quad (23)$$

From utility maximization, we have the following first order condition for all goods:

$$\frac{\partial U}{\partial C_i} = \lambda.Q_i \quad (24)$$

with λ the marginal utility of income. Total differentiation of the above first order condition gives:

$$\Delta \frac{\partial U}{\partial C_i} = \Delta\lambda.Q_i + \lambda.\Delta Q_i + \Delta\lambda.\Delta Q_i \quad (25)$$

Substituting (24) and (25) into (23) and next the resulting expression into (22), we get:

$$EV = \sum_{i=1}^{I} Q_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta Q_i.\Delta C_i$$
$$+ 0.5.\Delta\lambda.\left(\sum_{i=1}^{I} Q_i.\Delta C_i. + \sum_{i=1}^{I} \Delta Q_i.\Delta C_i\right) \quad (26)$$
$$+ 0.5.\frac{\partial^2 E(P^0, U^0)}{\partial U^2}.(\Delta U)^2 + O(\Delta U, \Delta C)$$

Harberger (1971) suggests to use only the first two terms for measuring welfare (which is an second order approximation to the consumer surplus) and that the others may be neglected. But equation (26) makes clear that even with homothetic preferences (such as the last two terms of the last line vanish), this does not strictly correspond to EV. It remains terms involving changes in the marginal utility of income. These terms generalize to N commodities the difference between EV and the consumer surplus derived by Boadway and Bruce (1984, p. 218) in a single commodity context.

We now concentrate on the first two terms. Using total differentiation of equations 6, 7 and 9, we can express them as follows:

$$\sum_{i=1}^{I} Q_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta Q_i.\Delta C_i = \sum_{i=1}^{I} tc_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta tc_i.\Delta C_i$$
$$+ \sum_{i=1}^{I} ty_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta ty_i.\Delta Y_i$$
$$+ \sum_{i=1}^{I} tm_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta tm_i.\Delta M_i \qquad (27)$$
$$+ \sum_{i=1}^{I} P_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta P_i.\Delta Y_i$$
$$+ \sum_{i=1}^{I} PW_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i$$

On can already remark that the first three lines of the right hand side represent changes in one flow multiplied by the corresponding average tax. The purpose of subsequent derivations is to show that the terms of the last two lines can also be expressed in this form. In that respect, we first totally differentiate equation 11, rearrange terms to get:

$$\sum_{i=1}^{I} PW_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i = \Delta b - \sum_{i=1}^{I} \Delta PW_i.M_i - 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i \qquad (28)$$

Finally, in order to simplify the terms involving changes in producer prices and domestic productions (penultimate line of equation 27), we again use a Taylor series approximation to the primal production technologies, in the same manner as we did for preferences:

$$\begin{aligned} \Delta Y_i &= \sum_{f=1}^{F} \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F}\sum_{g=1}^{F} \frac{\partial^2 Y_i}{\partial X_{i,f}\partial X_{i,g}}.\Delta X_{i,f}.\Delta X_{i,g} + O(\Delta X) \\ &= \sum_{f=1}^{F} \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F} \Delta \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + O(\Delta X) \end{aligned} \qquad (29)$$

The first-order conditions of profit maximization are:

$$\frac{\partial Y_i}{\partial X_{i,f}} = \frac{W_f + tf_{i,f}}{P_i} \qquad (30)$$

Total differentiating these first order conditions gives:

$$\Delta \frac{\partial Y_i}{\partial X_{i,f}} = \frac{\Delta W_f + \Delta tf_{i,f}}{P_i} + \left(W_f + tf_{i,f}\right).\Delta\left(\frac{1}{P_i}\right) + \left(\Delta W_f + \Delta tf_{i,f}\right).\Delta\left(\frac{1}{P_i}\right) \qquad (31)$$

Introducing 31 into 29 and making appropriate arrangements, we obtain:

$$P_i.\Delta Y_i.\frac{1}{1 - 0.5.\frac{\Delta P_i}{P_i}} = \sum_{f=1}^{F} \left(W_f + tf_{i,f}\right).\Delta X_{i,f} + 0.5.\left(\Delta W_f + \Delta tf_{i,f}\right).\Delta X_{i,f} + 0(\Delta X) \qquad (32)$$

The left hand side of this last equation can also be expressed using another Taylor approximation:

$$P_i.\Delta Y_i.\frac{1}{1-0.5.\frac{\Delta P_i}{P_i}} = P_i.\Delta Y_i. + 0.5.\Delta P_i.\Delta Y_i. + 0.25.\frac{\Delta P_i.\Delta P_i.\Delta Y_i}{P_i} + 0(\Delta P) \tag{33}$$

Combining (32) and (33) and summing over all production activities, we get:

$$\begin{aligned}\sum_{i=1}^{I} P_i.\Delta Y_i. + 0.5.\Delta P_i.\Delta Y_i = \sum_{f=1}^{F}\left(W_f + tf_{i,f}\right).\Delta X_{i,f} + 0.5.\left(\Delta W_f + \Delta tf_{i,f}\right).\Delta X_{i,f} \\ - 0.25.\sum_{i=1}^{I}\frac{\Delta P_i.\Delta P_i.\Delta Y_i}{.P_i} + 0(\Delta P, \Delta X)\end{aligned} \tag{34}$$

Finally we get our EV decomposition by plugging 27, 28 and 34 in 26 and rearranging terms for facilitating the discussion:

$$\begin{aligned} EV = {} & \Delta b \\ & + \sum_{f=1}^{F} W_f.\Delta x_f + 0.5.\sum_{f=1}^{F}\Delta W_f.\Delta x_f \\ & + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F}\sum_{i=1}^{I}\Delta tf_{i,f}.\Delta X_{i,f} \\ & + \sum_{i=1}^{I} ty_i.\Delta Y_i + 0.5.\sum_{i=1}^{I}\Delta ty_i.\Delta Y_i \\ & + \sum_{i=1}^{I} tc_i.\Delta C_i + 0.5.\sum_{i=1}^{I}\Delta tc_i.\Delta C_i \\ & + \sum_{i=1}^{I} tm_i.\Delta M_i + 0.5.\sum_{i=1}^{I}\Delta tm_i.\Delta M_i \\ & - \sum_{i=1}^{I} M_i.\Delta PW_i - 0.5.\sum_{i=1}^{I}\Delta M_i.\Delta PW_i \\ & + 0.5.\Delta\lambda.\left(\sum_{i=1}^{I} Q_i.\Delta C_i. + \sum_{i=1}^{I}\Delta Q_i.\Delta C_i\right) \\ & + 0.5.\frac{\partial^2 E\left(P^0, U^0\right)}{\partial U^2}.\left(\Delta U\right)^2 - 0.25.\sum_{i=1}^{I}\frac{\Delta P_i.\Delta P_i.\Delta Y_i}{P_i} \\ & + O\left(\Delta U, \Delta C, \Delta P, \Delta X\right)\end{aligned} \tag{35}$$

This expression must be compared to expression 21 which gives the EV decomposition in the usual first-order approximation. The first thing to note is that expression 35 was obtained using all CGE equations with the exception of the income equation (equation 10) while expression 21 makes use of all CGE equations, with the exception of the balance of payments condition (equation 11). As stated previously, this has no incidence of the resolution of the CGE model and hence the computation of welfare. The second thing to note is that the proposed decomposition is path independent, so that the explanation of welfare impacts of a given policy reform package is unique. The third thing to note is that the evolution of the marginal utility of income enters as an additional term rather than a multiplicative term. This make things more understandable: we can divide the decomposition of EV between a Marshallian part (corresponding to CS and observables) and a Hicksian one (not directly observable, only with modeling). The fourth thing to note is that the "Marshallian" parts are very similar with all distortions evaluated at the average rather than the initial level. Graphically, this can be seen as computing triangles rather than rectangles. Obviously triangles may not be very good approximations if supply and demand functions are highly non-linear. That's the reason why third order terms appear at the bottom of expression (35). Harberger (1971) simply neglects all terms of order higher than three (that is, the last three terms in 35). This is an empirical issue to which we turn now in some "real" policy experiments.

2. Empirical illustrations

2.1. Empirical framework

In order to assess the benefits of our decomposition, we perform some illustrative simulations using one simple version of the well known GTAP CGE model (Hertel, 1997). This version is described in Rutherford and Paltsev (2000) and is static, perfectly competitive and with constant return to scale production technologies. This CGE model differs from the stylized model described in the preceding section on the following aspects (we simultaneously present the new notations).

This is a multi-country model (regions indexed by $r = 1,...,R$) where bilateral trade is modeled with a nested Armington structure using CES functions. It thus distinguishes imports and exports (denoted by $E_{i,r,s}$ for export of good from region r to s). We assume that domestic production and export are perfect substitute. At the import side, we assume that domestic consumers first combine, according to a CES function, imports from the different zones into an aggregate bundle (denoted by $MT_{i,r}$ with price $PM_{i,r}$). The latter is then combined, according to a new CES function, to domestic production to determine domestic demand.

In addition to the final demand derived from the maximization of a Cobb Douglas utility function, the domestic demand includes intermediate use by activities (denoted by $ID_{i,j,r}$ for the demand of good i by the activity j in region r), public demand (denoted by $GD_{i,r}$) and investment demand (denoted by $I_{i,r}$). Investment by commodity is assumed to be fixed in this static version. Public demands are derived from a Cobb Douglas public utility function and the macro-economic closure are such that public utility is fixed. Intermediate demands are proportional to the level of activity and a Cobb-Douglas production function relates activity level and primary factor inputs.

In this model, all distortions are represented by ad valorem taxes. They include all taxes described above and taxes on intermediate demands (denoted by $ti_{i,j,r}$), on exports (denoted by $te_{i,r,s}$) and on public consumption (denoted by $tg_{i,r}$). Finally the model includes an international transport sector responsible for trade flows between regions (denoted by $T_{i,r,s}$).

International transport services are assumed to be proportional to trade and are a Cobb Douglas aggregate of national transport services. The price of imports is thus given by the net price from the exporting country plus the associate transport cost to the importing country.

In this model, first order decomposition of EV for a given region is given by:

$$EV_r = \sum_{k=1}^{K} \frac{\frac{\partial E_r\left(P_r^0, U_r^k\right)}{\partial U}}{\frac{\partial E_r\left(P_r^k, U_r^k\right)}{\partial U}} . \left(\begin{array}{l} db_r^k + \sum_{f=1}^{F} W_{f,r}^k .dx_{f,r}^k + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f,r}^k .W_{f,r}^k .dX_{i,f,r}^k \\ + \sum_{i=1}^{I} ty_{i,r}^k .P_{i,r}^k .dY_{i,r}^k + tc_{i,r}^k .Q_{i,r}^k .dC_{i,r}^k \\ + \sum_{i=1}^{I}\sum_{s=1}^{R} tm_{i,r,s}^k .PW_{i,r,s}^k .dM_{i,r,s}^k \\ + \sum_{i=1}^{I}\sum_{j=1}^{J} ti_{i,j,r}^k .Q_{i,r}^k .dID_{i,j,r}^k \\ + \sum_{i=1}^{I}\sum_{s=1}^{R} te_{i,r,s}^k .P_{i,r}^k .dE_{i,r,s}^k \\ + \sum_{i=1}^{I} tg_{i,r}^k .Q_{i,r}^k .dGD_{i,r}^k \\ + \sum_{i=1}^{I}\sum_{s=1}^{R} .E_{i,r,s}^k .d\left(\left(1+te_{i,r,s}^k\right).P_{i,r}^k\right) - M_{i,r,s}^k .dPW_{i,r,s}^k \\ + \sum_{i=1}^{I}\sum_{s=1}^{R} T_{i,r,s}^k .dP_{i,r}^k \end{array} \right) \tag{36}$$

Few comments of this expression are in order. Terms in the three first lines are very similar to those previously identified. We only take into account for *ad valorem* taxes rather specific taxes. Subsequent terms represent the new distortions on intermediate demands, exports and public demands. Terms of trade effects must be now decomposed between price of exports and price of imports. Finally the last term is related to the selling of transport services to the international trade transport.

The Taylor decomposition of the same EV is given by:

$$
\begin{aligned}
EV_r = {} & \Delta b_r + \sum_{f=1}^{F} W_{f,r}.\Delta x_{f,r} + 0.5.\Delta W_{f,r}.\Delta x_{f,r} \\
& + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f,r}.W_{f,r}.\Delta X_{i,f,r} + 0.5.\Delta\left(W_{f,r}.tf_{i,f,r}\right).\Delta X_{i,f,r} \\
& + \sum_{i=1}^{I} ty_{i,r}.P_{i,r}.\Delta Y_{i,r} + 0.5.\Delta\left(ty_{i,r}.P_{i,r}\right).\Delta Y_{i,r} \\
& + \sum_{i=1}^{I} tc_{i,r}.Q_{i,r}.\Delta C_{i,r} + 0.5.\Delta\left(tc_{i,r}.Q_{i,r}\right).\Delta C_{i,r} \\
& + \sum_{i=1}^{I} tg_{i,r}.Q_{i,r}.\Delta GD_{i,r} + 0.5.\Delta\left(tg_{i,r}.Q_{i,r}\right).\Delta GD_{i,r} \\
& + \sum_{i=1}^{I}\sum_{s=1}^{R} tm_{i,r,s}.PW_{i,r,s}.\Delta M_{i,r,s} + 0.5.\Delta\left(tm_{i,r,s}.PW_{i,r,s}\right).\Delta M_{i,r,s} \\
& + \sum_{i=1}^{I}\sum_{s=1}^{R} te_{i,r,s}.P_{i,r}.\Delta E_{i,r,s} + 0.5..\Delta\left(te_{i,r,s}.P_{i,r}\right).\Delta E_{i,r,s} \\
& - \sum_{i=1}^{I}\sum_{s=1}^{R} M_{i,r,s}.\Delta PW_{i,r,s} + 0.5.\Delta M_{i,r,s}.\Delta PW_{i,r,s} \\
& + \sum_{i=1}^{I}\sum_{s=1}^{R} E_{i,r,s}.\Delta\left(\left(1+te_{i,r,s}\right).P_{i,r}\right) + 0.5.\Delta E_{i,r,s}.\Delta\left(\left(1+te_{i,r,s}\right).P_{i,r}\right) \\
& + \sum_{i=1}^{I}\sum_{s=1}^{R} T_{i,r,s}.\Delta P_{i,r} + 0.5.\Delta T_{i,r,s}.\Delta P_{i,r} \\
& + 0.5.\Delta\lambda_r.\left(\sum_{i=1}^{I} Q_{i,r}.\Delta C_{i,r}. + \sum_{i=1}^{I}\Delta Q_{i,r}.\Delta C_{i,r}\right) + 0.5.\frac{\partial^2 E\left(P_r^0, U_r^0\right)}{\partial U^2}.\left(\Delta U_r\right)^2 \\
& - 0.25.\left(\sum_{i=1}^{I}\frac{\Delta P_{i,r}.\Delta P_{i,r}.\Delta Y_{i,r}}{P_{i,r}} + \sum_{i=1}^{I}\frac{\Delta Q_{i,r}.\Delta Q_{i,r}.\Delta C_{i,r}}{Q_{i,r}} + \sum_{i=1}^{I}\frac{\Delta PM_{i,r}.\Delta PM_{i,r}.\Delta MT_{i,r}}{PM_{i,r}}\right) \\
& + O\left(\Delta U, \Delta C, \Delta P, \Delta Y, \Delta X, \Delta PM, \Delta MT\right)
\end{aligned}
\tag{37}
$$

Again this expression is very similar to 35. We only remark that in the penultimate line we have now three terms which correspond to three distinct functions and behavior: profit maximization, cost minimization of total consumption and finally cost minimization of total imports. These do not correspond to distortions. Like Harberger, we neglect them in the following numerical examples ; we also neglect them as well as the polynomial functions of order strictly higher than two (the last line of expression 37).

2.2. Simulations

In order to implement the CGE model, we use the GTAP 4.0 database which captures economic flow of the year 1995. Like Huff and Hertel (2001), we use a crude sectoral and regional aggregation. It features three produced sectors: food, manufactures and services, and three regions: the United States (USA), the European Union (EU) and the Rest of the World (ROW). We perform three experiments which focuses on trade instruments. In the first one, we decrease trade taxes (both import and export) by only 0.1% in all regions. This first simulation intends to show similarities between the two decompositions. The second experiment is more "policy-minded" and assume a 50% decrease of these taxes. Finally the last experiment is for testing purpose (the quality of approximations) where we assume full removal of all trade taxes. In the last two non marginal experiments, we test different paths when implementing the first order decomposition.

Welfare results of the first experiment are reported in Table 1. As expected, there are few differences between the two approaches This illustrates that our proposed approach encompasses current ones. On can still remark small deviations in the case of first order decomposition. For instance, EU welfare is 29.070 million US dollars and the first order decomposition gives a value of 29.079 while the Taylor based decomposition fits perfectly.

Results of the second experiment are reported in Table 2. The differences between the two approaches are much more pronounced. Let’s focus initially on the decomposition when the policy reform is fully implemented in one time. The first order approach does not accurately measure the welfare impacts. Above all, the first order decomposition overstates the welfare impacts by 2053 millions US dollars in case of the EU, which is equivalent to a 16.2% error. Corresponding figures for the USA and ROW are respectively 523 (9.5%) and 8822 (59.1%). Our Taylor based decomposition performs much better in this case. It first provides the correct measure of welfare (which is eased by the Cobb Douglas specification). Moreover the decomposition is quite close. The errors are now limited to 32 millions US dollars (0.2%) in case of the EU, to 118 millions US dollars (2.1%) in case of the USA and finally to 367 millions US dollars (2.4%) in case of the ROW. For the EU, the error is divided by a factor 64 ! Without any surprise, the contributions of each distortion to the global welfare effects are overstated with the first order decomposition. For instance, the alleviation of tariffs distortions in the EU amounts to 4128 millions US dollar according to this decomposition while it amounts to 3102 according to the Taylor decomposition (difference of 33%).

Let’s turn now to the case where this policy reform is implemented in two steps. There are obviously many ways to define it and we explore here three possibilities. The first path assumes that in the first step export taxes are reduced by 50% while tariffs are unchanged. The second path is symmetric to the first one by assuming only tariff reductions in the first step. Finally the third path assumes a 25% reduction of trade taxes in the first step. For these three paths, the second step is rationally defined in order to complete the simulation. As expected, decompositions are sensitive to the path choice. For instance, the alleviation of export taxes distortions in the EU takes the value of 2195, 2880 and 1896 millions US dollars. There is as much as 50% variation between the higher and the lower. Aggregate figures reported in Table 2 tends to suggest that the third linear path performs better than the two others (providing that the Taylor decomposition is a good candidate for comparison). However, when detailing further the decompositions, it appears that this is often but not always the case. For instance, we report in Table 3 the contributions of tariff distortions for each commodity. In two of nine cases (Manufactures in the USA and Services in the ROW), the third linear path is outperformed.

Results of the third large experiment are reported in table 4. They mainly confirm previous results and hence are not repeated here. We just underline that our decomposition performs still reasonably well for the EU and the USA. On the other hand, the approximation is poorer for the ROW. In order to discover the main reasons, we implement our decomposition step by step and find that the main issue lies in the second order approximation of the Armington-type import demand functions (derived from CES utility specifications). This is not surprising once we give a look to market effects and observe dramatic changes in trade flows (as much as 160% increase of imports of food from the USA to the RoW). As stated previously, our

Taylor based decomposition still remains an approximation and it may not fit perfectly in some rather extreme cases. When such cases occur, it is still attractive to reveal the places where more details are needed.

3. Concluding remarks

CGE models are widely used in order to measure the welfare effects of policy scenarios and thus to identify the winners and losers in each case. Recently CGE modelers quite systematically report some decompositions of welfare effects in order to understand why such or such effect appears. To date, these welfare decompositions are based on first order approximations of the CGE specifications. These locally-based decompositions may have a poor explanatory power from an empirical standpoint when one contemplates large policy shocks. The current device to circumvent this issue is to split the original shock into many smaller ones, where first order approximations are more adequate. Unfortunately this strategy raises another problem, mainly the path dependency of the welfare decomposition.

In that context, this paper proposes a new decomposition of welfare effects simulated by CGE models. Our approach overcomes previous issues and is based on Taylor series approximations of CGE specifications. Then it is a generalization of current ones which still allows to attribute changes in welfare to sources corresponding to the alleviation, or exacerbation, of existing market imperfections and distortions. Our decomposition approach is also attractive because that i) it can be applied to any globally regular representation of preferences, ii) it can be implemented in both level and linearized CGE models and iii) it eases the comparisons of welfare effects across individuals.

We implement our approach to the widely used GTAP CGE model. We simulate many experiments and find that our approach represents a substantial empirical improvement compared to standard practices. Moreover it performs well in most cases in the sense that the decompositions are quite close to the true welfare effects.

As usual, many works remain to do before providing definitive statements on the proposed approach. We suggest here two main directions. The first one, quite obvious, is to test the proposed approach on many policy scenarios simulated with “standard” CGE model. The second, much challenging, is to extend it to CGE models with more complex sources of market imperfections (for instance with imperfect competition, dynamic considerations, …) and more complex specifications (for instance with more flexible preference structures, production technologies, ….).

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Table 1. Welfare effects and decompositions of a 0.1% reduction of tariffs and export taxes (millions US dollars)

	EU	USA	ROW
True welfare (18)	29.070	8.783	40.953
First order decomposition			
One shock			
EV (first part of 21)	29.070	8.783	40.952
Total decomposition	29.079	8.785	40.980
Tariffs contribution	7.385	2.069	57.115
Export taxes contribution	5.045	2.041	0.890
Taylor series decomposition			
EV (26)	29.070	8.783	40.953
Total decomposition	29.070	8.783	40.952
Tariffs contribution	7.381	2.068	57.086
Export taxes contribution	5.042	2.040	0.890

Numbers in parentheses correspond to the defining equations in the first section.

Table 2. Welfare effects and decompositions of a 50% reduction of tariffs and export taxes (millions US dollars)

	EU	USA	ROW
True welfare	12669	5485	15125
First order decomposition			
One shock			
EV	12647	5448	14915
Total decomposition	14700	5971	23737
Tariffs contribution	4128	1149	33381
Export taxes contribution	2142	943	503
Path 1 reduces export taxes first			
EV	12612	5484	14844
Total decomposition	15425	6091	23670
Tariffs contribution	4120	1160	33618
Export taxes contribution	2880	1010	313
Path 2 reduces tariffs first			
EV	12688	5476	14840
Total decomposition	15875	6074	23413
Tariffs contribution	4471	1042	34081
Export taxes contribution	2195	963	503
Path 3 reduces trade taxes by 25%			
EV	12657	5484	15020
Total decomposition	13677	5747	19332
Tariffs contribution	3588	1003	28889
Export taxes contribution	1896	838	435
Taylor series decomposition			
EV	12669	5485	15125
Total decomposition	12701	5603	15492
Tariffs contribution	3102	874	25036
Export taxes contribution	1607	721	377

Table 3. Decomposition of the contribution of tariff distortions by commodities for a 50% reduction of tariffs and export taxes (millions US dollars)

	EU	USA	ROW
First order decomposition			
One shock			
Food	2782	321	16976
Manufactures	1344	828	16307
Services	1		98
Total	4128	1149	33381
Path 1 reduces export taxes first			
Food	2788	331	17274
Manufactures	1331	830	16246
Services	1		98
Total	4120	1160	33618
Path 2 reduces tariffs first			
Food	3129	367	18291
Manufactures	1341	676	15724
Services	1		65
Total	4120	1043	34081
Path 3 reduces trade taxes by 25%			
Food	2414	280	14603
Manufactures	1172	724	14200
Services	1		85
Total	3588	1003	28889
Taylor series decomposition			
Food	2093	245	12737
Manufactures	1008	**629**	12226
Services	1		**73**
Total	3102	874	25036

Table 4. Welfare effects and decompositions of a full removal of tariffs and export taxes (millions US dollars

	EU	USA	ROW
True welfare	21776	15586	10899
First order decomposition			
One shock			
EV	21668	15400	9862
Total decomposition	29978	16926	55867
Tariffs contribution	9317	2646	80151
Export taxes contribution	3728	1541	1192
Path 1 removes export taxes first			
EV	21540	15616	9557
Total decomposition	32140	18017	55260
Tariffs contribution	9297	2700	81051
Export taxes contribution	6004	2271	288
Path 2 removes tariffs first			
EV	21843	15557	9525
Total decomposition	35242	17871	53155
Tariffs contribution	10935	2209	82365
Export taxes contribution	3964	1652	1193
Path 3 reduces both by 50%			
EV	21726	15630	10367
Total decomposition	25811	16377	32419
Tariffs contribution	6741	1922	56883
Export taxes contribution	2937	1262	848
Taylor series decomposition			
EV	21776	15588	10901
Total decomposition	22042	16220	14969
Tariffs contribution	4676	1371	40075
Export taxes contribution	1871	799	596

Appendix 1.
The path dependency of welfare decomposition: A graphical presentation

This appendix shows in a highly simplified context the path dependency issue of welfare decompositions that are based on multi-step simulations. We assume one market (primary factor or good) whose supply is fixed and two demands/consumers (say 1 and 2). Let's consider that initially there are no distortions on this market. We would like to understand the welfare effects of a policy reform which introduces differentiated taxes on the two demands. Market and welfare impacts are fully reported in the table A.1. In this context, it is obvious that the final allocation may be obtained with only one tax (given by the absolute difference between the two taxes) but we maintain the introduction of both in order to illustrate the path dependency issue.

In order to explain the welfare loss from these taxes, one can examine first the impacts of the tax on demand 1 and next the additional impacts of the tax on demand 2. In the table below, we label it path 1. One can also examine the welfare impacts of the reform by looking first at the impacts of the tax on demand 2 and next the additional impacts of the tax on demand 1 (path 2). In our graphical illustration, it appears that tax 2 appears to be welfare improving with path 1 (by area fhjb') and is welfare decreasing with path 2 (by area obb"). Tax one is always welfare decreasing but the magnitude varies with the path. Thus the two "extreme" path decompositions lead to very different explanations and thus do not help much to understand the welfare impacts of the policy reform.

The decomposition we propose in this paper is as follows. Tax 1 is welfare decreasing by half of the area b"hbs while tax 2 is welfare increasing by half the area b"jbt. Total welfare is still decreasing by the area hbj.

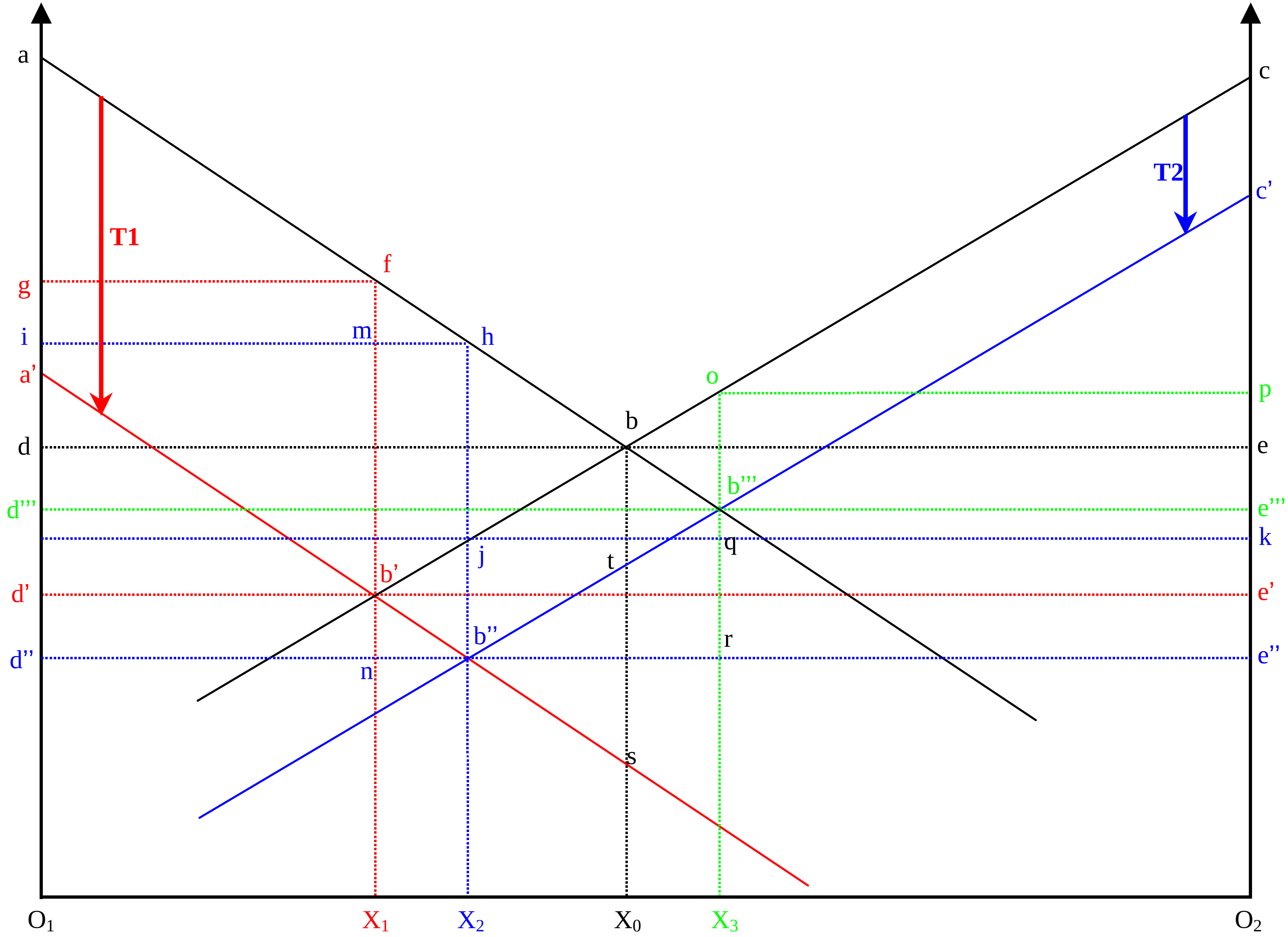
Table A.1. Market and Welfare impacts of the policy reform.

a. Decomposition according to path 1.

	Initial (A)	With T1 (B)	T1 impacts (B)-(A)	With T2 (C)	T2 impacts (C)-(B)	T1,T2 impacts (C)-(A)
<u>Markets</u>						
Demand 1	01X0	01X1	-X0X1	01X2	X1X2	-X0X2
Demand 2	02X0	02X1	X0X1	02X2	-X1X2	X0X2
Price	01d	01d'	-dd'	01d"	-d'd"	-dd"
<u>Welfare of</u>						
Consumer 1	abd	afg	-gfbd	ahi	gfhi	-ihbd
Consumer 2	cbe	cb'e'	bee'b'	cjk	-b'jke'	bekj
"Producer"	O1de02	01d'e'O2	-dee'd'	01e"d"O2	-d'e'e"d"	-dee"d"
Budget	0	gfb'd'	gfb'd'	ihb"d"+ jke"b"	mhb"n+ b"c"e'j	ihb"d"+ jke"d"
Total	01abc02	01abc02 -fbb'	-fbb'	O1abc02 -hbj	fhjb'	-hbj

b. Decomposition according to path 2.

	Initial (A)	With T2 (B)	T2 impacts (B)-(A)	With T1 (C)	T1 impacts (C)-(B)	T1,T2 impacts (C)-(A)
<u>Markets</u>						
Demand 1	01X0	O1X3	X0X3	01X2	-X2X3	-X0X2
Demand 2	02X0	02X3	-X0X3	02X2	X2X3	X0X2
Price	01d	01d’”	-dd’”	01d”	-d”d’”	-dd”
<u>Welfare of</u>						
Consumer 1	abd	ab’”d’”	dbb’”d’”	ahi	-ihb’”d’”	-ihbd
Consumer 2	cbe	cop	-obep	cjk	jopk	bekj
“Producer”	O1de02	d’”e’”0201	-dee’”d’”	01e”d”O2	-d’”e’”e”d”	-dee”d”
Budget	0	ope’”b’”	ope’”b’”	ihb“d“+ jke“b“	jqrb“+ b“hid“	ihb”d”+ jke”d”
Total	01abc02	01abcO2 -obb’”	-obb”	O1abc02 -hbj	-hbf+obb”	-hbj



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n
s
O1
X1
X2
X0
X3
O2

Appendix 2:
Welfare decomposition in a multiple-household CGE model

The analytical framework developed in the main text considers only one representative consumer for the economy. However applications of multiple household CGE models are mounting ; accordingly we briefly describe in this appendix the welfare decompositions in this context.

Incorporating many households in a CGE model is a great challenge, mainly in terms of data collection (both economic flows and elasticities). On the other hand, behavioral specifications may remain quite simple. In this appendix, we assume that there are H households (indexed by h) who consume goods and who own some primary factors. For simplicity, we assume that they all face the same price of goods and factors. In fact households differ with respect to factor endowments and preferences. In our stylized CGE model, they are introduced by modifying equations (3), (4), (8), (9) and (10) as follows:

$$C_{i,h} = C_{i,h}(Q, R_h) \tag{3'}$$

$$R_h = E_h(Q, U_h) \tag{4'}$$

$$\sum_i X_{i,f} = x_f = \sum_{h=1}^{H} x_{f,h} \tag{8'}$$

$$C_i = \sum_{h=1}^{H} C_{i,h} = Y_i + M_i \tag{9'}$$

$$R_h = \sum_{f=1}^{F} W_f . x_{f,h} + \theta_h . \left(\sum_{f=1}^{F} \sum_{i=1}^{I} tf_{i,f} . X_{i,f} + \sum_{i=1}^{I} ty_i . Y_i + tc_i . C_i + tm_i . M_i + b \right) \tag{10'}$$

where θ_h is the share of the product of net taxes that the household h receives (pays) and we obviously have $\sum_{h=1}^{H} \theta_h = 1$. With this framework, welfare decomposition equation equivalent to 21 is given by:

$$\begin{aligned} EV_h = & \sum_{k=1}^{K} \frac{\dfrac{\partial E_h(P^0, U_h^k)}{\partial U_h}}{\dfrac{\partial E_h(P^k, U_h^k)}{\partial U_h}} . \left(\begin{aligned} & db^k + \sum_{f=1}^{F} W_f^k . dx_f^k + \sum_{f=1}^{F} \sum_{i=1}^{I} tf_{i,f}^k . dX_{i,f}^k \\ & + \sum_{i=1}^{I} ty_i^k . dY_i^k + tc_i^k . dC_i^k + tm_i^k . dM_i^k - dPW_i^k . M_i^k \end{aligned} \right) \\ & - \sum_{l \neq h} \sum_{k=1}^{K} \frac{\dfrac{\partial E_h(P^0, U_h^k)}{\partial U_h}}{\dfrac{\partial E_h(P^k, U_h^k)}{\partial U_h}} . \frac{\dfrac{\partial E_l(P^k, U_l^k)}{\partial U_l}}{\dfrac{\partial E_l(P^k, U_l^k)}{\partial U_l}} . EV_l \end{aligned} \tag{21'}$$

Hence the welfare of a given household is still explained by the alleviation/exacerbation of current distortions minus the welfare of other households weighted by the relative marginal utility of income at each step in the decomposition.

Equation equivalent to 35 is the following one:

$$
\begin{aligned}
EV_h &= \Delta b \\
&+ \sum_{f=1}^{F} W_f.\Delta x_f + 0.5.\sum_{f=1}^{F} \Delta W_f.\Delta x_f \\
&+ \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F}\sum_{i=1}^{I} \Delta tf_{i,f}.\Delta X_{i,f} \\
&+ \sum_{i=1}^{I} ty_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta ty_i.\Delta Y_i \\
&+ \sum_{i=1}^{I} tc_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta tc_i.\Delta C_i \\
&+ \sum_{i=1}^{I} tm_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta tm_i.\Delta M_i \\
&- \sum_{i=1}^{I} M_i.\Delta PW_i - 0.5.\sum_{i=1}^{I} \Delta M_i.\Delta PW_i \\
&- 0.25.\sum_{i=1}^{I} \frac{\Delta P_i.\Delta P_i.\Delta Y_i}{P_i} \\
&- \sum_{l\neq h} EV_l \\
&+ 0.5.\begin{pmatrix} \Delta\lambda_h.\left(\sum_{i=1}^{I} Q_i.\Delta C_{i,h} + \sum_{i=1}^{I} \Delta Q_i.\Delta C_{i,h}\right) \\ -\sum_{l\neq h}\Delta\lambda_l.\left(\sum_{i=1}^{I} Q_i.\Delta C_{i,l} + \sum_{i=1}^{I} \Delta Q_i.\Delta C_{i,l}\right) \end{pmatrix} \\
&+ 0.5.\left(\frac{\partial^2 E_h\left(P^0, U_h^0\right)}{\partial U_h^2}.(\Delta U_h)^2 - \sum_{l\neq h} \frac{\partial^2 E_l\left(P^0, U_l^0\right)}{\partial U_l^2}.(\Delta U_l)^2 \right) \\
&+ O(\Delta U, \Delta C, \Delta P, \Delta X)
\end{aligned}
\qquad (35')
$$

The welfare decomposition for a given household is still negatively dependent of the welfare for other household, this time in a more direct way.

Finally it is interesting to remark that both decompositions can not attribute the distortionary effects to any particular household. This is unsurprising because these effects depend on market equilibrium which in turn depend on the behavior of all agents.